Stabilization models and structures for move of very maneuverable flying objects

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Abstract: - One presents a dynamic model of the very maneuverable flying objects (A), which expresses the dependence between the vector formed by angles of A regarding aerodynamic trihedron and the vector of angular velocities of A or the vector of linear acceleration components with respect to the trihedron related to A. Also, leading structures (stabilization of movement) are presented. These are made by control loops after angles, angular velocities and linear accelerations and an adaptive control loop with neuronal network for dynamic inversion errors compensation of the non-linear function which describes unknown system of the dynamic model of A. Adaptive command is projected upon stability theory using a Liapunov function.

Key-Words: dynamic model, adaptive command, neuronal network

1 Introduction

The A's movement control takes into account the values of the A's angles regarding aerodynamic trihedron and angular velocities and accelerations sensors utilization (placed on trihedron axis related to A). Dynamic model is made by two sub-systems: one of them is described by a well known non-linear function (f_1) and the other is described by a proximate known or unknown non-linear function (f_2).

Control law synthesis is based on dynamic inversion (the f_2 inversion). The control law has components expressed as functions of state variables and an adaptive component. This is obtained with a neuronal network with the role of f_2 inversion error compensation.

The control and stabilization of A's movement in non-linear description are closely to real flight conditions than the linear variants. The learning capacity of the neuronal networks in control of the non-linear systems is taken into account.

2 Spatial movement models of the flying objects

The following equations express dependences between linear accelerations a_x, a_y, a_z and angular velocities $\omega_x, \omega_y, \omega_z$ regarding with trihedron related to flying machine A. These variables are available because of the accelerometers and gyrometers.

Let $oxyz$ be the trihedron related to A with ox - the longitudinal axis, oy - the lateral axis and oz rectangular to ox and oy and $ox_a y_a z_a$ -

aerodynamic trihedron; V is the flying velocity, α - attack angle, β - side-slip angle (fig.1). For $ox_a y_a z_a$ and $oxyz$ superpose the following coordinates transformations are made

$$ox_a y_a z_a \xrightarrow[\dot{\beta}(oz_a)]{\beta} ox'_a y z_a \xrightarrow[\dot{\alpha}(oy)]{\alpha} oxyz. \quad (1)$$

Acceleration $\vec{a}$ is expressed with formula

$$\vec{a} = \dot{\vec{V}} + \vec{\omega} \times \vec{V}, \quad (2)$$

with

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z, \quad (3)$$

$$\vec{\omega} = \vec{\omega}_x + \vec{\omega}_y + \vec{\omega}_z - \left(\dot{\vec{\alpha}} + \dot{\vec{\beta}}\right). \quad (4)$$

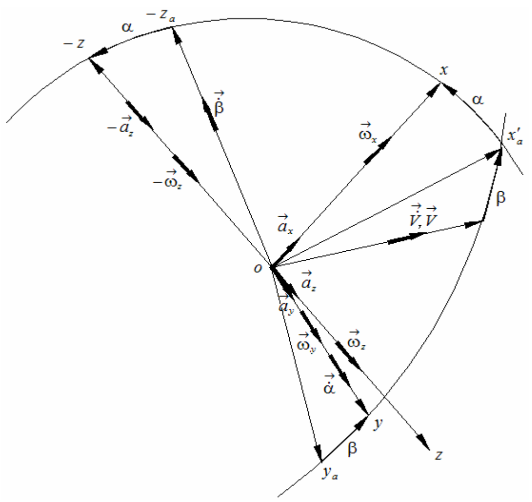

Fig.1

From equation (2) one obtains

$$\vec{a}_x + \vec{a}_y + \vec{a}_y = \dot{\vec{V}} + \left(\vec{\omega}_x \times \vec{V}\right) + \left(\vec{\omega}_y - \dot{\vec{\alpha}}\right) \times \vec{V} + \left(\vec{\omega}_z - \dot{\vec{\beta}}\right) \times \vec{V}. \quad (5)$$

Through projection on $ox_a y_a z_a$ axes one obtains

$$\begin{aligned} &\left[a_x \cos\alpha - a_z \sin(90^\circ + \alpha)\right]\cos\beta + a_y \sin\beta = \dot{V}, \\ &\left[a_x \cos\alpha - a_z \sin(90^\circ + \alpha)\right]\sin(90^\circ + \beta) + a_y \cos\beta = -\left[\omega_x \sin\alpha - \omega_z \cos\alpha + \dot{\beta}\right]V, \\ &-(a_x \sin\alpha - a_z \cos\alpha) = \left[\omega_x \cos\alpha - \omega_z \sin(90^\circ + \alpha)\right]V \sin\beta - (\omega_y - \dot{\alpha})\cos\beta \cdot V. \end{aligned} \quad (6)$$

