Command laws for rockets' motion stabilisation

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Abstract: - This paper deals with models of the rockets' motion around mass centre, command laws of vertical and horizontal planes trajectory, stabilisation laws of the roll motion of the rockets and target control laws. One also studies the way to choose of the parameters of general command laws which assures state variables simultaneous cancellation. Some automatic control systems, using the previous laws, are also presented.

Key-Words: rocket, command law, target, control.

1 Models of the rocket's motion around mass centre in vertical and horizontal planes

In fig.1 one presents the rocket's motion (A) around mass centre in vertical plane block diagram, with transfer operators, where: θ - the trajectory slope in vertical plane, α - attack angle, V - flight velocity, δ - the ribbed deflection in horizontal plane (vertical plane command), T_v - the time constant of A [1],

$$k_\delta = \frac{m_\delta}{J_z}, 2\xi\omega_0 = \frac{m_{\dot{\vartheta}}}{J_z} + \frac{V}{T_v},$$
$$\omega_0^2 = \frac{m_\alpha}{J_z} + \frac{m_{\dot{\vartheta}}}{J_z T_v} + \frac{g\sin\theta}{VT_v} + \frac{\dot{T}_v}{T_v^2}, \tag{1}$$

where m_δ - command coefficient, J_z - inertial moment with respect to lateral axis, $m_{\dot{\vartheta}}$ - dynamic damp couple coefficient, m_α - static stabilisation coefficient, ξ - damp coefficient, ω_0 - proper pulsation.

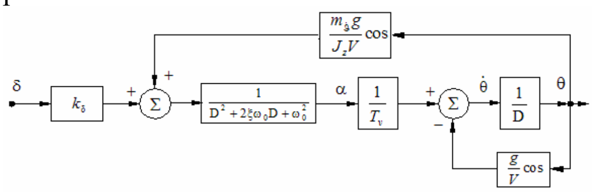

Fig.1

Similar to the block diagram in fig.1, one can obtain the block diagram, with transfer operators, in fig.2. This block diagram corresponds to the plane's horizontal motion; $\delta \to \delta', \alpha \to \alpha', \theta \to \theta'$; the two feed-back links are missing here, because of the missing of the proportional terms with respect to gravitational acceleration (g) miss. Thus, transfer operator for the horizontal plane motion is

$$H_\theta(\mathrm{D}) = \left(\frac{\theta'}{\delta'}\right)(\mathrm{D}) = \frac{k_\delta}{T_v \mathrm{D}\left(\mathrm{D}^2 + 2\xi\omega_0\mathrm{D} + \omega_0^2\right)}. \tag{2}$$

2 Automatic command of the flight trajectory

In fig.2 one presents the block diagram, with transfer operators, of the automatic control system of the trajectory slope (θ and θ'), where $\overline{\theta}$ and $\overline{\theta'}$ expresses the desired slope angles for the two planes (vertical and horizontal); E.E.1 and E.E.2 are actuators (execution elements) for the commands δ and δ', U_1 and U_2 - command variables.

$$U_1 = \ddot{\theta} - k\frac{\dot{\theta}^2}{\Delta\theta}, \tag{3}$$

$$U_2 = \dot{\theta}' - ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}, \tag{4}$$

where $\Delta\theta = \overline{\theta} - \theta$, $\Delta\theta' = \overline{\theta'} - \theta'$ and a, k - coefficients whose variation domains are presented below. For a steady state regime

$$\ddot{\theta} = k\frac{\dot{\theta}^2}{\Delta\theta}, \tag{5}$$

$$\dot{\theta}' = ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}, \tag{6}$$

which express command laws of the trajectory for two planes (vertical and horizontal). For an appropriate choosing of the parameter k, the variables $\Delta\theta, \dot{\theta}$ and $\ddot{\theta}\left(\theta \to \overline{\theta}, \dot{\theta} \to 0, \ddot{\theta} \to 0\right)$ becomes simultaneously null. Simultaneously with $\Delta\theta$'s cancellation, $\Delta\theta'$ and $\ddot{\theta}\left(\theta' \to \overline{\theta'}, \dot{\theta}' \to 0\right)$ becomes null. All these statements will be demonstrated below for a general case.

