Analysis of Energy Storage Devices to Enhance Power System Oscillation Stability – Part I Theory

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Abstract:-This paper proposes a novel theoretical method in studying the capability of an energy storage system installed in a power system to enhance system oscillation stability. The proposed method is developed based on the well-know equal-area criterion and small-signal stability analysis. In the paper, some useful analytical conclusions are presented. Simulation results of an example power system installed with a Battery Energy Storage System (BESS) are given and the extension to more complicated case of multi-machine power systems is also discussed briefly. This is part I of the paper.

Key-words: - Energy Storage Systems (ESS), Battery Energy Storage Systems (BESS), power system oscillations, equal-area criterion

1 Introduction

Recent advance in new materials for energy storage and power electronics technology has made the energy storage systems (ESS) a new option to regulate and control modern power systems that have experienced dramatic changes recently. ESS can provide powerful means to rapidly vary real power and reactive power to improve system reliability and power quality. Commonly used energy storage systems are the Superconducting Magnetic Energy Storage (SMES), Battery Energy Storage Systems (BESS), Advanced Capacitors (AC) and Flywheel Energy Storage (FES) [1]. One important aspect of applications of the ESS in power systems is to enhance power system oscillation stability.

In [2] and [3], improvement of power system stability by using Flywheel Energy Storage (FES) is investigated with positive results presented. In [4] and [5], integration of FACTS and ESS is thoroughly discussed and comparison of FACTS and ESS in improving power system stability is made with constructive suggestions proposed. In [6] and [7] implementation of Battery Energy Storage Systems (BESS) to enhance power system stability is studied and an example of field application is given, indicating a bright future of ESS in power system applications. Especially in [2]-[7], capability of ESS in providing power systems positive damping is addressed as one of the important features of ESS applications in power systems. It is confirmed and demonstrated through either simulation or field test that ESS can significantly improve power system oscillation stability.

This paper presents analytical results on the capability of an ESS installed in a power system to suppress system oscillations. That provides an essential understanding and explanation on why and how the ESS can improve power system oscillation stability with some useful conclusions obtained. The analytical method used in the paper is a novel small-signal approach based on the well-known equal-area criterion for a simplified case of the ESS installed in a power system, giving insight and guidance into the investigation of applying the ESS in more complicated power systems. All analytical conclusions are demonstrated by the results of computation and simulation, which are further confirmed by an example to apply a BESS in a power system to damp power system oscillations. Issues, such as the limitation of capacity of energy storage devices, suppression of large-disturbance oscillations and extension of the study in multimachine power systems, are also addressed in the paper.

The organization of the paper is as follows. Firstly the capability of ESS in damping power system oscillations is studied by using a new small-signal analytical method proposed in the paper. Secondly some useful conclusions are drawn based on the analytical results that are demonstrated by simulation results. Those two sections are presented in part I of the paper. Thirdly an example power system installed with a BESS is presented to examine and verify theoretical results obtained in the paper. Finally, the extension of investigation to multimachine power systems, suppression of large-disturbance power system oscillations and a new concept of distributed damping of power system oscillations considering the capacity limitation of the ESS are given in the paper. Those two sections are presented in part II of the paper. Reference and conclusions are also given in part II of the paper.

2 Energy storage device integrated into power systems

Considering a simple single-machine infinite-bus power system as shown by Figure 1, where a shunt energy storage device is installed at the busbar s. The energy storage device can be a FES, BESS, SMES, AC or their combination. When it is modelled in the power system, at the power system level, it can be represented by an AC voltage, $\overline{V}_C$, that is the voltage at the AC terminal of the VSC (Voltage Source Converter) of the energy storage device [9]-[10].

