A Fuzzy Logic Approach to Robust Cost Functions in Adaptive Systems.

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Abstract: - This paper presents a fuzzy logic approach to combine cost functions in adaptive systems. The proposed approach is based on the application of a "soft" threshold to switch between two different cost functions. A generic analysis is carried out, being the Huber's function studied as a particular case. A channel equalization problem is used in order to benchmark the results achieved by the different cost functions.

Key-Words: - fuzzy systems, adaptive systems, learning cost.

1 Introduction

Adaptive systems are a fundamental tool in digital signal processing. The general scheme of such a system is shown in Fig. 1. An adaptive system consists of the following parts [1]:

- The signals that are processed by the filter: the input signal, x(n); the output signal, y(n); the desired signal, d(n); and the error signal, e(n), which is the difference between the desired and the output signals.
- The structure that defines how the output signal of the filter is computed from its input signal (adaptive filter in Fig 1). This filter can be an FIR, IIR, Volterra filter, etc.
- The parameters within this structure, which are the coefficients that define the transfer function of the adaptive filter.
- The adaptive algorithm that describes how the parameters are at each time instant. This algorithm is based on the minimization of a cost function, so that the minimum corresponds to the optimum performance of the system (zero error).

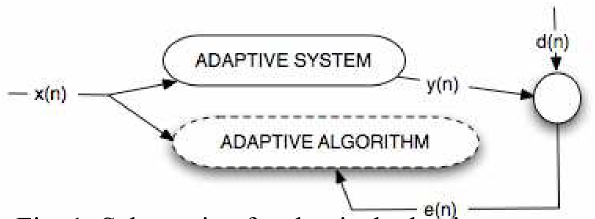

Fig. 1: Schematic of a classical adaptive system.

The analysis of different cost functions is a common research activity that can be extended to other processing fields such as artificial neural networks [2], [3]. The identification of the best cost function for a given problem may increase the performance of the adaptive system. Nonetheless, it is difficult to choose an optimal cost function since each function shows a different behaviour depending on the input and desired signals. For instance, the quadratic cost function is optimal when the error distribution is normal, but it is not optimal with impulsive noise [3]. In order to circumvent this problem, cost functions, which are the result of combining elementary cost functions, are used. The theoretical basis for this combination comes from the field of robust statistics [4]. In this work, we propose the use of fuzzy logic to obtain an optimal combination of cost functions.

The remainder of the paper is outlined as follows. In Section 2, the theoretical development of our approach is shown. Section 3 shows the results achieved in a channel equalization problem in which impulsive noise appears in the transmission; experimental results show the suitability of our approach. We end up the paper with some conclusions in Section 4.

2 Combination of Cost Functions Using Fuzzy Logic

Please, The use of cost functions derived from robust statistics is advisable in many adaptive systems [4]. Some of these functions are obtained by combining two cost functions that present different behaviours according to the error committed by the adaptive system:

$$J = \begin{cases} f(e) \text{ if } |e| > \beta \\ g(e) \text{ if } |e| \leq \beta \end{cases} \quad (1)$$

where f(e) and g(e) are error functions, and the commutation between the two cost functions is given by a crisp threshold β. This type of functions are widely used in adaptive signal processing, and especially, in the field of neural networks [4]. Some examples are given in Table 1.

Talvar	$J = \begin{cases} 0.5 \cdot e^2 & \|e\| \leq \beta \\ 0.5 \cdot \beta^2 & otherwise \end{cases}$
Huber	$J = \begin{cases} 0.5 \cdot e^2 & \|e\| \leq \beta \\ \beta \cdot \|e\| - 0.5 \cdot \beta^2 & otherwise \end{cases}$
Hampel	$J = \begin{cases} \frac{\beta^2}{\pi} \cdot \left[1 - \cos\left(\frac{\pi \cdot e}{\beta}\right)\right] & \|e\| \leq \beta \\ \frac{2 \cdot \beta^2}{\pi} & otherwise \end{cases}$

Table 1. Examples of combinations of cost functions.

The crisp commutation does not properly work when a wrong choice of the parameter β is made, since it may lead to slowing down the learning process (in the case of the Talvar's function, it leads to stopping the learning process).

In this work, we propose to use the approach presented in [5], to implement the combination of cost functions. In order to obtain a fuzzy combination, (1) is changed to:

$$J = \begin{cases} f(e) \ if \ |e| \ is \ high \\ g(e) \ if \ |e| \ is \ low. \end{cases} \quad (2)$$

In accordance with fuzzy logic, the linguistic terms "high" and "low" should be defined using membership functions [6]. Fig. 2 shows two typical membership functions, $\mu_1(e)$ y $\mu_2(e)$, that define the membership degree to the functions f(e) and g(e).

