Application of Eigensolution-Free Method in a Real Large-scale Power System

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Abstract - The eigensolution-free method is proposed to calculate index of modal observability and controllability of large-scale power systems to study power system oscillation stability. This paper presents the work of applying the eigensolution-free method in a real large-scale power system in China. Because the method does not need to carry out eigensolution of the power system, a large amount of computational cost is saved. In addition, the numerical difficulty of high-dimensional computation is successfully avoided.

Keywords - Controllability, Observability, Eigensolution-Free method, Power system oscillation stability, Selection of stabilizer installing locations and feedback signals

1 Introduction

Generators operate synchronously with each other connected by transmission lines in power systems. The rotors of generators may swing against each other under disturbances, that leads to the so-called low frequency oscillation with oscillation frequency usually between 0.1~2.5Hz. Generally speaking, the oscillation is due to the variations of real power loads and the lack of system damp. Hence it is also named power oscillation or electromechanical oscillation. PSS (power system stabilizer) and FACTS-based stabilizer are two types of commonly-used devices to damp the low frequency oscillation. The damping effect significantly depends on the selection of the stabilizers' installing locations. Hence it has been an active topic of research since 1980s to select the most effective installing locations of stabilizers.

Eigensolution-Free method in the selection of stabilizer's installing locations proposed in [1] has its unique advantages in practice compared with typical modal control analysis method. It simplifies the calculations of the controllability and observability of the low frequency oscillation modes. It is especially useful for large-scale power system. This paper will firstly review the method in [1] and then gives an example of application in a real large-scale power system.

2 Power system modal observability, controllability theory and eigensolution-free method

The linearized Heffron-Philips model of a power system is

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = \left[\Delta\delta^T, \ \ \Delta\omega^T, \ \ \Delta E_q'^{\,T}, \ \ \Delta E_{qe}{}^T\right]^T$

$$\Delta\delta = \left[\Delta\delta_1, \ \ \Delta\delta_2, \ \ \cdots, \ \ \Delta\delta_n\right]^T$$

$$\Delta\omega = \left[\Delta\omega_1, \ \ \Delta\omega_2, \ \ \cdots, \ \ \Delta\omega_n\right]^T$$

$$\Delta E_q' = \left[\Delta E_{q_1}', \ \ \Delta E_{q_2}', \ \ \cdots, \ \ \Delta E_{q_n}'\right]^T$$

$$\Delta E_{qe} = \left[\Delta E_{qe_1}, \ \ \Delta E_{qe_2}, \ \ \cdots, \ \ \Delta E_{qe_n}\right]^T$$

The input matrix ***B***(controlling matrix) depends on the selection of input signals.

The output equation of power system is:

$$y = Cx + Du \tag{2}$$

where the output matrix ***C*** (observing matrix) generally selects Δw to be output signals and the direct matrix ***D*** is usually zero.

By introducing new system state variable vector z,

$$x = \phi\, z \tag{3}$$

where, ϕ is the right eigenvector matrix of state matrix ***A***, state equation of the power system is converted to

$$\dot{z} = \phi^{-1} A \phi\, z + \phi^{-1} Bu = \phi^{-1} A \phi\, z + B'u \tag{4}$$

$$y = C\phi\, z + Du = C'z + Du \tag{5}$$

where ϕ^{-1} is the left eigenvector matrix of state matrix $\boldsymbol{A}$ and

$B' = \phi^{-1} B$ is the modal controllability matrix and $C' = C\phi$ modal observability matrix. $\boldsymbol{\varphi}^{-1} \boldsymbol{A} \boldsymbol{\varphi}$ is a diagonal matrix and the ith diagonal element λ_i is the ith mode of the system.

Providing that the ith row of matrix $\boldsymbol{B'}$ is zero, the input signals will not affect the ith mode of the power system. In this situation, the ith mode is uncontrollable. Similarly, providing that the ith column of matrix $\boldsymbol{C'}$ is zero, the variable z_i will not affect the formation of the output signals. In this situation, the ith mode is unobservable. To mode λ_i , the kth generator has stronger controllability if $|B_{ik}|$ is bigger; the lth generator has stronger observability and it is easier for the mode λ_i to be observed there if $|C_{li}|$ is bigger.

The conventional modal control analysis method stated above is based on eigenvalue and eigenvector calculations. Therefore the system eigen-equation must be solved first. For the applications in large-scale power systems, due to the computational complexity and numerical difficulty of eigensolution of high-dimensional matrix, the conventional method is not very favourable.

Eigensolution-Free method proposed in [1] is based on the following three assumptions

1) The oscillation mode of interest should be lightly damped. That is $\lambda_i \approx j\omega_i$.
2) The oscillation frequency ω_i that we are interested in is known.
3) At least one of the sensitive generators to the oscillation modes of interest should be known, being assumed to be the jth generator in the system.