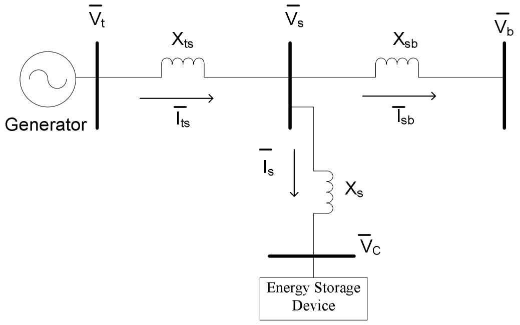

Figure 1 An energy storage device in shunt integration to single-machine infinite-bus power system

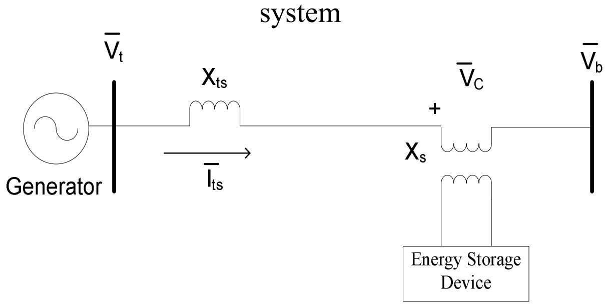

Figure 2 An energy storage device in series integration to single-machine infinite-bus power system

From Figure 1 we have

$$\overline{V}_S = jX_{sb}\overline{I}_{sb} + \overline{V}_b = jX_{sb}[\overline{I}_{ts} - (\frac{\overline{V}_S - \overline{V}_C}{jX_S})] + \overline{V}_b$$

$$= jX_{sb}\overline{I}_{ts} - \frac{X_{sb}}{X_S}\overline{V}_S + \frac{X_{sb}}{X_S}\overline{V}_C + \overline{V}_b$$

and hence

$$\overline{V}_S = \frac{jX_{sb}}{1+\frac{X_{sb}}{X_S}}\overline{I}_{ts} + \frac{X_{sb}}{X_S(1+\frac{X_{sb}}{X_S})}\overline{V}_C + \frac{\overline{V}_b}{1+\frac{X_{sb}}{X_S}}$$

That gives

$$\overline{V}_t = jX_{ts}\overline{I}_{ts} + \overline{V}_S = j(X_{ts} + \frac{X_S X_{sb}}{X_S + X_{sb}})\overline{I}_{ts}$$

$$+ \frac{X_{sb}}{X_S + X_{sb}}\overline{V}_C + \frac{X_S}{X_S + X_{sb}}\overline{V}_b = jX\overline{I}_{ts} + \overline{V} \quad (1)$$

Where

$$X = (X_{ts} + \frac{X_S X_{sb}}{X_S + X_{sb}})$$

$$\overline{V} = \frac{X_{sb}}{X_S + X_{sb}}\overline{V}_C + \frac{X_S}{X_S + X_{sb}}\overline{V}_b = a\overline{V}_C + b\overline{V}_b$$

Eq.(1) shows that the integration of the shunt energy storage device is electrically equivalent to a similar system when the energy storage device is installed and integrated to the transmission line in series, as the case shown by Figure 2. From Figure 2 we have

$$\overline{V}_t = jX_{ts}\overline{I}_{ts} + \overline{V}_C + \overline{V}_b = jX_{ts}\overline{I}_{ts} + \overline{V} \quad (2)$$

Hence, in the following discussion, we will focus on the system integrated with the shunt energy storage device, since electrically Eq.(1) and (2) are same.

If we consider that the generator is represented simply by a fixed EMF E' behind a reactance X', from Eq.(1) we can have the following voltage equation

$$jE' = j(X'+X)\overline{I}_{ts} + \overline{V}$$

Phasor diagram based on Eq.(1) and (3) is shown by Figure 3, from which we have

$$P_{ts} = \frac{E'}{X'_\Sigma}V\sin\delta' = \frac{E'}{X'_\Sigma}[bV_b\sin\delta + aV_c\sin(\delta-\gamma)] \quad (3)$$

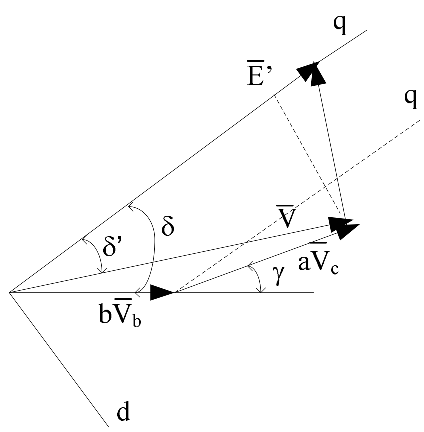

Figure 3 Phasor diagram

Hence linearizing Eq.(3), we obtain the variation of the active power delivered along the transmission line to be

