Signal Detection by Generalized Detector in Compound-Gaussian Noise

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Abstract: - This paper handles the problem of detecting signals with known form and structure and unknown or random amplitude and phase in the presence of compound-Gaussian noise with known spectral density in remote sensing systems. The generalized detector (GA) based on the generalized approach to signal processing (GASP) in noise is investigated in [1]–[5]. The structure of the GA detector is independent of the disturbance amplitude probability density. Based on this result, the threshold setting, which is itself independent on both the noise distribution and the signal parameters, ensures a constant false alarm rate (CFAR). The detection performance analysis shows that the GA detector outperforms the optimum Neyman-Pearson (NP) receiver and the conventional square-law detector.

Key-Words: - Generalized detector, Constant false alarm rate, Detection performance, Gaussian noise, Radar.

1 Introduction

This paper deals with a detection of signals with known form and unknown amplitude and phase embedded in non-Gaussian noise that is characteristic of any remote sensing system. Non-Gaussian processes play a central role in statistical signal processing because of the often-dominant effects of this type of noise and interference in several situations like clutter, man-made, and natural environment noise and underwater acoustics, electromagnetic, remote sensing and acoustic scattering. Unlike the Gaussian disturbance, such a noise is impulsive in that it is characterized by significant probabilities of large interference levels. Among the marginal probability distribution densities that have been found appropriate for modeling impulsive noise, we quote the generalized Gaussian, the generalized Cauchy, and the contaminated normal, which in turn subsumes Middleton Class-A univariate probability distribution density [6]. In radar and sonar applications, for instance, in remote sensing systems, the amplitude probability distribution density is commonly used, with particular attention to the Weibull and *K*-distributions [7]–[9]. In the course of the theoretical investigations and experimental modeling, we consider the non-Gaussian noise as a compound-Gaussian process in the form of product between a Gaussian, possibly complex, process and a non-negative stochastic process [10]. Experimental study of high-resolution radar systems in remote sensing shows that clutter returns within the limits of time intervals of the order of the usual processing time consist of a Gaussian signal modulated by a slowly varying non-negative stochastic process. The modulating signal can be thought constant but random within the limits of observation intervals. Similar model was used in [11],[12] for the atmospheric noise. From a theoretical viewpoint, the compound-Gaussian process arises from the well-known random walk model where the noise is given by the sum of a large number of contributions. If the number of such contributions is modeled as a random variable and its mean value is taken arbitrarily large, a generalized form of the central limit theorem ensures the conver-gence of the sum to a compound-Gaussian variable, [13] and [14]. Finally, in the radar detection scenario, the compound-Gaussian model is the most widely accepted and experimental verified. We study the performance of the GA detector when operating in the compound-Gaussian noise. Under analysis, we use the approach discussed in [15]. In synthesizing the GA detector, we use the completely received waveform.

2 Problem Statement

The detection problem consists of testing the hypothesis H_0, i.e., a "no" signal, versus the alternative H_1, i.e., a "yes" signal. Specifically, under the use of the GASP in noise we can write [2]–[5]:

$$H_0 : \begin{cases} x(t) = \xi(t); \\ y(t) = \eta(t); \end{cases} \quad t \in [0, T] \tag{1}$$

$$H_1 : \begin{cases} x(t) = \alpha\psi_1(t) + \xi(t); \\ y(t) = \eta(t); \end{cases} \quad t \in [0, T], \tag{2}$$

