Application for Symbolic Analysis of Linear Circuits Including Switched Circuits

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Abstract: This paper presents a tool for symbolic analysis of electronic circuits – continuous-time circuits and periodically switched linear (PSL) circuits. Applied method of analysis is based on modeling of periodically switched networks by mixed s, z description. The method is implemented in program Maple™ in the form of Maple package named PraSCAn. Current version of the package provides all basic analysis of linear continuous-time circuits as well as of multiphase PSL circuits. The paper presents time domain analysis of idealized switched circuits in more detail. Described algorithm uses closed-form solution in time domain for continuous time which is implemented in the package. A system of www interface based on PraSCAn package for electronic circuit analysis via internet is presented in the next part of the article. Some results are shown in the end of the paper.

Key–Words: SC circuits, SI circuits, Maple, Internet Application

1 Introduction

Analysis of electric circuits is necessary not only for computing of circuit properties but also understanding their principles. To simplify development of synthesis and optimization methods for example in filter design, it is useful to have a powerful tool for symbolic analysis of circuit parameters. Modern systems use besides continuous-time circuits also periodically switched circuits – switched capacitors (SC) or switched currents (SI). It is advantageous to have a tool capable to analyze both type of circuits. In addition it is optimal, when such a tool is combined with suitable mathematical software, able to do symbolic computations. From this point of view, mathematical software Maple was found as a suitable environment for solving this task. We have designed special library of functions named PraSCAn (Prague Switched Circuits Analyzer) enables symbolic and semisymbolic analysis of continuous-time as well switched linear circuits in Maple.

The widely used programs for circuit analysis depends on local software installation and some of them require the user to know syntax of many commands as in case of PraSCAn package. It was the reason for development of a www user interface to the program – Maple with the package. These programs must enable batch processing which is necessary condition for our solution. The interface is based on PHP scripts and mediates communication between user and the program – creates input to processing program from user requests and prepares outputs to user from processing program [7]. The mentioned system has been already used for creation of www pages for electric filter design through internet, see [6, 10].

2 PraSCAn Package

PraSCAn package is a library of functions for MAPLE which facilitates the symbolic analysis of electric circuits [8]. The package goes from SCSyrup package [4, 5] for frequency analysis idealized SC and SI circuits. The SCSyrup package was completely rewritten – analysis of continuous-time circuit was implemented as well as functions for real switched circuit (including resistors of switches and other nonidealities) [9]. The implementation of frequency domain analysis of two-phase PSL circuit was carried out according to algorithm demonstrated in [3]. The input syntax for circuit description is near the same as in Spice programs including independent sources which have been also newly implemented. The time domain analysis of idealized switched circuits has been now implemented according to the following description.

2.1 Time domain analysis of idealized swithed circuits

The analysis starts from the matrix description of the circuit. There is used modified nodal equations based on following set of equations

$$\begin{aligned}\mathbf{W}_1(n) &= \mathbf{T}_{11}\mathbf{X}_1(n) - \mathbf{T}_{1k}\mathbf{X}_k(n-1),\\ \mathbf{W}_2(n) &= \mathbf{T}_{21}\mathbf{X}_1(n) - \mathbf{T}_{22}\mathbf{X}_2(n),\\ \mathbf{W}_3(n) &= \mathbf{T}_{32}\mathbf{X}_2(n) - \mathbf{T}_{33}\mathbf{X}_3(n),\\ &\vdots\\ \mathbf{W}_k(n) &= \mathbf{T}_{kk-1}\mathbf{V}_{k-1}(n) - \mathbf{T}_{kk}\mathbf{X}_k(n),\end{aligned} \tag{1}$$

where $\mathbf{W}_1$ is vector input sources (voltages, currents or charge respectively) , $\mathbf{X}_1$ is vector of unknown variables (voltages or current), $\mathbf{T}_{ii}$ matrix describes switched circuit in i-th phase and $\mathbf{T}_{ii-1}$ is matrix of energy transfer from phase $i-1$ to phase i. k is number of phases and n is discrete variable. These equation can be written in matrix form and we get after applying the $\mathcal{Z}$ transform formula 2.

