Analysis of HLM on PISA 2003 based on Networked Readiness Index of Countries and Mathematics Learning of Students

YUAN-HORNG LIN
Department of Mathematics Education
National Taichung University
140 Min-Shen Rd., Taichung City 403, Taiwan
TAIWAN

CHIA-TAI CHANG
Graduate Institute of Educational Measurement and Statistics
National Taichung University
140 Min-Shen Rd., Taichung City 403, Taiwan
TAIWAN

HSIAO-FANG LIN
Institute of Teaching Art
MingDao University
369 Wen-Hua Rd., Peetow, ChangHua 523, Taiwan
TAIWAN

ZE-SHINE WU
Department of Comparative Education
National Chi Nan University
University Rd., Puli, Nantou 545, Taiwan
TAIWAN

Abstract: The purpose of this study is to investigate the PISA 2003 database by Hierarchical Linear model (HLM). The plan of PISA 2003 aimed at the evaluation of mathematics learing of 15-year-old students. The sample included 41 countries. There will be estimation errors if all the task-takers were put in the same level when it went to analyze all the casual relation between the variables of countries and students. Therefore, HLM of two levels is used to analyze the PISA 2003 database. The unit in level-1 is students. The variables belonging to the level-1 are mathematics cognition, mathematics view and mathematics thinking. As to the level-2, the unit is countries and the variable is Networked Readiness Index (NRI). There are four submodels in this study. Based on the four submodels, the relationship between mathematics learning of students and NRI of countries are clearly understood. Finally, some suggesstions and recommendations for future research are provided.

Key-Words: PISA, HLM, mathematics learning, networked readiness index, assessment.

1 Introduction

With the unremittingly educational innovation, educational psychology researchers in the world pay more and more attention to emphasize the influential factors on learning achievements. They concern students how to use the knowledge learned from school to solve the problems in their life. Also, they emphasize the values of international assessment for students so that it can provide suggestions for the education policies.

The Organization for Economic Co-operation and Development (OECD) created Programme for International Student Assessment (PISA) in 1997. PISA 2000 is the first series of triennial assessments, and the emphasis in PISA 2000 is on the assessment reading literacy. PISA 2003 represents a continuation of data strategy and the assessed domains build on those used in PISA 2000. The focus of PISA 2003 is on mathematics literacy, defined as the capacity of students to identify, understand and engage in mathematics and make well-founded judgements about the role that mathematics plays in life. The prblem-solving skills in PISA 2003 are defined as the ability of students to use cognitive processes to solve real cross-disciplinary problems where the solution path is not obvious [12].

As to PISA, the first cycle was in 2000 and the second was in 2003. The third cycle was on the assessment of scientific literacy in 2006. The following is the sketch of key features in PISA 2003. First, PISA 2003 is an internationally standardized assessment that is jointly developed by 41 countries and administered to 15-year-old students in educational programmes. The contents emphasize the understanding of complicated situations. Second, it is a paper-and-pencil test, lasting a total two hours for each student. The test is a maxture of multiple-choice items and questions requiring students to construct their own responses [11].

One intension of PISA is to build the longitudinal database and the trace of international assessment is a distinguishing feature. However, most of the data analysis methodologies on PISA 2003 are basic statistics descriptions. The advanced information for the comparison of the relationships between countries and students are lmited. Thanks to the features of hierarchical linear model (HLM), the comparisons of relationships between variables of different levels will be clearly revealed [3]. Therefore, HLM is used to analyze the PISA 2003 database in this study. Exploatory study of two levels model will be applied. The unit of level-1 is students and the unit of level-2 is countries.

2 Literature Review

2.1 The Structural Feature of Regression Model

Ordinal Least Square (OLS) regression models assume that residuals are independent, normally distributed and constant variance. However, when data are collected using a “cluster” sampling method, like the case in PISA 2003, the residuals will be unlikely to be independent [5]. For instance, it is expected that mathematics cognition of students within the same country to be more similar than would be the case in a simple random of students. The reason is that students within the same country are likely to share the common variables of countries. Hence, one shortcoming of OLS regression is that the standard error will be too small when it is used to estimate relationships on clustered data. Two traditional techniques of estimating unbiased standard errors for clusters data are bootstrap and jackknife procedure. Another approach is the multilevel model which makes assumptions about the correlation structure of the individual observations [14].