$$\begin{aligned} &\dot{V} = (a_x \cos\alpha + a_z \sin\alpha)\cos\beta + a_y \sin\beta, \\ &\dot{\alpha} = \omega_y - (\omega_x \cos\alpha + \omega_z \sin\alpha)\mathrm{tg}\beta + \frac{-a_x \sin\alpha + a_z \cos\alpha}{V\cos\beta}, \\ &\dot{\beta} = \omega_x \sin\alpha - \omega_z \cos\alpha - \frac{(a_x \cos\alpha + a_z \sin\alpha)\sin\beta - a_y \cos\beta}{V}. \end{aligned} \quad (7)$$

To these one adds the moments equilibrium equations

$$\begin{aligned} &\dot{\omega}_x = \frac{M_x}{J_{xx}}, \\ &\dot{\omega}_y = \frac{M_y}{J_{yy}} + \left(1 - \frac{J_{xx}}{J_{yy}}\right)\omega_x \omega_z, \\ &\dot{\omega}_z = \frac{M_z}{J_{zz}} - \left(1 - \frac{J_{xx}}{J_{yy}}\right)\omega_x \omega_y, \end{aligned} \quad (8)$$

where M_x, M_y, M_z are the aerodynamic moments which operate round ox, oy, oz axes.

$$\begin{aligned} &M_x = M_x^{\beta}\beta + M_x^{\omega_x}\omega_x + M_x^{\omega_z}\omega_z + M_x^{\delta_e}\delta_e + M_x^{\delta_d}\delta_d, \\ &M_y = M_y^{\alpha}\alpha + M_y^{\omega_y}\omega_y + M_y^{\delta_p}\delta_p, \\ &M_z = M_z^{\beta}\beta + M_z^{\omega_x}\omega_x + M_z^{\omega_z}\omega_z + M_z^{\delta_e}\delta_e + M_z^{\delta_d}\delta_d; \end{aligned} \quad (9)$$

the coefficients of the angular variables represent variation speeds (slopes) of the aerodynamic moments regarding to respective angular variables (stability derivates).

Equations (7) and (8) are used especially in the case of very maneuverable aircrafts and in the case of agile rockets with big attack and side-slip angles. For a very good control of the agile air – air rockets' inclination, in [1] and [2] an aerodynamic roll angle is used; it verifies equation

$$\dot{\gamma} = \frac{\omega_x \cos\alpha + \omega_z \sin\alpha}{\cos\beta} + \frac{a_x \sin\alpha - a_z \cos\alpha}{V}\mathrm{tg}\beta \quad (10)$$

and the angular variables are grouped in the vectors

$$x^T = [\alpha \quad \beta \quad \gamma], \omega^T = [\omega_x \quad \omega_y \quad \omega_z]. \quad (11)$$

The second and the third equation (7) and equation (10) may be expressed under the vectorial form

$$\dot{x} = T(x)\omega + a_f, \quad (12)$$

where

$$T(x) = \begin{bmatrix} -\cos\alpha\mathrm{tg}\beta & 1 & -\sin\alpha\mathrm{tg}\beta \\ \sin\alpha & 0 & -\cos\alpha \\ \cos\alpha/\cos\beta & 0 & \sin\alpha/\cos\beta \end{bmatrix},$$

$$a_f = \begin{bmatrix} \dfrac{-a_x \sin\alpha + a_z \cos\alpha}{V\cos\beta} \\ -\dfrac{(a_x \cos\alpha + a_z \sin\alpha)\sin\beta - a_y \cos\beta}{V} \\ \dfrac{a_x \sin\alpha - a_z \cos\alpha}{V}\mathrm{tg}\beta \end{bmatrix}. \quad (13)$$

Equation (12) is equivalent with the following equations system, in which a component u_x of the pseudo -command is distinguished [3]

$$\dot{x} = u_x, u_x = T(x)\omega + a_f. \quad (14)$$

Similarly, equation system (8) may be described by equations in which another component u_ω of the pseudo-command is distinguished

$$\dot{\omega} = u_\omega, u_\omega = f(z, \omega, \delta), \delta^T = [\delta_e \quad \delta_p \quad \delta_d]. \quad (15)$$