$$\begin{bmatrix}\mathbf{W}_1(z)\\ \mathbf{W}_2(z)\\ \vdots\\ \mathbf{W}_k(z)\end{bmatrix} = \begin{bmatrix}\mathbf{T}_{11} & \dots & 0 & -\mathbf{T}_{1k}\\ -\mathbf{T}_{21} & \mathbf{T}_{22} & \dots & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & \dots & -\mathbf{T}_{kk-1} & z\mathbf{T}_{kk}\end{bmatrix} \begin{bmatrix}\mathbf{X}_1(z)\\ \mathbf{X}_2(z)\\ \vdots\\ \frac{\mathbf{X}_k(z)}{z}\end{bmatrix} \tag{2}$$

The whole matrix **T** is then decomposed into its LU factors and the vector **X** is found by forward and back substitution [1]. Particular transfer functions $F_{WX\phi P}(z)$ can be easy computed, whereas W labels index of input source, X labels index of output variable, ϕ labels phase of excitation and P phase where output variable is computed.

PraSCAn takes into account three types of exciting signals:

1. original time function of the source,
2. time function modified by Sample&Hold circuit held over the switching phase (SH) and
3. time function modified by Sample&Hold circuit held over the whole switching period (SHT).

Now the relevant sequences of samples for each input source are constructed according to concrete exciting types and phase where the particular source is active. $\mathcal{Z}$ transform of the each sequences of samples (sources) $S_{W\phi}(z)$ is then computed. Sample sequences of output signals can be computed by formula 3, in case that input source is modified by SH or SHT circuit (item 2 or 3).

$$Os_{XP}(n) = \mathcal{Z}^{-1}\left\{\sum_{\phi=1}^{k}\sum_{W} F_{WX\phi P}(z) S_{W\phi}(z)\right\}, \tag{3}$$

where Os_{XP} is output signal sequence for X variable (computed nodal voltage or current) in phase P.

Output signal time function $O_{XP}(t)$ we get by substitution $n = \text{floor}(tf_c)$ to formula (3), where floor is function which returns greatest integer less than or equal to the argument, t is time and f_c is clock frequency.

The situation is more difficult when the original signal is applied (item 1) in case that input signal is changing in process of phase time. The input signal can leaks directly to the output and then the output is also changing in process of phase time. It can be derived that it comes on for transfer function $F_{WX\phi P}(z)$ which is possible to express in following form

$$F_{WX\phi P}(z) = K_{WX\phi P} + \frac{\text{rem}_{WX\phi P}(z)}{D_{F_{WX\phi P}}(z)} \qquad \forall\, \phi = P, \tag{4}$$

where $K_{WX\phi P}$ is quotient of the transfer function which is constant independent on z, $\text{rem}_{WX\phi P}(z)$ is the reminder function and where $D_{F_{WX\phi P}}(z)$ is the denominator of the transfer function $F_{WX\phi P}(z)$.

Output signal function is then given by two parts – by part $O_{F_{XP}}(t)$ from the discrete transfer function and by part $O_{L_{XP}}(t)$ given by direct way due to leakage of input signal.

$$O_{XP}(t) = O_{F_{XP}}(t) + O_{L_{XP}}(t) \tag{5}$$

The part $O_{F_{XP}}(t)$ for given variable X (voltage or current) and output phase P can be computed using formula (3), where transfer function $F_{WX\phi P}$ given by (4) is replaced by its second part only, i.e. without quotient $K_{WX\phi P}$. The part $O_{L_{XP}}(t)$ caused by leakage of input signal can be expressed as

$$O_{L_{XP}}(t) = \sum_{W} K_{WX\phi P}\, s_W(t) \qquad \forall\, \phi = P, \tag{6}$$

where $s_W(t)$ is time function of corresponding input source.

The presented formulas were implemented into PraSCAn package in mathematical program Maple. Elementary SC network with bilinear resistor illustrated on figure 1 was chosen first as an example for above described method presentation.