$$\Delta P_{ts} = \Delta P_{tsdelta} + \Delta P_{control} \qquad (4)$$

where

$$\Delta P_{tsdelta} = \frac{E'_0}{X'_\Sigma}[bV_{b0}\cos\delta_0 + aV_{c0}\cos(\delta_0 - \gamma_0)]\Delta\delta$$
$$= C_{tsdelta}\Delta\delta$$
$$\Delta P_{control} = \frac{E'_0}{X'_\Sigma} aV_{c0}\sin(\delta_0 - \gamma_0)\Delta V_c$$
$$- \frac{E'_0}{X'_\Sigma} aV_{c0}\cos(\delta_0 - \gamma_0)\Delta\gamma$$

The second component in ΔP_{ts} is $\Delta P_{control}$, the forced variation of active power due to the control by the energy storage device, which can affect the damping of power system oscillations.

3 Voltage control implemented by the energy storage device

A voltage control function can be implemented by the energy storage device to regulate the voltage at the busbar where it is installed, V_s. This voltage control can be implemented by either changing V_c, γ or both through controlling the VSC. If it is implemented by controlling V_c only with fixed $\gamma = \gamma_0$, for the simplicity of analysis, we assume it to be a proportional controller. That is

$$V_c = V_{c0} + K_{volV}(V_{sref} - V_s)$$

Linearization of the above equation gives

$$\Delta V_c = -K_{volV}\Delta V_s \qquad (5)$$

From Eq.(I) in Appendix I we have

$$\Delta V_c = \frac{-K_{volV}C_1}{1 + K_{volV}C_2}\Delta\delta \qquad (6)$$

Hence substituting Eq.(6) to (4) we can obtain

$$\Delta P_{control} = -\frac{E'_0}{X'_\Sigma} aV_{c0}\sin(\delta_0 - \gamma_0)\frac{K_{volV}C_1}{1 + K_{volV}C_2}\Delta\delta$$
$$= C_{delta}\Delta\delta \qquad (7)$$

If the voltage control is implemented by controlling γ only with fixed $V_c = V_{c0}$, that is

$$\gamma = \gamma_0 + K_{vol\gamma}(V_{sref} - V_s)$$

Linearizing the above equation and using Eq.(I) in Appendix I, we can obtain

$$\Delta\gamma = \frac{-K_{vol\gamma}C_1}{1 + K_{vol\gamma}C_3}\Delta\delta \qquad (8)$$

Hence substituting Eq.(8) into (4) we can obtain

$$\Delta P_{control} = \frac{E'_0}{X'_\Sigma} aV_{c0}\cos(\delta_0 - \gamma_0)\frac{K_{vol\gamma}C_1}{1 + K_{vol\gamma}C_3}\Delta\delta$$
$$= C_{delta2}\Delta\delta \qquad (9)$$

If the voltage control is implemented by changing both V_c and γ, that involves exchange of active and reactive power between the power system and the energy storage device, we have

$$V_c = V_{c0} + K_{volV}(V_{sref} - V_s)$$
$$\gamma = \gamma_0 + K_{vol\gamma}(V_{sref} - V_s)$$

Linearizing the above equations and from Eq.(I) in Appendix I we have

$$\Delta V_c = \frac{-K_{volV} C_1}{1 + K_{volV} C_2} \Delta\delta - \frac{K_{volV} C_3}{1 + K_{volV} C_2} \Delta\gamma$$

$$\Delta\gamma = \frac{-K_{vol\gamma} C_1}{1 + K_{vol\gamma} C_3} \Delta\delta - \frac{K_{vol\gamma} C_2}{1 + K_{vol\gamma} C_3} \Delta V_c \qquad (10)$$

From details given in Appendix II, we should have

$$\Delta P_{control} = C_{delta3} \Delta\delta \qquad (11)$$

Eq.(7), (9) and (11) indicate that in all three cases, the forced variation of active power delivered along the transmission line by the voltage control implemented by the energy storage device is proportional to the deviation of the rotor angle □. That is

$$\Delta P_{ts} = \Delta P_{tsdelta} + \Delta P_{control} = C_{delta} \Delta\delta \qquad (12)$$

Damping effect of the voltage control implemented by the energy storage device on the small-signal oscillations of the power system can be analysed based on the equal-area criterion as illustrated in Figure 4 and 5 as follows.