where $[0, T]$ is the observation interval; $x(t)$ is the baseband equivalent of received waveform; $\xi(t)$ is the baseband equivalent of the noise; $\eta(t)$ is the additional (reference) and uncorrelated with the noise $\xi(t)$ baseband equivalent of the noise, too, having the same statistical parameters as the noise $\xi(t)$, since the noise $\xi(t)$ and $\eta(t)$ are obtained at the input of the GA detector from common noise $n(t)$, in a general case, the statistical parameters of the noise $\xi(t)$ and $\eta(t)$ are differed, how we can do this is discussed in greater detail in [2], [3], and [5]; $\psi_1(t)$ is the signature of the transmitted pulse; $\alpha = A\exp(j\theta)$ is the complex (nuisance) parameter accounting for propagation and scattering effects; θ is the phase of the transmitted signal. *It is known a priori that a "no" signal is obtained in the additional (reference) noise* $\eta(t)$. The parameter α is modeled as an unknown parameter if no *a priori* information on its statistics is available; otherwise, it is assumed to be a complex random variable with known distribution law. As to the noise $n(t)$ (consequently, the noise $\xi(t)$ and $\eta(t)$, it is modeled as a sample function from a compound-Gaussian process with known autocorrelation function; otherwise, the noise $n(t)$ can be written in the form $n(t) = \mu(t) \times g(t)$, consequently $\xi(t) = \mu(t) \times g_1(t)$ and $\eta(t) = \mu(t) \times g_2(t)$, in general case, where $\mu(t)$ and $g(t)$ are independent random processes representing the modulating and Gaussian components, respectively. Within the limits of the observation interval, the process $\mu(t)$ can be thought of, for all practical purposes, as a random constant so that the overall disturbance process degenerates into a Gaussian process with random mean square value, i.e.,

$$n(t) = \mu g(t)\,;\ \xi(t) = \mu g_1(t)\,;\ \eta(t) = \mu g_2(t)\,; \qquad (3)$$

where $t \in [0, T]$. We assume, with no loss of generality for situations of practical interest, that the random variable μ has unit root mean square value and that $g(t)$ is the base-band equivalent of a wide-sense stationary bandpass random process (naturally, this statement is true for $g_1(t)$ and $g_2(t)$). We note that the processes (3) are closed with respect to affine transformations. In particular, linearly filtering such a process yields again a Gaussian process with random root mean square value proportional to μ but with different correlation function. Finally, we note that the model (3) encompasses Gaussian processes as the special case of nonrandom root mean square value.

3 Generalized Detector

At first, we assume that α is known. This hypothesis is not realistic, especially for the radar or sonar problem in remote sensing systems, but deserves some attention for several reasons. Firstly, if a uniformly most powerful test exists, then it does not depend on the parameter, and the GA detector is still optimum for α unknown. Secondly, if the uniformly most powerful tests exist, the performance of the most powerful test is an upper bound on how well any test could does. Thus, the performance of the GA detector for a perfectly known signal will be used to assess the detection loss of any other receiver The likelihood ratio for known α is the starting point for studying the cases of both unknown and random α. For the derivation of the GA detector, we resort to the usual approach based on projecting the received vector along the first M functions of an orthonormal basis and then letting $M \to \infty$. Having chosen $F = \{\psi_i(t)\}_{i=1}^{\infty}$ whose first component is the signature of transmitted pulse, the coefficients of the expansion of the received waveform according to the GASP [1]–[5] are,

$$H_1: \begin{cases} x(i) = \langle x(t), \psi_i(t) \rangle = \begin{cases} \alpha + \xi(1), & i = 1; \\ \xi(i), & i > 1; \end{cases} \\ y(i) = \langle y(t), \psi_i(t) \rangle = \eta(i); \end{cases} \qquad (4)$$

$$H_0: \begin{cases} x(i) = \langle x(t), \psi_i(t) \rangle = \xi(i); \\ y(i) = \langle y(t), \psi_i(t) \rangle = \eta(i), \end{cases} \qquad (5)$$

where $\langle \cdot, \cdot \rangle$ denotes the dot product. Since the noise $n(t)$ is white, the coefficients $\xi(i)$ and $\eta(i)$ given μ form sequences of conditionally Gaussian, uncorrelated variables having zero-mean and conditional variances $\frac{N_0}{2}\mu^2$ both for $\xi(i)$ and $\eta(i)$ [5], where $\frac{N_0}{2}$ is the power spectral density of $g(t)$, (as well as of $n(t)$ due to the unit root mean square value of μ) independently of the basis. Indeed, we used such degrees of freedom in choosing, as a basis, a set of complex functions whose first term is the signal signature $\psi_1(t)$. Therefore, the likelihood ratio can be evaluated as