The circuit is excited by sinusoidal input voltage $v_i(t) = \sin(2\pi t)$, clock frequency was $f_c = 10$ and capacitor values was $C_1 = 0.1$, $C_2 = 1$. The time

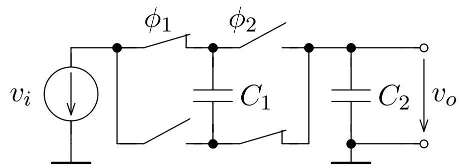

Figure 1: Elementary SC circuit.

response output voltage v_o was calculated using presented method by means of PraSCAn package. Transfer function F_{io11} and F_{io21} – from input in phase 1 and 2 to output in phase 1, were obtained in that form

$$F_{io11} = \frac{11z + 9}{121z - 81}, \quad F_{io21} = \frac{20}{121z - 81}.$$

Formulas for input signal sequence in phase 1 and 2 ($v_{i1}(n)$ and $v_{i2}(n)$) are given by

$$v_{i1}(n) = v_i\big((n{+}1/2)/f_c\big), \quad v_{i2}(n) = v_i\big((n{+}1)/f_c\big)$$

and their $\mathcal{Z}$ transform by

$$S_{i1}(z) = \frac{\cos(1.257)\, z\,(z+1)}{z^2 - 2\,z\cos(0.6284) + 1},$$
$$S_{i2}(z) = \frac{\cos(0.9426)\, z^2}{z^2 - 2\,z\cos(0.6284) + 1}.$$

The F_{io11} transfer function can be expressed in form (4), i.e. input signal leakes directly to output signal with transfer 1/11.

$$F_{io11} = \frac{1}{11} + \frac{180}{11(121z - 81)}$$

$\mathcal{Z}$ transform of discrete part of output signal in phase 1 can be calculated from as

$$O_{F_{o1}}(z) = -\frac{184.9z^2 + 55.62z}{(-z^2 + 1.618z - 1)(121z - 81)}.$$

The result in first phase is expressed by following formula. It is evident that analytical time function includes three parts.

$$O_{o1}(t) = v_{o1}(t) = \Big(0.3693\, 0.6694^{\mathrm{floor}(10t)} + \\ + 0.4914 \sin\big(0.6283\, \mathrm{floor}(10t) - 0.8505\big) + \\ + 0.09091 \sin(6.283t)\Big)\, \mathrm{sp}(t) \quad (7)$$

The first part is a transient component which goes off, the second part is the main (sinusoidal) steady state part given by discrete signal $O_{F_{o1}}$ and the third part is the direct leakage of input signal ($1/11v_i$). Coefficient $\mathrm{sp}(t) = \mathrm{floor}(10t) - \mathrm{floor}(10t + 0.5) + 1$ is function for phase selection. Analysis of more complex example is presented in section 4.

3 Internet Application

The application is based on www interface which utilizes www client-server conception. The computation and interface programs run on the server and a user uses an arbitrary graphic client i.e. standard www browser (Internet Explorer, Mozilla Firefox, Opera, etc.) for results displaying only.

The server runs under operation system Linux. The analysis of required circuit is solved using PraSCan package in Maple program. It runs using batch-processing which is necessary for utilization in the interface where the programs are called by the PHP scripts [7]. According to client requests the results are presented by dynamically created www pages. These pages are provided to the client by means of HTTP server Apache. The described application of circuit analysis was realized according this model.

Input requests are inserted in forms in www browser. The program in JavaScript tests the validity of these requests before sending them to the server where they are tested, too. Input files for Maple are generated from input requests by PHP scripts and results are saved in separated files. The PHP scripts process these files and create the structure of dynamic www pages, which are sent to the client. It is needed to solve many other problems, for example, to distinguishing simultaneously connected users, deleting temporary files and directories, etc. These tasks are solved by cookies and session variables (PHP). Next programs are used for additional functions – GnuPlot program is used for graphs droving and typographical system LaTeX for protocol of analysis creation.