Function f has two components, as we can see from (8) and (9)

$$u_\omega = f(x, \omega, \delta) = F(x, \omega) + G \cdot \delta; \quad (16)$$

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \dfrac{M_x^{\beta}\beta + M_x^{\omega_x}\omega_x + M_x^{\omega_z}\omega_z}{J_{xx}} \\ \dfrac{M_y^{\alpha}\alpha + M_x^{\omega_y}\omega_y}{J_{yy}} + \left(1 - \dfrac{J_{xx}}{J_{yy}}\right)\omega_x\omega_z \\ \dfrac{M_z^{\beta}\beta + M_z^{\omega_x}\omega_x + M_z^{\omega_z}\omega_z}{J_{yy}} + \left(1 - \dfrac{J_{xx}}{J_{zz}}\right)\omega_x\omega_y \end{bmatrix}, \quad (17)$$

$$G=\begin{bmatrix} \dfrac{M_x^{\delta_e}}{J_{xx}} & 0 & \dfrac{M_x^{\delta_d}\delta_d}{J_{xx}} \\ 0 & \dfrac{M_y^{\delta_p}\delta_p}{J_{yy}} & 0 \\ \dfrac{M_z^{\delta_e}}{J_y} & 0 & \dfrac{M_z^{\delta_e}}{J_y} \end{bmatrix}. \tag{18}$$

In the particular case of longitudinal move $(\omega_x=\omega_z=\beta=0)$ equations (7) and (8) becomes

$$\dot{\alpha}=\omega_y+\frac{a_x\sin\alpha-a_z\cos\alpha}{V\cos\beta}, \dot{\omega}_y=\frac{M_y}{J_{yy}}, \tag{19}$$

where

$$M_y=M_y^{\alpha}\alpha+M_y^{\omega_y}\omega_y+M_y^{\delta_p}\delta_p. \tag{20}$$

As a consequence $x=\alpha$, $\omega=\omega_y$, $T(x)=1$ and $a_f=(a_x\sin\alpha-a_z\cos\alpha)/V\cos\beta$; equation (12) becomes

$$\dot{\alpha}=\omega_y+a_f, \tag{21}$$

and equations (14), (15) and (16) becomes

$$\dot{\alpha}=u_x, u_x=\omega_y+a_f; \tag{22}$$

$$\dot{\omega}_y=u_\omega, u_\omega=f(V,H,\alpha,\omega_y,\delta_p); \tag{23}$$

$$f(V,H,\alpha,\omega_y,\delta_p)=F(V,H,\alpha,\omega_y)+G\cdot\delta_p \tag{24}$$

with

$$F=F_y=\frac{M_y^{\alpha}\alpha+M_y^{\omega_y}\omega_y}{J_{yy}}, G=\frac{M_y^{\delta_p}}{J_{yy}}. \tag{25}$$

From (14) one results

$$\omega_c=T^{-1}(x)(u_x-a_f), \tag{26}$$

where u_x is the pseudo-command, which may be chosen

$$u_x=K_x\tilde{x}, \tilde{x}=\bar{x}-x, \tag{27}$$

with $\bar{x}$ - leading command. From (16) one obtains

$$\delta_c=G_c^{-1}(u_\omega-F_c)=\hat{f}^{-1}(x,\omega,u_\omega), \tag{28}$$

with pseudo-command

$$u\omega=K_\omega\tilde{\omega}=u_c-u_a, \tag{29}$$

where $\tilde{\omega}=\omega_c-\omega$ and u_a - the adaptive command for inversion error's compensation.

3 Stabilization structures for flying objects' movement

Control block scheme of the closed loop system is presented in fig.2.

Another control structure may be obtained using stability theory with Liapunov functions if the leading object (A) may be described by the non – linear equations system [2]

$$\begin{aligned}\dot{x}_1&=f_1(x_1)+h_1(x_1)x_2,\\ \dot{x}_2&=f_2(x_1,x_2,u),\end{aligned} \tag{30}$$

used by the system from fig.3.

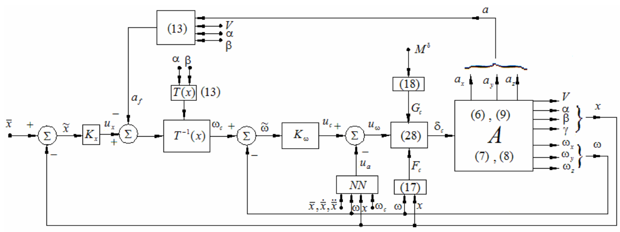

Fig.2

The impose vector $\bar{x}_2$ is

$$\bar{x}_2=q_1(\tilde{x}_1,t). \tag{31}$$

This law must assure the stability of the variable $\tilde{x}_1$ regarding variable z (fig.3);

$$\tilde{x}_2=\bar{x}_2-x_2=q_1(\tilde{x}_1,t)-x_2. \tag{32}$$

The second sub-system (described by the second equation (30)), in the lack of the external perturbations, may be described by equation