$$\Lambda[x(t) \mid \alpha] = \lim_{M \to \infty} \Lambda_M[x_M(t) \mid \alpha] = \lim_{M \to \infty} \frac{f_{\mathbf{X}_M \mid H_{1,\alpha}}(\mathbf{X}_M)}{f_{\mathbf{Y}_M \mid H_0}(\mathbf{Y}_M)},$$

where $\mathbf{X}_M$ and $\mathbf{Y}_M$ are the vectors containing the first M coefficients of the received waveform on the subspace spanned by the first M functions of the basis, $f_{\mathbf{X}_M \mid H_{1,\alpha}}(\mathbf{X}_M)$ and $f_{\mathbf{Y}_M \mid H_0}(\mathbf{Y}_M)$ are the likelihood functions for H_0 and H_1, respectively. Due to the compound-Gaussian nature of the noise (3) and (4), (5)

$$\Lambda_M[x_M(t) \mid \alpha] = \frac{\int_0^{\infty} \mu^{-2M} \exp\left\{-\frac{2}{N_0\mu^2}\left[\,|x(1)-\alpha|^2 + \sum_{i=2}^{M} |\xi(i)|^2\right]\right\} f(\mu)\,d\mu}{\int_0^{\infty} \mu^{-2M} \exp\left[-\frac{2}{N_0\mu^2} \sum_{i=2}^{M} |\eta(i)|^2\right] f(\mu)\,d\mu} \qquad (6)$$

where $f(\mu)$ is the probability distribution density of the random variable μ. Now, letting in the statistical sense

$$\begin{cases} Z_{M-1} = \frac{1}{M-1}\sum_{i=2}^{M} |\xi(i)|^2 = \frac{1}{M-1}\sum_{i=2}^{M} |\eta(i)|^2 \, ; \\ z = \frac{2Z_{M-1}}{N_0 \mu^2} \end{cases} \qquad (7)$$

and substituting in both integrals of (6) yields

$$\Lambda_M[x_M(t)\,|\,\alpha] = \frac{\int_0^\infty z^{M-1.5} e^{-(M-1)z} \exp\left[-\frac{|x(1)-\alpha|^2 z}{Z_{M-1}}\right] f\left(\sqrt{\frac{Z_{M-1}}{N_0 z}}\right) dz}{\int_0^\infty z^{M-1.5} e^{-(M-1)z} \exp\left[-\frac{|\eta(1)|^2 z}{Z_{M-1}}\right] f\left(\sqrt{\frac{Z_{M-1}}{N_0 z}}\right) dz} .$$

The likelihood ratio is now obtained by evaluating the limit for $M \to \infty$. We use the result

$$\lim_{M\to\infty} \frac{M^M}{\Gamma(M)} z^{M-1} e^{-Mz} u(z) = \lim_{M\to\infty} f_M(z) = \delta(z-1) , \quad (8)$$

where $u(\cdot)$ denotes the unit step function; $f_M(z)$ is the probability distribution density of a Gamma variable with the mean equal to 1 and the variance equal to $\frac{1}{M}$ As M diverges, such a variable converges in mean square sense to its statistical average, implying the convergence of $f_M(z)$ to the delta function $\delta(z-1)$. Thus, the likelihood ratio for known signal is

$$\Lambda[x(t)\,|\,\alpha] = \exp\left\{-\frac{\mathrm{Re}[-2<x(t),\psi_1(t)>\alpha^* + <x(t),\psi_1(t)><x(t),\psi_1(t)>]}{Z_\infty} + \frac{\mathrm{Re}[<y(t),\psi_1(t)><y(t),\psi_1(t)>]}{Z_\infty}\right\} , \quad (9)$$

wherein Z_∞ is the limit of the sample mean square value of difference of the sequences $\{y(i)\}_{i=2}^{\infty}$ and $\{x(i)\}_{i=2}^{\infty}$, namely