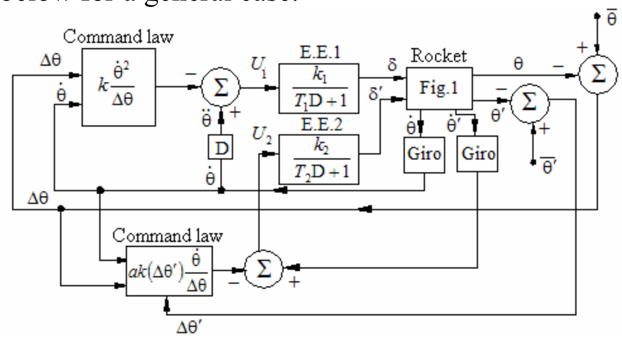

Fig.2

3 Rocket's roll motion stabilisation

For an appropriate precision of the automatic control process, rockets with symmetrical distribution of the empennage and wings have stabilisation and maintaining φ to the zero value systems. Under disturbances action, the rocket may rotate around its longitudinal axis. Thus, the vertical and horizontal channel may interfere, the control errors increases [1], [2]. The systems in fig.3 and in fig.4 assures simultaneous cancellation of the variables $\varphi, \dot{\varphi}$ and $\ddot{\varphi}$. The command law for the system in fig.3 is the following one

$$\ddot{\varphi} = k\frac{\dot{\varphi}^2}{\varphi}. \tag{7}$$

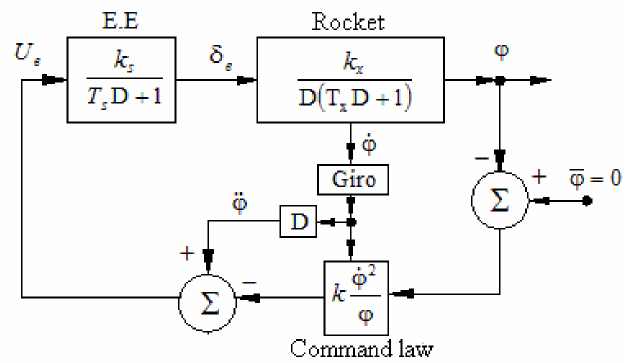

Fig.3

Command law for the system in fig.4 is equivalent to the one given by Eq. (7); a supplementary phase coordinate z_0 is introduced [1].

Thus, if $T_x \to T_M$ (T_x - time constant of the rocket's motion around longitudinal axis, while T_M - model time constant), then the variable becomes

$$z_1 = \frac{1}{T_M D + 1}U_e = \frac{1}{T_M D + 1}\frac{T_x D + 1}{k_x k_s}\dot{\varphi} \cong \frac{1}{k_c}\dot{\varphi}, \tag{8}$$

where $k_c = k_x k_s$ and

$$z_0 = \frac{1}{D}\left(U_e - z_1\right) = \frac{1}{D}U_e - \frac{1}{k_c}\varphi \tag{9}$$

or

$$\dot{z}_0 = U_e - \frac{1}{k_c}\dot{\varphi}. \tag{10}$$

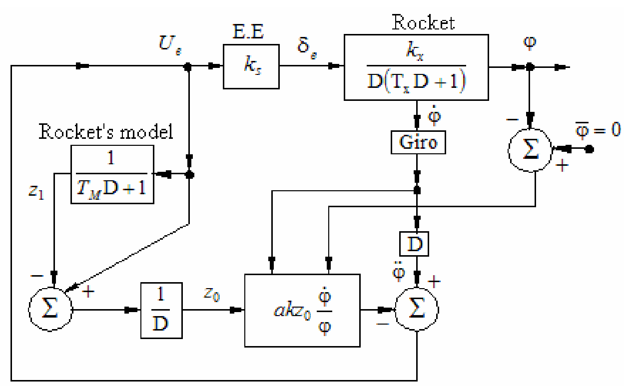

Fig.4

From the equation that expresses the rocket's motion around its longitudinal axis

$$T_x\ddot{\varphi} + \dot{\varphi} = k_c U_e \tag{11}$$

it results

$$\frac{T_x}{k_c}\ddot{\varphi} = U_e - \frac{1}{k_c}\dot{\varphi}. \qquad (12)$$

Comparing equations (10) and (12), it results

$$\dot{\varphi} = \frac{k_c}{T_x} z_0 = a z_0, a = \frac{k_c}{T_x}. \qquad (13)$$

Thus, command law (7) becomes

$$\ddot{\varphi} = a k z_0 \frac{\dot{\varphi}}{\varphi}. \qquad (14)$$

The variables $\varphi, \dot{\varphi}, \ddot{\varphi}$ and z_0 simultaneously tend to zero.

4 Rocket's flight trajectory command and roll motion stabilisation

Such a control system is the one in fig.5. Command laws are

$$\ddot{\theta} = k\frac{\dot{\theta}^2}{\Delta\theta}, \qquad (15)$$

$$\dot{\theta}' = ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}, \qquad (16)$$

$$\dot{\varphi} = bk\varphi\frac{\dot{\theta}}{\Delta\theta}. \qquad (17)$$

The angular variables $\varphi, \dot{\varphi}, \dot{\theta}, \dot{\theta}'$ simultaneously tend to zero together with $\Delta\theta$ and $\Delta\theta'\left(\theta \to \overline{\theta}, \theta' \to \overline{\theta}'\right)$.