The system linearized state space equation of (1) is rearranged to be

$$\begin{bmatrix} \Delta\dot{\delta}_j \\ \Delta\dot{\omega}_j \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & \boldsymbol{0} \\ -k_j & -d_j & \boldsymbol{A}^T{}_{J23} \\ \boldsymbol{A}_{J31} & \boldsymbol{A}_{J32} & \boldsymbol{A}_{J33} \end{bmatrix} \begin{bmatrix} \Delta\delta_j \\ \Delta\omega_j \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} 0 \\ -B_{J2} \\ \boldsymbol{B}_{J3} \end{bmatrix} \Delta u_k \tag{6}$$

$$y_k = [C_{J1}, C_{J2}, \boldsymbol{C}_{J3}{}^T] \begin{bmatrix} \Delta\delta_j \\ \Delta\omega_{ji} \\ \boldsymbol{z} \end{bmatrix}$$

The observability and controllability of the mode $\lambda_i = -\sigma_i + j\omega_i$ are:

$$b_i = \boldsymbol{W_i}^T \boldsymbol{B}, \quad c_i = \boldsymbol{C}^T \boldsymbol{V_i} \tag{7}$$

where $\boldsymbol{V}_i$ and $\boldsymbol{W_i}^T$ are right and left eigenvectors of matrix $\boldsymbol{A}$ to mode $\lambda_i = -\sigma_i + j\omega_i$.

$$\begin{aligned} &\boldsymbol{A}\boldsymbol{V}_i = \lambda_i \boldsymbol{V_i} \\ &\boldsymbol{W_i}^T \boldsymbol{A} = \lambda_i \boldsymbol{W_i}^T \\ &\boldsymbol{W_i}^T \boldsymbol{V_j} = 1, if\ i = j, 0\ otherwise \end{aligned} \tag{8}$$

From equation (7) and (8), we can obtain:

$$\begin{aligned} &\begin{bmatrix} w_{i1} & w_{i2} & \boldsymbol{W_{i3}}^T \end{bmatrix} \begin{bmatrix} 0 & \omega_0 & \boldsymbol{0} \\ -k_j & -d_j & -\boldsymbol{A}_{J23} \\ \boldsymbol{A}_{J31} & \boldsymbol{A}_{J32} & \boldsymbol{A}_{J33} \end{bmatrix} \\ &= \lambda_i \begin{bmatrix} w_{i1} & w_{i2} & \boldsymbol{W_{i3}}^T \end{bmatrix} \end{aligned} \tag{9}$$

Hence

$$\begin{aligned} b_i(\lambda_i) &= \begin{bmatrix} w_{i1} & w_{i2} & \mathbf{W_{i3}}^T \end{bmatrix} \begin{bmatrix} 0 \\ -B_{J2} \\ \mathbf{B_{J3}} \end{bmatrix} \\ &= -[B_{J2} + \mathbf{A_{J23}}(\lambda_i \mathbf{I} - \mathbf{A_{J33}})^{-1} \mathbf{B_{J3}}] w_{i2} \\ &= -K_{cj}(\lambda_i) w_{i2} \end{aligned}$$

(10)

Taking the same procedure we can have

$$c_i(\lambda_i) = [C_{J1}, C_{J2}, \mathbf{C_{J3}}] \begin{bmatrix} v_{i1} \\ v_{i2} \\ \mathbf{V_{i3}} \end{bmatrix}$$

$$= [(\frac{\omega_0}{\lambda_i} C_{J1} + C_{J2}) + \mathbf{C_{J3}}(\lambda_i \mathbf{I} - \mathbf{A_{J33}})^{-1}$$

$$(\mathbf{A_{J31}} \frac{\omega_0}{\lambda_i} + \mathbf{A_{J32}})] v_{i2} = K_{oj}(\lambda_i) v_{i2}$$

(11)

From equation (10) and (11), we can get the residue index to be:

$$R_i(\lambda_i) = K_{cj}(\lambda_i) K_{oj}(\lambda_i) v_{i2} w_{i2} = -K_{cj}(\lambda_i) K_{oj}(\lambda_i) P_j \quad (12)$$

For damping the ith mode, if we choose the installing locations of a stabilizer between location A and B, when $|b_{iA}(\lambda_i)|\ |c_{iA}(\lambda_i)| > |b_{iB}(\lambda_i)||c_{iB}(\lambda_i)|$, A is better than B. Therefore, it is the ratio of multiplication of $\frac{|b_{iA}(\lambda_i)|}{|b_{iB}(\lambda_i)|}\frac{|c_{iA}(\lambda_i)|}{|c_{iB}(\lambda_i)|}$ that determines the selection of best installing locations and feedback signals.