$$Z_\infty = \lim_{M\to\infty} \frac{1}{M-1}\sum_{i=2}^{M} |\eta(i)|^2 - \lim_{M\to\infty} \frac{1}{M-1}\sum_{i=2}^{M} |\xi(i)|^2 \quad (10)$$

with mean square convergence. Note that $\{x(i)\}_{i=2}^{\infty}$ and $\{y(i)\}_{i=2}^{\infty}$ are the uncorrelated coefficients of the projections $x_\perp(t)$ and $y_\perp(t)$ of the received waveform orthogonal to the space spanned by the useful signal. It can be shown that

$$\lim_{M\to\infty} \frac{1}{M-1}\sum_{i=2}^{M} |\xi(i)|^2 = \lim_{M\to\infty} \frac{1}{M-1}\sum_{i=2}^{M} |\eta(i)|^2 = \frac{N_0}{2}\mu^2 \, . \quad (11)$$

Thus, Z_∞ is the error-free measurement of difference of random power spectral densities of the compound-Gaussian noise $\xi(i)$ and $\eta(i)$ based on the received waveform. In conclusion, the likelihood ratio test for completely known signal is

$$\frac{\mathrm{Re}[2<x(t),\psi_1(t)>e^{-j\theta} - <x(t),\psi_1(t)><x(t),\psi_1(t)> + <y(t),\psi_1(t)><y(t),\psi_1(t)>]}{Z_\infty} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_G + 0.5|\alpha| \, , \quad (12)$$

where γ_G is the threshold to be set according to the desired probability of false alarm P_{FA}. The detector based on the test (11) will be called as the generalized (GA) detector. Evidently, the test (11) requires averaging infinitely many coefficients. The performance of the GA detector is an upper bound for any other receiver operating under the same signal model and in the presence of a compound-Gaussian noise.

3.1 Generalized Likelihood Ratio Test

When no the uniformly most powerful test exists, a suitable procedure to deal with the case of unknown parameters is to estimate them under both hypotheses and then use these estimates in the likelihood ratio; if we resort to the maximum likelihood estimates, the corresponding tests is the so-called GLRT [16]. Note that the likelihood ratio (11) can be written as

$$\Lambda[x(t)\,|\,\alpha] = \exp\left[-\frac{\|x_1(t)-\alpha\psi_1(t)\|^2}{Z_\infty} + \frac{\|y_1(t)\|^2}{Z_\infty}\right] , \quad (13)$$

where $\|\cdot\|$ is the norm, $x_1(t)$ and $y_1(t)$ are the projections of the received waveform along $\psi_1(t)$. Then, as in the case at hand, the only hypothesis H_1 is composite, and the GLRT is obtained by maximizing the likelihood ratio (13) with respect to α. In other words, the GLRT for the GASP is the tests

$$\max_\alpha \exp\left[-\frac{\|x_1(t)-\alpha\psi_1(t)\|^2}{Z_\infty} + \frac{\|y_1(t)\|^2}{Z_\infty}\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_G \, . \quad (14)$$

In (14), and throughout the paper γ_G has the same significance as in (12), namely, the threshold to be set according to the desired P_{FA}. Naturally, its numerical value varies since the test statistic is different. Maximizing the functional on the left-hand side of (14) is equivalent to minimizing the square norm at the exponent of the first factor, which is easily seen to vanish for

$$\alpha = \hat{\alpha} = <x(t), \psi_1(t)> \quad (15)$$

so that the GLRT, after simple manipulations, takes a form

$$\frac{2\|x(t),\psi_1(t)\|^2 - <x(t),\psi_1(t)><x(t),\psi_1(t)> + <y(t),\psi_1(t)><y(t),\psi_1(t)>]}{Z_\infty} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_G \quad (16)$$

The detector implementing the test (16) will be referred to as the GA GLRT detector