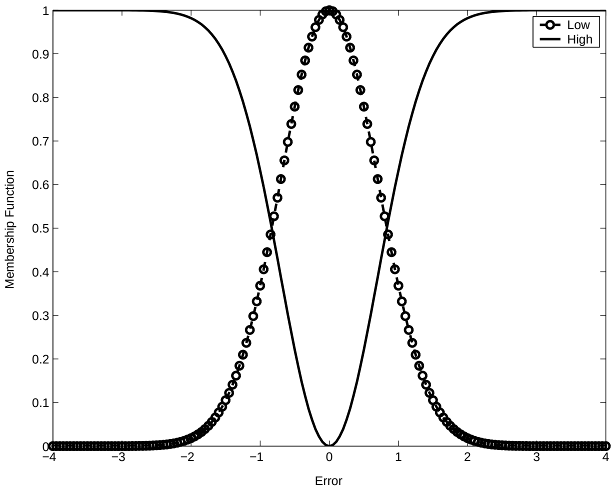

Fig. 2. Membership functions of the linguistic terms "high" and "low".

Therefore, the cost function given in (2), can be defined as:

$$J = \mu_1(e) \cdot f(e) + \mu_2(e) \cdot g(e) \quad (3)$$

This function is a generalization of the expression given in (1), where the membership functions worth 1 in a certain range and 0 in the rest of values. Moreover, (3) allows the production of many different cost functions J, since there are many different fuzzy membership functions.

Taking into account the minimum condition when the error equals zero:

$$\left.\frac{d\mu_1(e)}{de} \cdot f(e) + \frac{d\mu_2(e)}{de} \cdot g(e)\right|_{e=0} = -\left[\frac{df(e)}{de} \cdot \mu_1(e) + \frac{dg(e)}{de} \cdot \mu_2(e)\right]_{e=0}$$

(4)

If the elementary cost functions, f(e) and g(e), have a minimum in the origin, as it occurs in the cost functions shown in Table 1, the second term of (4) is equal to zero, so:

$$\left.\frac{d\mu_1(e)}{de} \cdot f(e) + \frac{d\mu_2(e)}{de} \cdot g(e)\right|_{e=0} = 0 \quad (5)$$

As there are only two membership functions, the following condition can be proposed:

$\mu_1(e) + \mu_2(e) = 1$ (6)

This leads to:

$$\left.\frac{d\mu_1(e)}{de} \cdot [f(e) - g(e)]\right|_{e=0} = 0 \quad (7)$$

If the membership function has a minimum in the origin, then (7) will be true. One of the functions which has a minimum in the origin is the Gaussian function:

$$\mu(x) = e^{-\beta \cdot x^2} \qquad (8)$$

Fig. 3 shows an example, in which the Mean-Square Error (MSE), the Mean-Absolute Error (MAE) and a fuzzy combination of these functions (MSE and MAE), are compared using this membership function for different values of the parameter β.

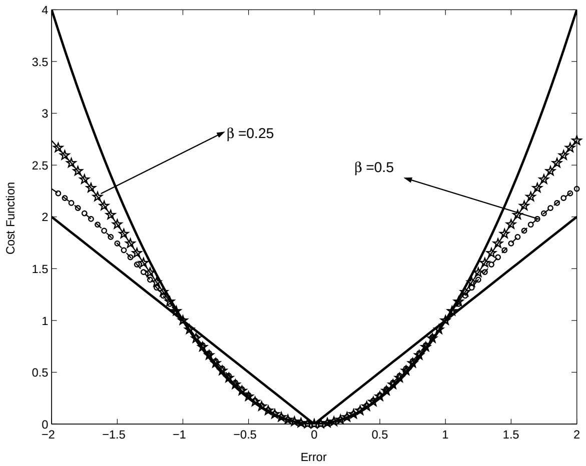

Fig. 3. Comparison of the quadratic cost function, the absolute-error cost function (solid lines) and our fuzzy approach

Fig. 3 shows that the proposed fuzzy cost function presents an intermediate behaviour between the two elementary cost functions. This behaviour is controlled by the parameter β.

3 Experimental Results.

A channel equalization problem (Fig. 4) is used in order to illustrate the capabilities of the proposed approach.