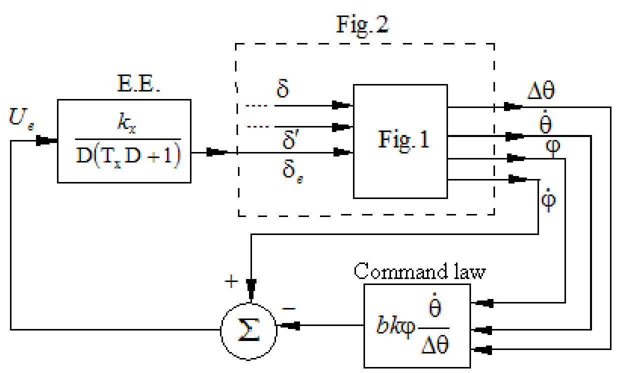

Fig.5

5 Rocket's flight trajectory command and target control

The structure of such a system is presented in fig.6, where δ_m is the engine input command signal, r - the distance from A to T (target).

Command laws are (5), (6) and

$$\dot{r} = ckr\frac{\dot{\theta}}{\Delta\theta}. \qquad (18)$$

For φ and $\dot{\varphi}$ cancellation, a supplementary law (17) is needed.

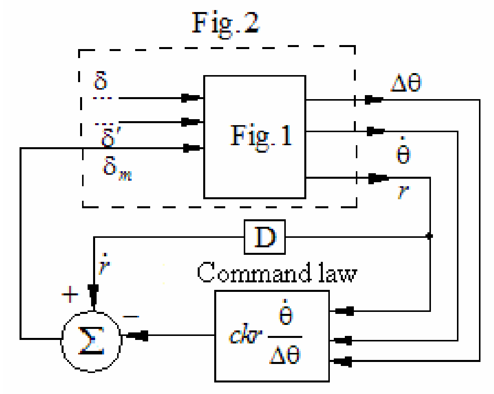

Fig.6

6 Command laws analysis

One considers the variable z, which describes the equation

$$\ddot{z} = f(z, \dot{z}), \qquad (19)$$

where f has such a chosen form in order to $\ddot{z}$ tends to zero when $z = \dot{z} = 0; f$ may have the form $f = k\dot{z}^2 / z$ and, consequently, equation (19) becomes [3]

$$\ddot{z} = k\frac{\dot{z}^2}{z}, \qquad (20)$$

where k is a non-dimensional proportionality coefficient .

Integrating equation (20), one obtains

$$\frac{\dot{z}}{\dot{z}_0} = \left(\frac{z}{z_0}\right)^k, \frac{z}{z_0} = \left[(1-k)\frac{\dot{z}_0}{z_0}\left(t_f - t\right)\right]^{\frac{1}{1-k}}, \qquad (21)$$

where $z_0 = z(0), \dot{z}_0 = \dot{z}(0)$ and t_f - time moment when the three variables become null. If one chooses $k > 1$, then the time interval till the three variables become null is very big and if one chooses $k \le 1/2$ then, if $t \to t_f$, z^2 tends to zero, while $\dot{z}$ (linear) tends to zero with a big slope $\frac{1}{2}\frac{\dot{z}_0}{z_0}$ [1]. If one chooses $k \in \left(\frac{1}{2}, 1\right)$ one obtains some changes. Thus, for $k = 0{,}7 \cong 2/3$, one observes that z^3 tends to zero, $\dot{z}^2$ tends to zero, while $\ddot{z}$ tends to zero (with a slope equal to 2/3) [3].

The laws (5), (7) have the form (20), where z is $\Delta\theta$, for equaation (5), respectively φ for Eq. (7).

Another command law, correlated with (19), has the form

$$\dot{q} = g(q, z, \dot{z}), \tag{22}$$

which may be chosen [3]

$$\dot{q} = akq\frac{\dot{z}}{z}, \tag{23}$$

with $a = \text{const.}$ Integrating (for $z(0) = z_0$ and $q(0) = q_0$) it results the equation

$$\frac{q}{q_0} = \left(\frac{z}{z_0}\right)^{ak}, \tag{24}$$

which expresses the fact that q and z are simultaneously null, together with $\dot{q}$ and $\dot{z}$.

The laws (6) and (16), (17), (18) have the form (23), where z is $\Delta\theta', \varphi, r$.

Other command law's forms are presented in [4] and [5].

7 Conclusion

The author starts from the models of rocket's motion around mass centre in vertical and horizontal plane and around longitudinal axis. Different structures of automatic command and stabilisation of the rocket's motion, using control laws that assure simultaneous cancellation of some state variables, are also presented in this paper.

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