In Figure 4, the group of parallel lines is the linearization of P_{ts} - δ curves for small-signal analysis. At the steady state, the system operates at point (δ_0 , P_{ts0}) on the linearized P_{ts} - δ curve in Figure 4. We assume that the oscillation starts from point a in Figure 4 that is due to a small disturbance. At point a, $\Delta\delta$ >0 and the resulted increase of P_{ts} has two parts as shown in Eq.(12) illustrated by Figure 4. The first part is $\Delta P_{tsdelta}$ in Eq.(12), caused directly by $\Delta\delta$ and the second is $\Delta P_{control}$ due to the voltage control of energy storage device, as shown in Eq.(12). When point a moves down with decreasing P_{ts} , from δ_1 to δ_0 the movement is along the line $\Delta P_{ts} = C_{delta} \Delta\delta$ on a group of continuous P_{ts} - δ lines above the line $\Delta P_{tsdelta}$ because $\Delta\delta > 0$. This forms area A_1. From δ_0 to δ_2, the movement is still along the line $\Delta P_{ts} = C_{delta} \Delta\delta$ but on a group of continuous P_{ts} - δ lines below the line $\Delta P_{tsdelta}$ because $\Delta\delta < 0$ and it stops only when arrives at point b where $A_2 = A_1$. Obviously, we should have $\delta_2 = \delta_1$ as the result of $A_2 = A_1$. Similarly, when point b moves up, it will not stop until it arrives at point c where $A_3 = A_2$, resulting in $\delta_3 = \delta_2$ as shown by Figure 5.

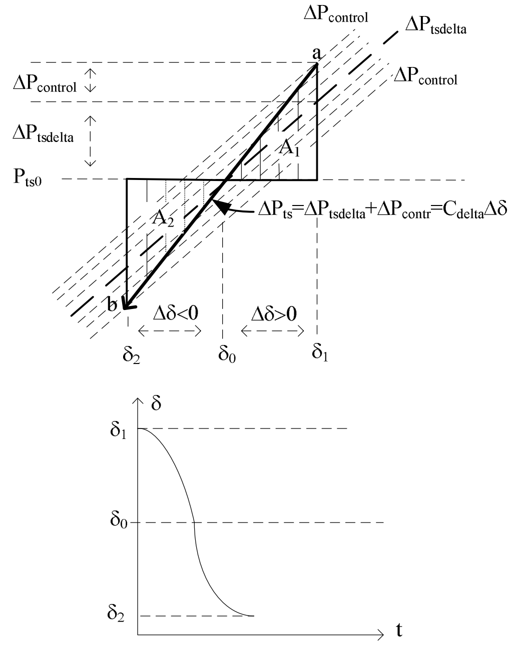

Figure 4 Analysis of damping effect of the voltage control implemented by energy storage device on power system oscillation (1)

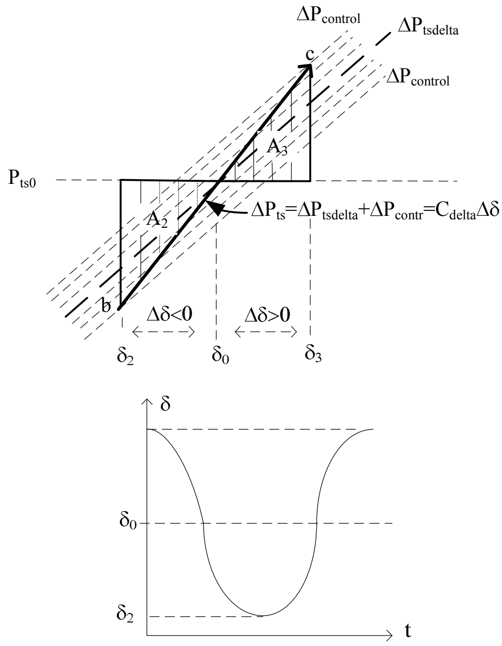

Figure 5 Analysis of damping effect of the voltage control implemented by energy storage device on power system oscillation (2)

The analysis above indicates that the voltage regulation implemented by the energy storage device generates a forced deviation of active power delivered along the transmission line proportional to the deviation of power angle, that contributes no damping to power system oscillations. Hence we can conclude that the voltage control implemented by the energy storage device will have little influence on the damping of power system oscillations.