The controlling vector **B** and the output vector $\mathbf{C}^T$ will change with the different selections of installing locations and feedback signals. However, the open-loop system matrix ***A*** will not change. That is:

$$v_{i2A} = v_{i2B}\ ,\quad w_{i2A} = w_{i2B} \quad (13)$$

From equation (11) and (12), we can get:

$$\frac{|b_{iA}(\lambda_i)|}{|b_{iB}(\lambda_i)|}\frac{|c_{iA}(\lambda_i)|}{|c_{iB}(\lambda_i)|} = \frac{|K_{biA}(\lambda_i)|}{|K_{biB}(\lambda_i)|}\frac{|K_{ciA}(\lambda_i)||w_{i2A}|}{|K_{ciB}(\lambda_i)||w_{i2B}|}\frac{|v_{i2A}|}{|v_{i2B}|}$$

$$= \frac{|K_{biA}(\lambda_i)|}{|K_{biB}(\lambda_i)|}\frac{|K_{ciA}(\lambda_i)|}{|K_{ciB}(\lambda_i)|} \quad (14)$$

The controllability $|b_i(\lambda_i)|$ and observability $|c_i(\lambda_i)|$, the index to measure the effectiveness of the stabilizers so as to select the installing locations and feedback signals, can be replaced by $|K_{bi}(\lambda_i)|$ and $|K_{ci}(\lambda_i)|$. Since in most cases, the oscillation mode of interest is lightly damped, $\lambda_i \approx j\omega_i$

$$|K_{bi}(\lambda_i)|\ |K_{ci}(\lambda_i)| \approx |K_{bi}(j\omega_i)|\ |K_{ci}(j\omega_i)| \quad (15)$$

Therefore, the modal control analysis by use of $|K_{bi}(\lambda_i)|\ |K_{ci}(\lambda_i)|$ is eigensolution free.

3 Test of eigensolution in a real large-scale power system examples

The eigensolution-free method proposed in [1] was demonstrated in a small power system in [1]. In this section, we will give results of testing it in a real large-scale power system. The tested system is a provincial power system in China with150 generators. Table 1-7 show the results of eigensolution analysis for selecting the installing locations in the power system for some lightly damped oscillation modes. For comparison, results of using the conventional method are also presented.

From the results presented in tables below, it can be seen that selection made by the eigensolution-free method is exactly same as that made by the conventional method

Table 1 mode 1 -0.000264+j10.297624

Bus name	Conventional method	Eigensolution-free
Gan Dongjin1#	1	1
Gan Dongjin2#	0.919282	0.916108

Table 2 mode 2 -0.003691+j7.817868

bus name	Typical method	Eigensolution-free
Gan Dongjin2#	1	1
Gan Dongjin1#	0.800776	0.795052
Gan Wan'an3#	0.509840	0.467385
Gan Wan'an4#	0.509840	0.467385
Gan Wan'an1#	0.229635	0.210997

Table 3 mode 3 -0.013503+j8.881832

bus name	Typical method	Eigensolution-free
Gan Zhelin3#	1	1
Gan Wan'an5#	0.120754	0.117752
Gan Zhelin5#	0.117684	0.108348
Gan Fenyi7#	0.104567	0.087921
Gan Zhelin1#	0.037257	0.037253

Table 4 mode 4 -0.027888+j8.415011

bus name	Typical method	Eigensolution-free
Gan Wan'an5#	1	1

Gan Wan'an3#	0.107613	0.100381
Gan Wan'an4#	0.107613	0.100381
Gan Wan'an2#	0.026426	0.024408
Gan Wan'an1#	0.021286	0.019830

Table 5 mode 5 -0.047967+j7.682415

bus name	Typical method	Eigensolution-free
Gan Wan'an1#	1	1
Gan Dongjin2#	0.238606	0.171410
Gan Dongjin1#	0.193703	0.164898
Gan Wan'an2#	0.121561	0.139184
Gan Wan'an4#	0.103880	0.099361
Gan Wan'an3#	0.103880	0.099361
Gan Wan'an5#	0.017337	0.016659

Table 6 mode 6 -0.437879+j7.421339

bus name	Typical method	Eigensolution-free
Gan Gui'erqi1#	1	1
Gan Gui'erqi2#	0.960338	0.896755
Gan Wan'an2#	0.653062	0.614935
Gan Zhelin5#	0.308348	0.243010
Gan Jiusanqi1#	0.250573	0.237940
Gan Jiusanqi2#	0.247190	0.058744
Gan Wan'an1#	0.132535	0.058379
Gan Dongjin2#	0.061440	0.055110
Gan Dongjin1#	0.051210	0.045848
Gan Wan'an4#	0.049057	0.013331
Gan Wan'an3#	0.049057	0.008396
Gan Fenyi7#	0.022908	0.008396

Table 7 mode 7 -0.985128+j11.309529

bus name	Typical method	Eigensolution-free
Gan Jingdezhen5#	1	1
Gan Jingdezhen3#	0.092646	0.081072
Gan Jinggangshan2#	0.016967	0.014204
Gan Jinggangshan1#	0.015801	0.011189
Gan Gui'erqi2#	0.009508	0.001171
Gan Gui'erqi1#	0.007248	0.001095

4 Conclusions

Eigensolution-Free method, a new approximate modal control analysis method, has been tested in a real large-scale power system in this paper, which is compared with conventional modal control analysis method. Results presented in this paper confirm that the eigensolution-free method can be used in a large-scale power system where the conventional method would become impossible if the system is very large.

References:

[1] H.F. Wang, Selection of Robust Installing Locations and Feedback Signals of FACTS-based Stabilizers in Multi-machine Power System, IEEE Transactions on Power Systems, Vol.14, No.2, May 1999

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