3.2 Signal with Random Amplitude

Consider the case of random α. Assume that the phase θ is uniformly distributed within the limits of the interval $[0, 2\pi]$ and the amplitude A has a chi distribution with $2m$ degrees of freedom, namely

$$f_A(z) = 2(\tfrac{m}{\overline{A^2}})^m \tfrac{z^{2m-1}}{\Gamma(m)} \exp\left[-\tfrac{m}{\overline{A^2}} z^2\right] u(z),\ m > 0. \quad (17)$$

The parameter m rules the depth of the amplitude fluctuation, i.e., the lower the shape parameter m is, the wider the fluctuation spans. Special cases are $m = 1$ and $m \to \infty$. The first case corresponds to the Rayleigh-distributed amplitude and the second case corresponds to the nonrandom amplitude. Averaging out the phase in (9) yields the likelihood ratio for the case of signal with random phase and known amplitude

$$\Lambda[x(t) \mid A] = \exp(-\tfrac{A^2}{Z_\infty}) I_0\left[\tfrac{2|A<x(t),\psi_1(t)>|^2}{Z_\infty} - \tfrac{<x(t),\psi_1(t)><x(t),\psi_1(t)> - <y(t),\psi_1(t)><y(t),\psi_1(t)>}{Z_\infty}\right] \quad (18)$$

where $I_0(\cdot)$ is the modified Bessel function of first kind and order zero. The GA detector for random energy is obtained by averaging out the amplitude in (18), which yields

$$\Lambda[x(t)] = \frac{m^m}{\left(m + \frac{\overline{A^2}}{Z_\infty}\right)^m} M\left\{m, 1, \frac{\overline{A^2}}{Z_\infty} \times \left[\frac{2|<x(t),\psi_1(t)>|^2}{\left(m+\frac{\overline{A^2}}{Z_\infty}\right)Z_\infty} - \frac{<x(t),\psi_1(t)><x(t),\psi_1(t)> - <y(t),\psi_1(t)><y(t),\psi_1(t)>}{\left(m+\frac{\overline{A^2}}{Z_\infty}\right)Z_\infty}\right]\right\} \quad (19)$$

where $M(a,b,z)$ is the confluent hypergeometric series [17]. Note that only for $m = 1$, the likelihood reduces to an elementary form. In fact, as $M(1,1,x^2) = e^{x^2}$ [17], (21) simplifies to

$$\Lambda[x(t)] = \frac{1}{1+\frac{\overline{A^2}}{Z_\infty}} \exp\left\{\frac{\frac{\overline{A^2}}{Z_\infty}}{1+\frac{\overline{A^2}}{Z_\infty}} \times \left[\frac{2|<x(t),\psi_1(t)>|^2}{Z_\infty} - \frac{<x(t),\psi_1(t)><x(t),\psi_1(t)> - <y(t),\psi_1(t)><y(t),\psi_1(t)>}{Z_\infty}\right]\right\} \quad (20)$$

and the generalized detector admits the sufficient statistic

$$\frac{\frac{\overline{A^2}}{Z_\infty}}{1+\frac{\overline{A^2}}{Z_\infty}} \cdot \frac{2|<x(t),\psi_1(t)>|^2 - <x(t),\psi_1(t)><x(t),\psi_1(t)> + <y(t),\psi_1(t)><y(t),\psi_1(t)>}{Z_\infty} - \ln\left[1 + \frac{\overline{A^2}}{Z_\infty}\right] \quad (21)$$

The sufficient statistic (21) is the same as for the GA GLRT detector but for a weighting factor and a bias term it cannot be further simplified since both the scale factor and the bias term in (21) are themselves data-dependent quantities (the presence Z_∞). Similar considerations apply also to the general case of (19).