HLM is also called the multilevel model. It appears in diverse literatures under a variety of titles. For example, scholars often refer to it as multilevel linear model in sociological research, and mixed-effects models or random-effects models are common terms in biometric research. In educational research, hierarchical linear model is the common term.

2.2 The International Assessment and its Influential Variables

Many institutions engage in the assessment of international learning achievement and attitude for students. For example, SITES 2006 (Second Information on Technology in Education Study 2006), TEDS (Teacher Education and Development Study), PIRLS 2006 (Progress in International Reading Literacy Study 2006) and TIMSS-R (Third International Mathematics and Science Study-Repeat, or named TIMSS 1999) are well-known international assessment. PISA is an international assessment under the plan of OECD. The contents of PISA are different from the other international assessment. It is not the test on specific knowledge in the textbook, but integrates all the field knowledge in reading, mathematics, science to the life problems. The plan of PISA tries to understand the 15-year-old students’ ability in solving life problems by cognition skills after they finish junior high school education. Hence, mathematics learning and problem solving by using mathematics skills are core capabilities.

Mathematical cognition assessment of PISA 2003 includes three dimensions: process, content and context. And the contents of items include quantity, space and shape, change and relationships, and uncertainty. The basic conception is an encompassing set of phenomena and concepts that make sense and can be encountered within and across a multitude of different situations [12].

There have been already some research outcomes in PISA. Williams examined cross-national variation in rural mathematics cognition among 15-year-old students in 24 industrialized nations. He found that rural mathematics scores were significantly lower than in urban and medium-size communities in 14 of 24 nations. However, some students in urban communities scored lowest. Usually it is believed that students in urban score higher than those of rural area. The reason is that lower SES results in lower achievement. But would this conception infer to every nations or conditions? Williams got the further results that once SES was controlled and rural locations predicted mathematics scores in only 4 of 24 nations [6]. This research indicates that there are other variables needed to clarify the casual relationships in the international database.

In the 1990s, several international data sets were disseminated, demanding more hierarchical data analyses at various organization levels [2]. Statistics analyses of multilevel models provided quite a little information for education outcomes [7]. Nevertheless, family background and school resources are commonly concerns which may have effects on the learning achievement. Many studies also have aimed at the background variables of family and school in order to understand how these variables carry effects on mathematics achievements of students [15] [9]. However, seldom research aimed at the relationship of nation background and mathematics achievement. Moreover, effects of these findings have been less consistent in large cross-national studies.

Although there were some researches which investigated the PISA database using HLM, these researches focus on other achievement test (e.g. reading achievement) and they did not involve all the countries in the PISA database [4]. Therefore, it is feasible to investigate the relationship between countries background and students mathematics cognition by HLM.

2.3 Basic Features of Hierarchical Linear Model

With proper mathematical presentations, HLM can be specified in both hierarchical and combined forms [1]. If there are J $(j = 1,2,3,\cdots,J)$ nations from a population and n_j $(i = 1,2,3,\cdots,n_j)$ students within nation j, countires are units of level-2 and students are units of units of level-1. An example of hierarchical form for the two levels HLM is exemplified in equation (1) (2) (3).

Level-1 (e.g. students):

$$Y_{ij} = \beta_{oj} + \beta_{1j} X_{ij} + r_{ij} \quad (1)$$

Y_{ij} is level-1 criterion variable (e.g. students mathematics cognition) and X_{ij} is a level-1 predictive variable (e.g. student mathematics view). β_{oj} and β_{1j} are level-1 coefficients and r_{ij} is a level-1 random effects. Besides, the centering of X_{ij} here is natrual metric. In some cases, grand-mean centering or group-centering are chosen for X_{ij}.

Level-2 (e.g. countries):

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j} \quad (2)$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j} \quad (3)$$

W_j is a level-2 predictive variable (e.g. country NRI). γ_{00}, γ_{01}, γ_{10} and γ_{11}, are level-2 coefficients and they are also called fixed effects. u_{0j} and u_{1j} are level-2 random effects.

The model in combined form will be as it is in equation (4).

$$Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} W_j + \gamma_{11} X_{ij} W_j + u_{0j} + u_{1j} X_{ij} + r_{ij} \quad (4)$$

The following assumptions are required for the above combined model [14].

$$E(r_{ij}) = 0 \text{ and } Var(r_{ij}) = \sigma^2$$

$$E\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } Var\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{bmatrix}$$

$$Cov(u_{0j}, r_{ij}) = Cov(u_{1j}, r_{ij}) = 0$$

σ^2 is the level-1 variance and τ_{00}, τ_{01}, τ_{10} and τ_{10} are level-2 variance-covariance components.