4 Damping control implemented by the energy storage device

A damping control function can also be implemented by the energy storage device to improve power system oscillation stability. For the simplicity of analysis, we assume a proportional damping control law is adopted and damping feedback signal is the rotor speed of the generator. If it is implemented by controlling V_c only with fixed $\gamma = \gamma_0$, γ only with fixed $V_c = V_{c0}$, or both V_c and γ, that is,

$$V_c = V_{c0} + K_{dampV}(\omega - 1)$$
$$\gamma = \gamma_0 + K_{damp\gamma}(\omega - 1)$$

we have

$$\Delta V_c = K_{dampV}\Delta\omega$$
$$\Delta\gamma = K_{damp\gamma}\Delta\omega$$

Substituting the above equations into Eq.(4), we can obtain

$$\begin{aligned}\Delta P_{control} &= [\frac{E'_0}{X'_\Sigma} aV_{c0}\sin(\delta_0 - \gamma_0)K_{dampV} \\ &- \frac{E'_0}{X'_\Sigma} aV_{c0}\cos(\delta_0 - \gamma_0)K_{damp\gamma}]\Delta\omega \quad (13) \\ &= C_{Ddelta}\Delta\omega\end{aligned}$$

From Eq.(13) we can see that with the damping function implemented by the energy storage device, the forced deviation of active power delivered along the transmission line due to the damping control is proportional to $\Delta\omega$, $\Delta P_{control} = C_{Ddelta}\Delta\omega$. This will contribute to the damping of power system oscillations, as the following analysis shows based on the equal-area criterion.

Again we assume that the operating point of the power system at the steady state is at (δ_0, P_{ts0}) on P_{ts} - δ curve. Due to a small disturbance, the operating point moves onto point a in Figure 6 where the oscillation will start from. At point a with an increase of $\Delta\delta$, the increase of P_{ts} is $\Delta P_{tsdelta}$ proportional to $\Delta\delta$ as shown in Eq.(4). When operating point a moves down, $\Delta\omega < 0$, $\Delta P_{control} = C_{Ddelta}\Delta\omega$ will be added on $\Delta P_{tsdelta}$ that results in the operating point moves below the line $\Delta P_{tsdelta}$ (assuming $C_{Ddelta} > 0$). As shown by Figure 6, the operating point will stop at point c where $\Delta\omega = 0$ and the operating point comes back to the line $\Delta P_{tsdelta} = C_{tsdelta}\Delta\delta$. That is because at c $\Delta P_{control} = C_{Ddelta}\Delta\omega$ =0. According to the analysis of Figure 4 and 5 above, without $\Delta P_{control} = C_{Ddelta}\Delta\omega$, area 'acd' should be equal to area 'dgf' that will lead to $|\delta_1| = |\delta_{2'}|$. With $\Delta P_{control} = C_{Ddelta}\Delta\omega$, not only area 'acd' is reduced to A_1 but also extra area is generated below the line $\Delta P_{tsdelta} = C_{tsdelta}\Delta\delta$ that is added to the required area 'dgf' as shown in Figure 6. Hence $A_2 = A_1$ will result in $|\delta_2| < |\delta_1| = |\delta_{2'}|$, which indicates the oscillation is damped.