3.3 Discussion

The GA detectors derived in this section deserve several comments. Note, that in all of the analyzed cases, the likelihood ratios are very similar to those derived in Gaussian noise environment [1]–[5] in that expressions are the same but for the presence of Z_∞ instead of the mean square value of the noise coefficients along this basis. More rigorously, in the compound-Gaussian environment, Z_∞ is the conditional, instead of the unconditional, common mean square of difference of the noise coefficients. Moreover, unlike the case of white Gaussian noise, it cannot be absorbed into the threshold, as it is data dependent. In fact, it is evaluated as the limit of the sample mean square of difference of the coefficients of the received waveform components $x_\perp(t)$ and $y_\perp(t)$ orthogonal to the space spanned by the useful signal. Although $x_\perp(t)$ and $y_\perp(t)$ are irrelevant in the case of white Gaussian noise with known power spectral density, when the disturbance is compound-Gaussian, $x_\perp(t)$ and $y_\perp(t)$ cannot only no longer be discarded, but they are used by the GA detector for measuring the random level $\frac{N_0}{2}\mu^2$ of the power spectral density of noise according to (11). The GA detector in the case of a perfectly known signal, as well as in the case of random phase and/or amplitude, requires averaging infinitely many coefficients of the observed waveform. On the contrary, the GA GLRT detector grants *a priori* a receiver independent of the unknown signal parameters. In addition, the GA GLRT detector turns out to be canonical in that it admits one, and the same sufficient statistic, whatever the compound-Gaussian clutter amplitude probability distribution density is. Precisely, it admits the same test variable of the GA detector but for the normalization to Z_∞ [2]. Remarkably, such a normalization factor ensures the constant false alarm rate (CFAR) with respect to the clutter amplitude probability distribution density and, in particular, to its power. In fact, under the hypothesis H_0 the random quantity μ^2 factors out from the ratio on the left-side of (16). Therefore, the detection threshold depends on the required P_{FA}, but it is otherwise independent of the non-Gaussian noise distribution. Finally, the distribution of the test statistic is independent of the signal phase θ, either if it is modeled as an unknown parameter or as a random variable. Consistent estimate can be obtained as the sample mean square value of the coefficients $x_\perp(t)$ and $y_\perp(t)$ A possible criticism against the above GA GLRT detector is that it is not directly implementable, as it shares with the GA detector the requirement of the perfect measurement of the noise random power spectral density level. This is not really a drawback, since the GA GLRT detector can be closely approximated by averaging only a finite number $M-1$ of coefficients. The resulting estimate of the noise random power spectral density level is consistent but no longer has zero mean square error for finite M. Such an approach amounts to implementing the test

$$\frac{2|<x(t),\psi_1(t)>|^2-<x(t),\psi_1(t)><x(t),\psi_1(t)>+<y(t),\psi_1(t)><y(t),\psi_1(t)>}{\frac{1}{M-1}\sum_{i=2}^{M}|\eta(i)|^2-\frac{1}{M-1}\sum_{i=2}^{M}|\xi(i)|^2}\underset{H_0}{\overset{H_1}{\gtrless}}\gamma_G \quad (22)$$

which converges to the GA GLRT detector as M diverges.The receiver implementing the test (21), whose diagram is shown in Fig.1, will be referred as the M -th order approximation of the GA GLRT (M-GA GLRT) detector. The preliminary (PF) and additi-

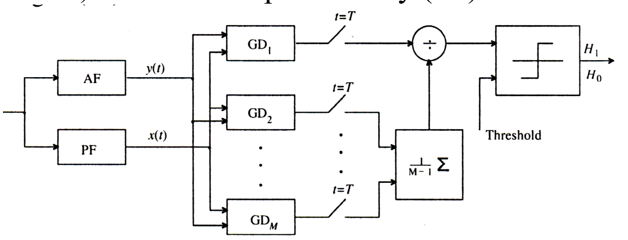

Figure 1. ***M*-GA GLRT detector.**

onal (AF) filters having detuned by central frequency characteristics are discussed in more detail in [2].