The application uses near all facilities of PraSCAn package in Maple program. Operating of the interface is very easy. It is interactive system supplemented by help pages in addition. The user can use the interface without any manual or study of syntax. The analysis can be very easy created and modified.

4 Obtained Results

Operation of the above described system – www application for electric circuit analysis is illustrated on following figures. These figures correspond to the screen copies of dynamic www pages, which have been generated during of particular circuit analysis. Figure 3 shows page for submission of circuit description for analysis of circuit from figure 2. A third-order elliptic SC lowpass ladder filter has been entered in this case [2]. Description of circuit topology is entered in text form. However it is not necessary to know syntax of particular elements. It is possible to use interactive table-based assignment for entering any supported element.

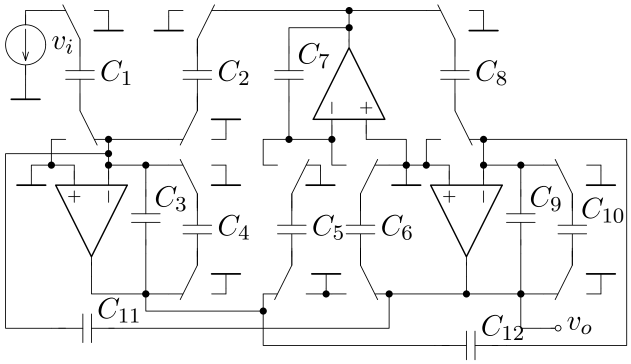

Figure 2: A third-order elliptic SC lowpass ladder filter.

Next figure 4 shows results of transient analysis of the circuit excited by sinusoidal signal. In that case time function of output voltage was displayed. The capacitor values was normalized so the time axis is normalized too (input signal frequency is 1). One can see that input signal leakes directly to output signal too.

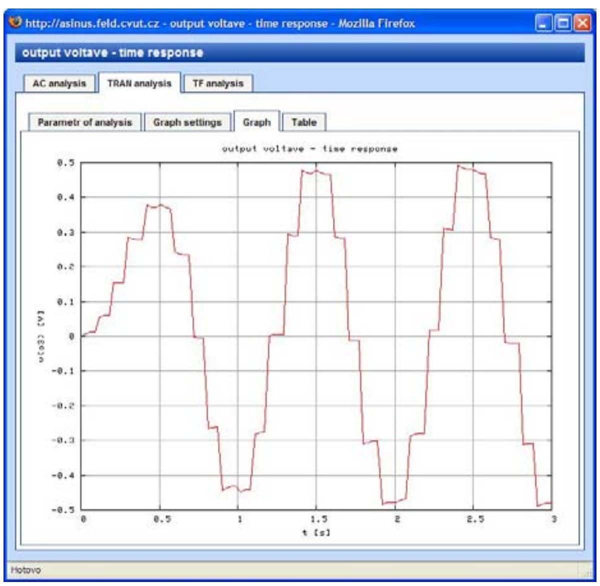

Figure 4: Time response of the SC lowpass filter from figure 2.

5 Conclusion

The PraSCAn package represents an universal tool for symbolic and semisymbolic analysis of continuous and discrete-time linearized circuits. Time responses are calculated with respect to real input signal character, i.e. including so called leakage effect. The PraSCAn package is being further developed to enable analysis of nonlinear circuits including electronic elements like diodes, transistors etc. It is necessary to finalize analysis of PSL circuits, especially circuits with switched currents.

The www application was created for the program to enable analysis of electric and electronic circuits for wide range of users. The interface of the application is based on www (client-server conception). The computation and interface programs run on the server and a user uses an arbitrary graphic client i.e. standard www browser (Internet Explorer, Mozilla Firefox, Opera etc.) only for displaying results. The aim of the interface is not to create application inclusive all facilities of original computation program – Maple with PraSCAn package for symbolic analysis of electric circuits. The interface should help the users to make analysis of electric and electronic circuits easy without any program installation and without learning of any command syntax. The interface is interactive and intuitive. This analysis is powered by Maple and its utilization is restricted by licence for this program.