The hierarchical form indicates that the level-1 regression coefficients (β_{oj} and β_{1j}) are random outcome variables at level-2. If u_{0j} and u_{1j} were null for every j, the combined model would be equivalent to an ordinary least squares regression model [1]. In the situation, HLM approach is not needed.

3 Method of Data Analysis

3.1 Data Source and Variables of Level-1

There are 41 countries participating in PISA 2003. These countries are depicted in Table 1. All the nations in Table 1 are members of the OECD, except those marked with an asterisk (*). The PISA in OECD was developed by an international team of people prominent in educational research and reform. The variables in level-1 are mathematics cognition, mathematics view and mathematics thinking. The mathematics cognition is the criterion variable.

Mathematics view and mathematics thinking are the predictive variables.

Table 1. The Countries of PISA 2003

Area (Number)	Countries	
Oceania (2)	Australia	New Zealand
Europe (25)	Austria	Belgium
	Denmark	Finland
	France	Germany
	Iceland	Hungary
	Czech Republic	Italy
	Ireland	Latvia*
	Luxembourg	Norway
	Netherlands	Greece
	Poland	Portugal
	Serbia and Montenegro*	Spain
	Sweden	Switzerland
	United Kingdom	Liechtenstein
	Slovak Republic	
Asia (8)	Macao-China*	Indonesia*
	Hong Kong-China*	Japan
	Korea	Russian Federation*
	Turkey	Thailand*
Africa (1)	Tunis*	
American (5)	Brazil*	Canada
	Mexico	United States
	Uruguay*	

3.2 Data Source and Variables of Level-2

The unit in level-2 is countries. It is assumed that NRI of countries will influence the student mathematics cognition. Besides, the varying influence of mathematics view and mathematics thinking on mathematics cognition for students will depend on NRI of countries.

NRI in 2003 is applied in this study and it is built by World Economic Forum (WEF). It measures the propensity for countries to exploit the opportunities offered by information and communication technology. NRI is a composite of three components: the environment for ICT (Information and Communication Technology) offered by a given nation or community, the readiness of the community's key stakeholders (indervіduals, businesses, and governments) to use ICT, and finally the usage of ICT among these stakeholders [8].

As to the original data resource of NRI, owing to the lack of complete indices for 3 countries, only 38 countries are included in HLM analysis.

4 Results

In this study, the unit in level-1 is students and the unit in level-2 is countries. HLM 6 software is used in analyze the structural data [13]. Four submodels will be the exploratory models. These submodels, running from simpler to the more complex, include the one-way ANOVA model with random effects, means-as-outcome regression, random coefficients regression model and model with non-randomly varying slopes [14].

4.1 One-way ANOVA model with random effects

The level-1 and level-2 equations are as follows.

Level-1:

$$Y_{ij} = \beta_{0j} + r_{ij} \quad (5)$$

Level-2:

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (6)$$

Y_{ij} is the mathematics cognition. The results is depicted in Table 2. It shows that there is significant mean difference among countries. The intraclass correlation coefficient is $\rho = 8.149/(8.149 + 70.381) = 10.38\%$. Therefore, the explained variance from countries is 10.38%. On the other hand, certain predictive variables may exist so that the difference of means for countries could be explained. Hence, the following submodel, means-as-outcome regression, will be investigated further.

Table 2. The Analysis of One-way ANOVA Model with Random Effects

Fixed Effect	Coefficient		Standard Error	
γ_{00}	12.646		.457	
Random Effect	Variance Component	df	χ^2	p-value
u_{0j}	8.149	37	34991.294	.000
r_{ij}	70.381			

4.2 Means-as Outcomes Regression

As to the predictive variable NRI in level-2, the level-1 and level-2 equations are as follows.

Level-1:

$$Y_{ij} = \beta_{0j} + r_{ij} \quad (7)$$

Level-2:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j} \quad (8)$$

Y_{ij} is the mathematics cognition of students and W_j is NRI of countries. The analysis result is depicted in Table 3. The coefficient is significant and it shows that predictive variable NRI can explain the mathematics cognition means of countries. Compared to Table 3, NRI can explain 44.54% of mathematics cognition means of countries because of $(8.149 - 4.519)/8.149 = 44.54\%$. The conditional intraclass correlation is $\rho = 4.519/(4.519 + 70.381) = 6.03\%$. Thus, NRI variable reduces the intraclass correlation from 10.38% to 6.03%.