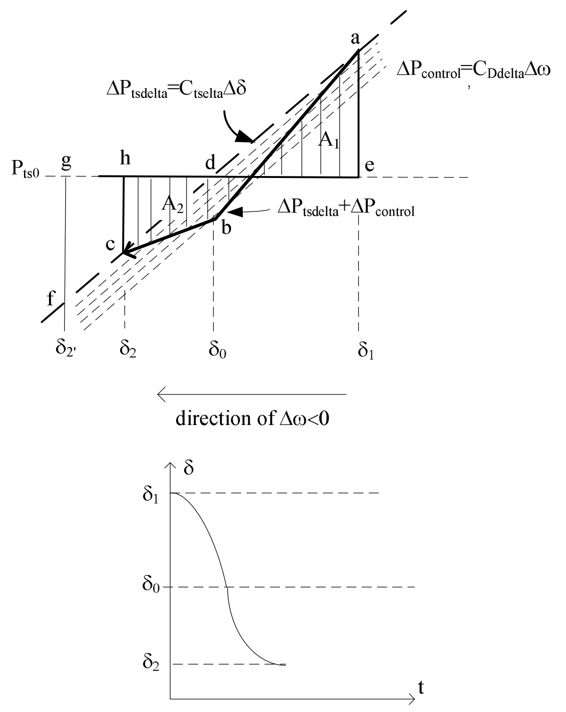

Figure 6 Analysis of damping control implemented by the energy storage device (1)

At operating point c, the accelerating area formed is 'cdh' that is smaller than A_2 in Figure 6. Furthermore, when the operating point moves up, it will moves above the line $\Delta P_{tsdelta} = C_{tsdelta}\Delta\delta$ due to the extra term $\Delta P_{control} = C_{Ddelta}\Delta\omega > 0$ because $\Delta\omega > 0$ as is shown in Figure 7. Hence the accelerating area is A_3 that is even smaller than area 'cdh'. In addition, the decelerating area increases and hence overall it results in $|\delta_3| < |\delta_2|$, indicating the oscillation is damped. In addition, from the analysis we can see that with a higher proportional coefficient C_{Ddelta}, the more decrease of $|\delta_i|$ (i=2,3) will be obtained and hence the better damping effect implemented by the energy storage device can be achieved.

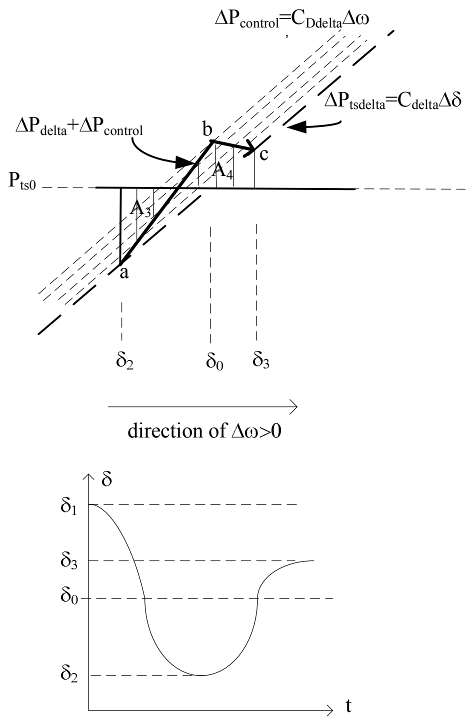

Figure 7 Analysis of damping control implemented by the energy storage device (2)

5 Damping effect analysis of the energy storage device with both voltage and damping control functions

Normally, the voltage control should be implemented via controlling V_c resulting in the exchange of reactive power exchange of the energy storage device with the power system. While damping control can be implemented by controlling γ through the exchange of active power or by adding onto the voltage control.

Considering the first case that the damping function is through controlling γ, i.e.

$$V_c = V_{c0} + K_{volV}(V_{sref} - V_s)$$
$$\gamma = \gamma_0 + K_{damp\gamma}(\omega - 1)$$

we have

$$\Delta V_c = -K_{volV}\Delta V_s$$
$$\Delta\gamma = K_{damp\gamma}\Delta\omega \tag{14}$$

From Eq.(I) in Appendix I we can obtain

$$\Delta V_c = \frac{-K_{volV}C_1}{1 + K_{volV}C_2}\Delta\delta - \frac{K_{volV}C_3}{1 + K_{volV}C_2}K_{damp\gamma}\Delta\omega \tag{15}$$

Substituting Eq.(14) and (15) into (4) we have

$$\Delta P_{control} = C_{controldelta}\Delta\delta + C_{Ddelta}\Delta\omega \tag{16}$$

where

$$C_{controldelta} = -\frac{E'_0}{X'_\Sigma}aV_{c0}\sin(\delta_0 - \gamma_0)\frac{K_{volV}C_1}{1 + K_{volV}C_2}$$
$$C_{Ddelta} = \frac{E'_0}{X'_\Sigma}aV_{c0}K_{damp\gamma}[\sin(\delta_0 - \gamma_0)\frac{K_{volV}C_3}{1 + K_{volV}C_2} + \cos(\delta_0 - \gamma_0)]$$