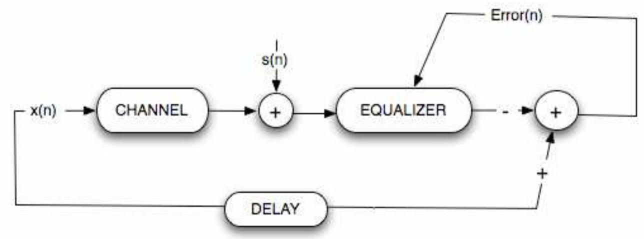

Fig. 4. Schematic of the channel equalization problem

Two different channels, previously proposed in [7], were considered in the simulations. The first one is represented in z-transform as $H_1(z) = 0.35 + z^{-1} - 0.35 \cdot z^{-2}$ and we used L = 15 (length of the adaptive system) and D = 8 (delay of the desired signal). This channel produces a misadjustment in the eigenvalues of the adaptive system input autocorrelation matrix of 1.45. The second channel is represented in z-transform as $H_2(z) = 0.35 + z^{-1} + 0.35 \cdot z^{-2}$, where a misadjustment of 28.7 is observed. We used 3000 input symbols for training and 5000 symbols for testing the performance, for a given Signal-To-Noise Ratio (SNR) between 5 and 20 dB (in 3 dB steps), being the noise an additive Gaussian noise. Moreover, we added an impulsive noise in order to simulate the behaviour of the equalizer in more difficult conditions. This impulsive noise always appeared in the same iterations in order to avoid that the simulation average lessened the effect of this noise. The impulse had a probability of appearing equal to 5%, and its amplitude was equal to 5. We used the membership function shown in (8), and the condition (6) was used to obtain the second membership function.

The parameter β is used as a threshold between the elementary cost functions in the Huber's function. It defines the amplitude of the membership functions in our approach. In the Huber's function, the parameter β actually controls the system's outliers: if the error is higher than β, then the update of the filter's coefficients depends on the absolute value of the error, which is a robust function with respect to outliers. Therefore, the error committed by the system is computed within a time window, and the following value is assigned to the parameter β of the Huber's function:

$$\beta = 2 \cdot \sigma_e^2$$

where σ_e is the standard deviation of the error. Thus, if the error committed by the adaptive system using the Huber's function is higher than this threshold, it can be considered as an outlier.

A similar discussion can be applied to the membership function shown in (8), so that the membership function μ_1 should equal zero if:

$$e^{-4} \approx 0 \Rightarrow \beta_{prop} = \frac{1}{\sigma_x^4}$$

This way, a robust behaviour can be achieved. Figs. 5 and 6 show the *BER (Bit Error Rate)* in the validation set, defined as follows:

$$BER = 10 \cdot \log_{10}\left(\frac{N_{error}}{N}\right)$$

where N=5000 and N_{error} is the number of errors committed by the equalizer.

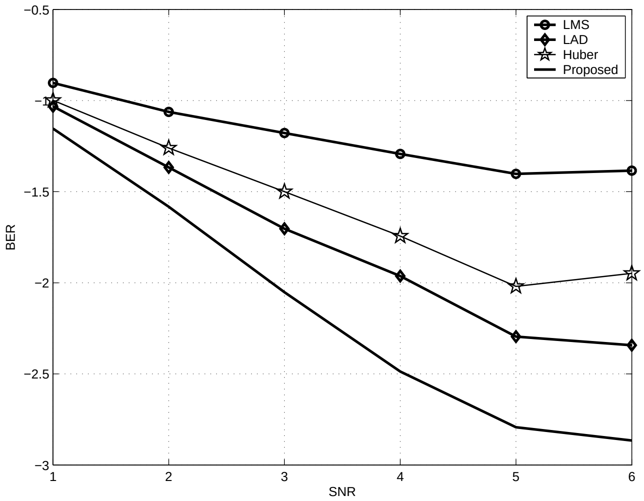

Fig. 5. BER vs. SNR obtained with the channel $H_1(z)$.

Fig. 5 shows that our approach is more robust than using either the quadratic cost function, LMS, or the absolute value of the error, LAD, or even the Huber's function. Moreover, the use of an adaptive parameter shows good results (it was not necessary to carry out many simulations to obtain an optimal parameter). Fig. 6 shows the same results but with a channel which produces a higher misadjustment in the eigenvalues of the adaptive system input autocorrelation matrix. An excellent behaviour of the proposed cost function is observed, as it was shown in the previous channel.

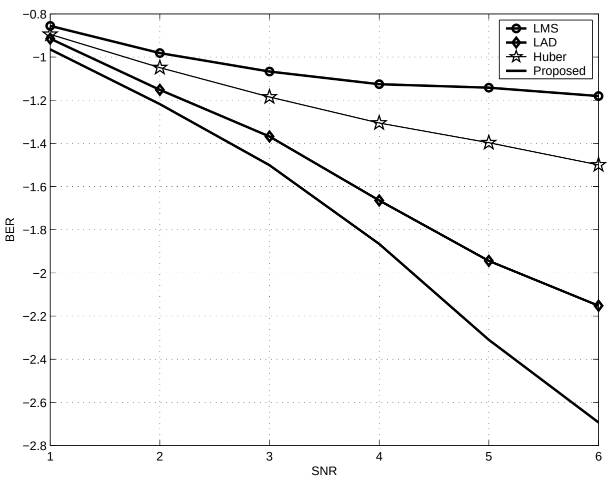

Fig. 6. BER vs. SNR obtained with the channel $H_2(z)$.

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4 Conclusion

In this communication, a fuzzy logic approach has been proposed in order to improve the combination of cost functions. This approach allows a wide range of possibilities to define cost functions. Our approach has been tested in a channel equalization problem, showing a more robust behaviour than other approaches based on a crisp threshold to define the cost function combination.

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