$$\dot{x}_2=v, v=f_2(x_1,x_2,u), \tag{33}$$

where error v is a pseudo-command. If the function f_2 is invertible than the dynamic inversion of f_2 may be approximately done; $u=f_2^{-1}(x_1,x_2,v)$. If f_2 is known than $f_2^{-1}f_2=1$ and if it is approximately known than the inversion of function f_2 is made with error $\varepsilon(x_1,x_2,u)$ and the first equation (33) becomes

$$\dot{x}_2=v+\varepsilon(x_1,x_2,u)+p, \tag{34}$$

where ε has the form

$$\varepsilon(x_1,x_2,u)=f_2(x_1,x_2,u)-\hat{f}_2(x_1,x_2,u)=\varepsilon(\tilde{x}_1,\tilde{x}_2,\bar{x}_1,\dot{\bar{x}}_1,v), \tag{35}$$

with $\hat{f}_2$ - calculated function.

The command law may be chosen as [2]

$$v=u_c+\dot{\bar{x}}_2+\bar{v}-u_a=K_2\tilde{x}_2+\dot{\bar{x}}_2+\bar{v}-u_a, \tag{36}$$

where u_c - the command in case $f_2^{-1}f_2=1$, K_2 - positive define matrix and u_a - adaptive command for the inversion error compensation ε, obtained from the Sigma neuronal network;

$$u_a=W^T\sigma(V^TI), \tag{37}$$

with σ - the activation function of the hidden layer (2), I - the input vector,

$$W^T=[b_i \quad w_{ij}], V^T=[c_i \quad v_{ij}], \tag{38}$$

b_i and c_i - bias, w_{ij} - the weights of connections between level 1 and 2, v_{ij} - the weights of connections between level 2 and 3.

Learning rule is obtained using stability theory of Liapunov [2]. Considering Frobenius norm of matrix A

$$\|A\|_F^2=\mathrm{tr}\{A^TA\}, \tag{39}$$

introducing the compact matrix

$$Z=\begin{bmatrix} W & 0\\ 0 & V\end{bmatrix}, \tag{40}$$

with $\|Z\|_F \le \bar{Z}$, choosing the input vector of the neuronal network

$$I^T = \begin{bmatrix} 1 & \tilde{x}_1^T & \tilde{x}_2^T & \bar{x}_1^T & \dot{\bar{x}}_1^T & \ddot{\bar{x}}_1^T & u_a^T & \|Z\|_F \end{bmatrix} \quad (41)$$

and standard Liapunov function

$$V_l = \frac{1}{2}\tilde{x}_2^T \tilde{x}_2 + \frac{1}{2}\mathrm{tr}\left(W^T \Gamma_W^{-1} W\right) + \frac{1}{2}\mathrm{tr}\left(V^T \Gamma_V^{-1} V\right), \quad (42)$$

from stability analysis one obtains the term $\bar{v}$ from (36)

$$\bar{v} = K_z \left(\|Z\|_F + \bar{Z}\right)\left(\|\tilde{x}_1\| + \|\tilde{x}_2\|\right) e_2, \quad (43)$$

where $K_z > 0$ and $e_2 = \tilde{x}_2 / \|\tilde{x}_2\|$.

The control system structure (PA-A) is presented in fig.3 (equivalent to the one from fig.4, where $\bar{v}$ is $\tilde{u}_\omega$).

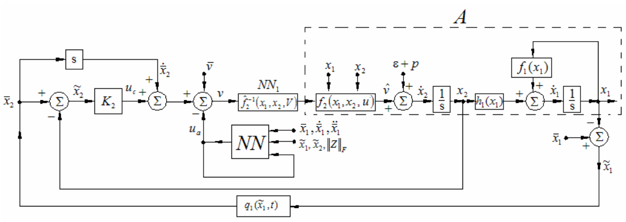

Fig.3

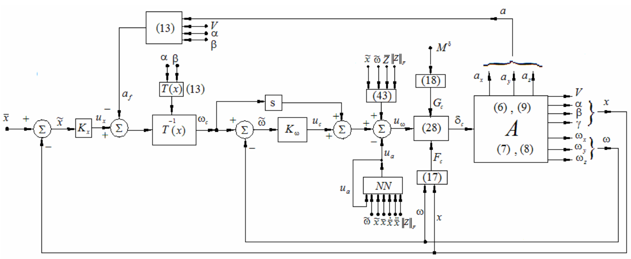

Fig.4

4 Conclusion

One presents some equivalent forms of models for A's movement as functions of A's angles related to aerodynamic trihedron, of angular velocities and linear accelerations.

Stabilization structures have some control loops after angles, angular velocities and linear accelerations and a control adaptive loop using a neuronal network for dynamic inversion error compensation of non-linear unknown function from model A. The adaptive command synthesis is based upon Liapunov function.

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