If the damping control is added on the voltage control of the energy storage device, i.e.

$$V_c = V_{c0} + K_{volV}(V_{sref} - V_s) + K_{dampV}(\omega - 1)$$

we have

$$\Delta V_c = -K_{volV}\Delta V_s + K_{dampV}\Delta\omega$$

From Eq.(I) in Appendix I we can obtain

$$\Delta V_c = \frac{-K_{volV}C_1}{1 + K_{volV}C_2}\Delta\delta - \frac{K_{dampV}}{1 + K_{volV}C_2}\Delta\omega \tag{17}$$

Substituting Eq.(17) into (4) we have

$$\Delta P_{control} = C_{controldelta}\Delta\delta + C_{Ddelta}\Delta\omega \tag{18}$$

where

$$C_{controldelta} = -\frac{E'_0}{X'_\Sigma} aV_{c0} \sin(\delta_0 - \gamma_0) \frac{K_{volV} C_1}{1 + K_{volV} C_2}$$

$$C_{Ddelta} = -\frac{E'_0}{X'_\Sigma} aV_{c0} \sin(\delta_0 - \gamma_0) \frac{K_{dampV}}{1 + K_{volV} C_2}$$

Substituting Eq.(16) or (18) into (4), we have that for both cases above

$$\Delta P_{ts} = \Delta P_{delta} + \Delta P_{Ddelta} = C_{delta} \Delta\delta + C_{Ddelta} \Delta\omega \quad (19)$$

A similar analysis can be carried out as in Figure 6 and 7 where the line $\Delta P_{tsdelta} = C_{tsdelta} \Delta\delta$ is replaced by $\Delta P_{delta} = C_{delta} \Delta\delta$ that is similar to the line $\Delta P_{ts} = C_{delta} \Delta\delta$ in Figure 4 and 5. The conclusion from the analysis will be that the portion of forced deviation of active power in Eq.(19), $C_{Ddelta} \Delta\omega$, contributes to the damping of power system oscillations. The higher C_{Ddelta} is, the more damping is provided by the damping control of the energy storage device.

Appendix I

From Eq.(3) and Figure 3 we have

$$I_{tsq} = \frac{V \sin\delta'}{X'_\Sigma} = \frac{bV_b \sin\delta + aV_c \sin(\delta - \gamma)}{X'_\Sigma} ,$$

$$I_{tsd} = \frac{E' - V\cos\delta'}{X'_\Sigma} = \frac{E' - bV_b \cos\delta - aV_c \cos(\delta - \gamma)}{X'_\Sigma}$$

From Eq.(1) and Figure 1 we can obtain

$$jE' = j(X' + X_{ts})\bar{I}_{ts} + \bar{V}_s = jX'_{ts\Sigma} \bar{I}_{ts} + V_{sd} + jV_{sq}$$

Hence

$$V_{sd} = X'_{ts\Sigma} I_{tsq} = \frac{X'_{ts\Sigma}}{X'_\Sigma}[bV_b \sin\delta + aV_c \sin(\delta - \gamma)]$$

$$V_{sq} = E' - X'_{ts\Sigma} I_{tsd} = E' - \frac{X'_{ts\Sigma}}{X'_\Sigma}(E' - V\cos\delta') = \frac{X'_\Sigma - X'_{ts\Sigma}}{X'_\Sigma} E' + \frac{X'_{ts\Sigma}}{X'_\Sigma}[bV_b \cos\delta + aV_c \cos(\delta - \gamma)]$$

That gives

$$\Delta V_{sd} = \frac{X'_{ts\Sigma}}{X'_\Sigma}[bV_{b0} \cos\delta_0 + aV_{c0} \cos(\delta_0 - \gamma_0)]\Delta\delta + \frac{X'_{ts\Sigma}}{X'_\Sigma} aV_{c0} \sin(\delta_0 - \gamma_0)\Delta V_c - \frac{X'_{ts\Sigma}}{X'_\Sigma} aV_{c0} \cos(\delta_0 - \gamma_0)\Delta\gamma$$

$$\Delta V_{sq} = -\frac{X'_{ts\Sigma}}{X'_\Sigma}[bV_{b0} \sin\delta_0 + aV_{c0} \sin(\delta_0 - \gamma_0)]\Delta\delta + \frac{X'_{ts\Sigma}}{X'_\Sigma} aV_{c0} \cos(\delta_0 - \gamma_0)\Delta V_c + \frac{X'_{ts\Sigma}}{X'_\Sigma} aV_{c0} \sin(\delta_0 - \gamma_0)\Delta\gamma$$