4 Performance Assessment

Note that the probability of detection P_D and the probability of false alarm P_{FA} for the GA detectors are discussed in more detail in [2] and [3]. Here we present the performance evaluated by Monte Carlo simulations based on $1000/P_D$ and $1000/P_{FA}$ independent trials for valuating P_D and P_{FA}, respectively. First, consider the case of nonrandom signal parameters. There is a need to require specifying the marginal probability distribution density of the compound-Gaussian noise. We assume a *K*-distribution that is among the most credited distributions for radar applications in remote sensing systems, but the results can be extended to other probability distribution densities in a straightforward manner. In the case at hand, the modulating variable μ has the chi distribution [15]

$$f_\mu(z)=\frac{2\nu^\nu}{\Gamma(\nu)}z^{2\nu-1}e^{-\nu z^2}, z\ge 0 \quad (23)$$

where $\Gamma(\cdot)$ is the Eulerian Gamma function, and ν is a share parameter, whereas the corresponding noise amplitude probability distribution density belongs to the *K* family

$$f_{|\xi|}(z)=f_{|\eta|}(z)=\frac{a^{\nu+1}}{2^{\nu-1}\Gamma(\nu)}K_{\nu-1}(az), z\ge 0; a,\nu>0, \quad (24)$$

where $K_\nu(\cdot)$ is the modified second-kind Bessel function, and α is a scale parameter. The performances of the GA (12), GA GLRT (16), and *M*-GA GLRT (22) detectors for some values of M, $P_{FA}=10^{-4}$, and $\nu=5$ are compared, in Fig.2, with the performances of the Neyman-Pearson (NP) and square-law detectors. The

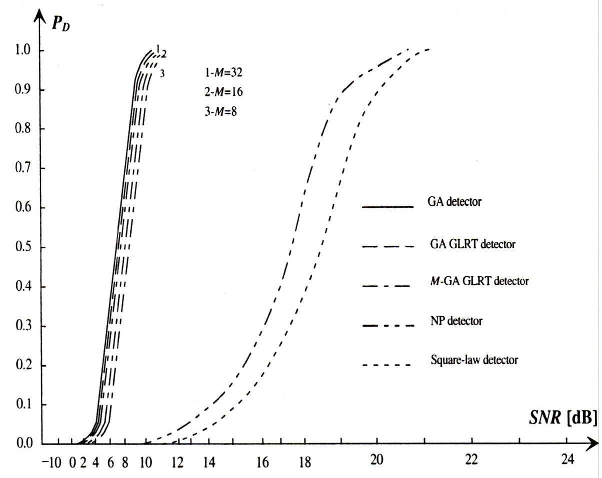

Figure 2. Performance for $P_{FA}=10^{-4}$ and $\nu=5$.

comparison shows that the GA, GA GLRT, and *M*-GA GLRT receivers largely outperform the NP and square-law detectors independently of ν. The loss of the *M*-GA GLRT detector with respect to the GA and GA GLRT receivers is defined as the increment signal-to-noise ratio (SNR) necessary for achieving the same detection performance. These losses can be read off from Fig.2 as the horizontal shift between the curves corresponding to the different receivers. Analysis of the curves shows that the loss of the GA GLRT detector with respect to the GA one is at most about 1 dB for different values of P_D and ν ,and it becomes less than 1 dB as $\nu\to\infty$. As to the loss of the *M*-GA GLRT detector with respect to the GA GLRT one, it is nearly independent of the noise shape parameter for a given *M*. For instance, the loss of the *8*-GA GLRT detector is about 1 dB, as can be read off from curves of Fig.2. Numerical computation not reported here show that the loss is practically independent of the particular value of P_{FA} as well. The effect is essentially a shift of all curves toward higher values of SNR for decreasing values of P_{FA}. Consider now the case of random α. Note that in the case of the GA, GA GLRT, and *M*-GA GLRT detectors, the P_{FA} is unaffected by the amplitude and phase joint probability distribution density since all of them are CFAR. Moreover, should only the phase be random, then the P_D does not change either; in fact, the probability distribution densities of the test statistics (12), (16), and (22) are independent of the signal phase. Therefore, the curves of Fig.2 are still valid. In this case, however, the performance of the NP detector for known signal is not longer the approximate term of comparison. Figure 3 shows the performances of the GA, GA GLRT, and *M*-GA GLRT detectors in the case of the useful signal with random parameter for $P_{FA} = 10^{-4}$, and $\nu = 5$. Figure 3 refers to $m \to \infty$, namely, the case where only the phase is a random variable with uniform probability distribution density. Figure 3 presents the case where $m = 1$, namely, uniform phase and Rayleigh amplitude. Curves for more constrained fluctuation lie between these two cases. In the same figure, for comparison purposes, the performances of the NP and square-law detectors are also reported. The performances of the GA, GA GLRT, and *M*-GA GLRT receivers are not affected by the phase distribution. In the case of non-fluctuated amplitude (Fig. 3), the losses of the GA GLRT and *M*-GA GLRT detector in comparison with the GA receiver are the same as in Fig.2.