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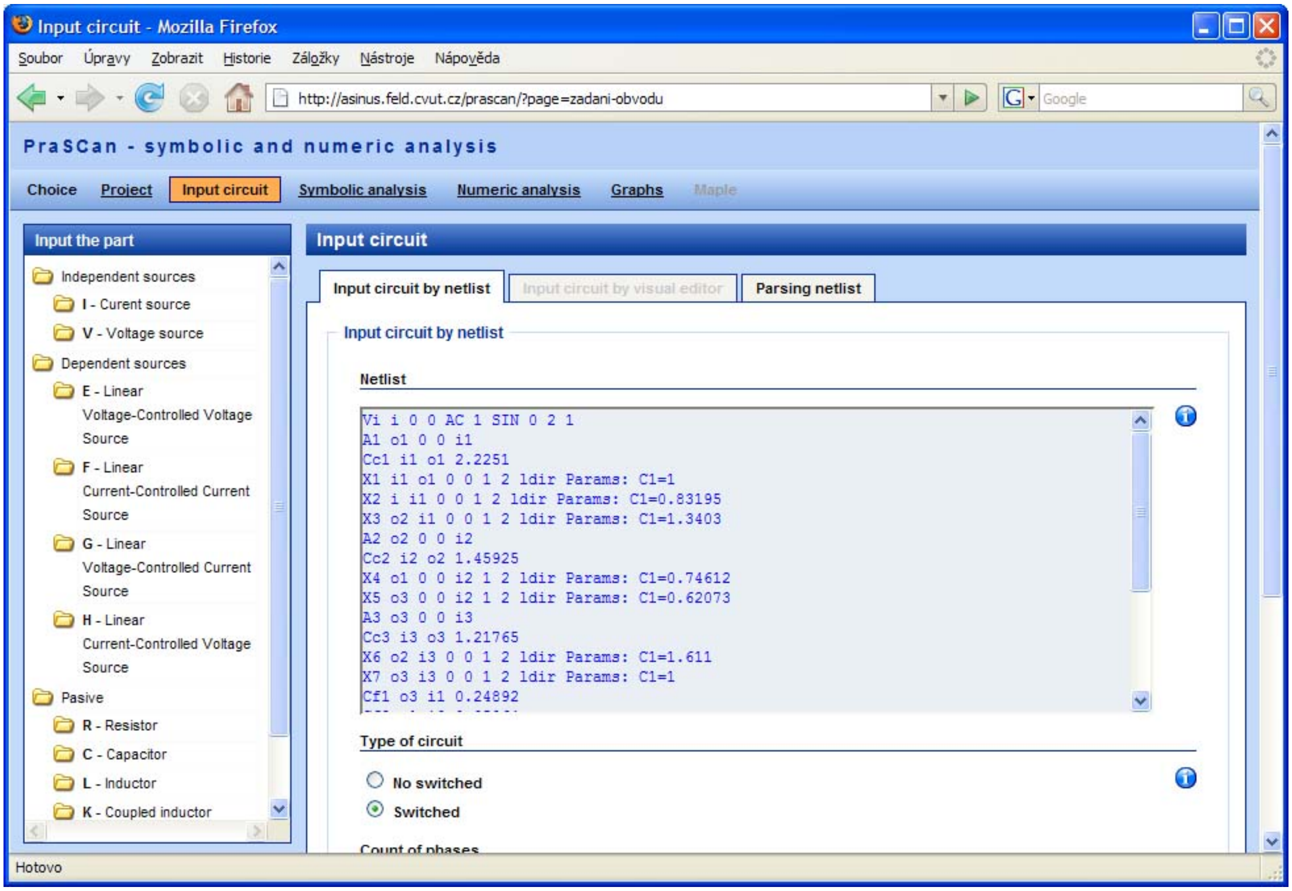

Figure 3: Page for submission of circuit description for the analysis.

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