Table 3. The Analysis of Means-as-Outcomes Regression (W_j is NRI)

Fixed Effect	Coefficient	Standard Error		p-value
γ_{00}	12.646	.336		.000
γ_{01}	2.887	.582		.001
Random Effect	Variance Component	df	χ^2	p-value
u_{0j}	4.519	36	15817.780	.000
r_{ij}	70.381			

4.3 Random Coefficient Regression Model

In this model, there are some predictive variables in level-1, but not in level-2. Mathematics view and mathematics thinking are possible factors which may influence mathematics cognition. Hence, mathematics view and mathematics thinking are predictive variables in level-1. The equations are as follows.

Level-1:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2i}X_{2ij} + r_{ij} \quad (9)$$

Level-2:

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (10)$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \quad (11)$$

$$\beta_{2j} = \gamma_{20} + u_{2j} \quad (12)$$

X_{1ij} is mathematics view and X_{2ij} is mathematics thinking. The analysis result is depicted in Table 4. All the coefficients in level-1 are statistically significant. Therefore, mathematics view and mathemarics thinking will influence mathematics cognition. As shown in Table 4, the higher mathematics view and mathematics thinking the students have, the higher mathematics cognition they will have. Mathematics view and mathematics thinking can explain 4.32% of mathematics cognition because of $(70.381 - 67.343)/70.381 = 4.32\%$.

Table 4. The Analysis of Random Coefficients Regression Model

Fixed Effect	Coefficient	Standard Error		p-value
γ_{00}	2.923	.884		.002
γ_{10}	.060	.019		.004
γ_{20}	.361	.019		.000
Random Effect	Variance Component	df	χ^2	p-value
u_{0j}	29.388	37	1188.341	.000
u_{1j}	.014	37	718.819	.000
u_{2j}	.013	37	817.089	.000
r_{ij}	67.343			

4.4 Model with Non-randomly Varying Slopes

Since mathematics view and mathematics thinking can influence mathematics cognition in the random coefficients regression model, NRI of countries in level-2 are considerd to be the predictor for advanced model. As to the predictive variable NRI in level-2, the equations are as follows.

Level-1:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + r_{ij} \quad (13)$$

Level-2:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j} \quad (14)$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j \quad (15)$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}W_j \quad (16)$$

X_{1ij} is mathematics view and X_{2ij} is mathematics thinking. W_j is NRI. The analysis result is depicted in Table 5. The coefficient γ_{01} shows that NRI provides positive effects on mathematics cognition. Consequently, the more NRI countries have, the more mathematics cognition their students will have. The coefficient γ_{11} shows that the influence of mathematics view on mathematics cognition varies with NRI of countries. Moreover, as NRI of countries increases, the influence of mathematics view on mathematics cognition will reduce. However, the influence of mathematics thinking on mathematics cognition does not vary with NRI.

Table 5. The Analysis of Model with Non-randomly Varying Slopes

Fixed Effect	Coefficient	Standard Error		p-value
γ_{00}	-9.240	2.385		
γ_{01}	2.830	.548		.001
γ_{10}	.191	.030		.000
γ_{11}	-.031	.007		.000
γ_{20}	.338	.024		.000
γ_{21}	.006	.005		.307
Random Effect	Variance Component	df	χ^2	p-value
u_{0j}	4.088	36	11961.111	.000
r_{ij}	67.801			

5 Conclusions

Based on the findings of this research, it shows that mathematics cognition among countries is different. NRI is the effective predictor of mathematics cognition. In addition, the higher mathematics view and mathematics thinking the students have, the higher mathematics cognition they have. The influence of mathematics view on mathematics cognition vary with NRI of countries.

The exploratory study of HLM analysis of PISA 2003 database demonstrates basic information about international assessment in this study. The step-by-step precedures for data analysis are followed so that the structural relationship of students and countries can be clarified [10]. Advanced research could aim at the findings of level-2 variables. On the other hand, three levels of HLM for PISA 2000, PISA 2003 and PISA 2006 is also a prospective study.

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