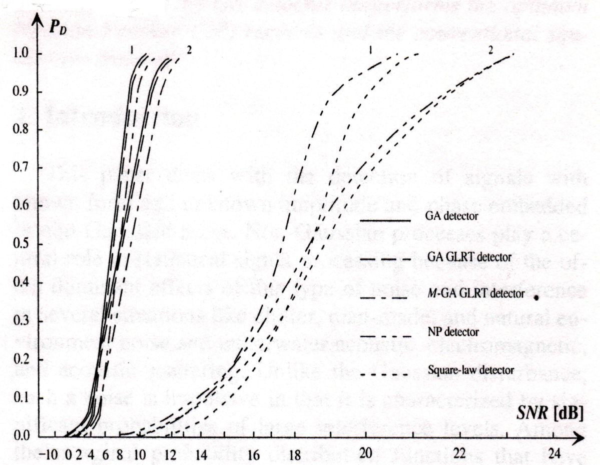

Figure 3. Performance for $P_{FA} = 10^{-4}$ and $\nu = 5$: $M = 16$; $1 - m \to \infty$; $2 - m = 1$.

In the case of uniformly distributed phase, the loss is about 0.5 dB and, hence, much lower than in the case of signal with unknown nonrandom parameters. Moreover, the larger ν is, the smaller the loss is because the complete ignorance of the phase information results in poorer performance. In addition, should the amplitude be Rayleigh distributed (Fig.3), the performances of the GA and GA GLRT detectors would be hardly distinguished. The influence of the shape parameter is analyzed in Fig.3, where the performances of the GA, GA GLRT, *M*-GA GLRT, NP, and square -law detectors are reported for $\nu = 5$ and still for the limit cases $m \to \infty$ and $m = 1$. In the absence of amplitude fluctuation, the loss of the GA GLRT with respect to the GA detector decreases with ν and as $\nu \to \infty$ two curves coincide; as in the presence of Gaussian noise, GA and GA GLRT detectors are equivalent [2] and [3]. Figure 3 also shows that if the amplitude fluctuates according to the Rayleigh model, the curves of the GA and GA GLRT receivers can hardly be distinguished for all values of ν. In other words, at least for moderately high P_D, the performance is ruled by the Rayleigh fluctuation law instead of by the noise probability distribution density. The performance analysis highlights that the loss of the *M*-GA GLRT detector with respect to the GLRT detector is essentially independent of the disturbance shape parameter, even in the case of Rayleigh fluctuating amplitude. In Fig. 3, the performances of the square-law receiver are also reported. In addition, in the presence of random parameters, the GA, GA GLRT, and *M*-GA GLRT detectors largely outperform the NP and square-law receivers. The gain is more relevant for the case of nonrandom amplitude and for smaller ν. Indeed, the performances are ruled by the Rayleigh fluctuation law; the larger ν becomes, the lower the non-Gaussian nature of the noise will be.

5 Conclusions

The use of the GASP in the compound-Gaussian noise allows us to design detectors largely outperforming the NP detector for known signal and the square-law one. The threshold setting does not require knowledge of the SNR but there is a need to average infinitely many noise coefficients. The *M*-GA GLRT detector has a completely canonical structure, which consists of the GA detector plus a parallel branch aimed at computing a difference of the common conditional mean square values of the noise coefficients. Interestingly, not only does this processor no longer require knowledge of the signal parameters but also it achieves CFAR with respect to the noise amplitude probability distribution density so that the detection threshold can be set once and for all based only on the systems parameters.

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