So we can have

$$\Delta V_s = \frac{1}{V_{s0}}(V_{sd0}\Delta V_{sd} + V_{sq0}\Delta V_{sq}) = C_1\Delta\delta + C_2\Delta V_c + C_3\Delta\gamma \quad (I)$$

where

$$C_1 = \frac{V_{sd0} X'_{ts\Sigma}}{V_{s0} X'_\Sigma}[bV_{b0} \cos\delta_0 + aV_{c0} \cos(\delta_0 - \gamma_0)] - \frac{V_{sq0} X'_{ts\Sigma}}{V_{s0} X'_\Sigma}[bV_{b0} \sin\delta_0 + aV_{c0} \sin(\delta_0 - \gamma_0)]$$

$$C_2 = \frac{V_{sd0} X'_{ts\Sigma}}{V_{s0} X'_\Sigma} aV_{c0} \sin(\delta_0 - \gamma_0) + \frac{V_{sq0} X'_{ts\Sigma}}{V_{s0} X'_\Sigma} aV_{c0} \cos(\delta_0 - \gamma_0)$$

$$C_3 = -\frac{V_{sd0}X'_{ts\Sigma}}{V_{s0}X'_{\Sigma}} aV_{c0}\cos(\delta_0 - \gamma_0) + \frac{V_{sq0}X'_{ts\Sigma}}{V_{s0}X'_{\Sigma}} aV_{c0}\sin(\delta_0 - \gamma_0)$$

Appendix II

From Eq.(10) we have

$$\Delta V_c = \frac{-K_{volV}C_1}{1+K_{volV}C_2}\Delta\delta + \frac{K_{volV}C_3}{1+K_{volV}C_2}\frac{K_{vol\gamma}C_1}{1+K_{vol\gamma}C_3}\Delta\delta + \frac{K_{volV}C_3}{1+K_{volV}C_2}\frac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}\Delta V_c$$

$$\Delta\gamma = \frac{-K_{vol\gamma}C_1}{1+K_{vol\gamma}C_3}\Delta\delta + \frac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}\frac{K_{volV}C_1}{1+K_{volV}C_2}\Delta\delta + \frac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}\frac{K_{volV}C_3}{1+K_{volV}C_2}\Delta\gamma$$

Hence we have

$$\Delta V_c = \frac{\dfrac{-K_{volV}C_1}{1+K_{volV}C_2} + \dfrac{K_{volV}C_3}{1+K_{volV}C_2}\dfrac{K_{vol\gamma}C_1}{1+K_{vol\gamma}C_3}}{1-\dfrac{K_{volV}C_3}{1+K_{volV}C_2}\dfrac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}}\Delta\delta = \frac{K_{volV}C_3K_{vol\gamma}C_1 - K_{volV}C_1(1+K_{vol\gamma}C_3)}{(1+K_{volV}C_2)(1+K_{vol\gamma}C_3) - K_{volV}C_3K_{vol\gamma}C_2}\Delta\delta$$

$$\Delta\gamma = \frac{\dfrac{-K_{vol\gamma}C_1}{1+K_{vol\gamma}C_3} + \dfrac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}\dfrac{K_{volV}C_1}{1+K_{volV}C_2}}{1-\dfrac{K_{vol\gamma}C_2}{1+K_{vol\gamma}C_3}\dfrac{K_{volV}C_3}{1+K_{volV}C_2}}\Delta\delta = \frac{K_{volV}C_1K_{vol\gamma}C_2 - K_{vol\gamma}C_1(1+K_{volV}C_2)}{(1+K_{volV}C_2)(1+K_{vol\gamma}C_3) - K_{volV}C_3K_{vol\gamma}C_2}\Delta\delta$$

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