A Numerical Analysis of the Loading Capacity of an Elbow

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Abstract: - The elbow under internal pressure represents a common part in many industrial types of equipment. Often an elbow, loaded with internal pressure, is supposed to hard working conditions. Sometimes these working conditions have just a result the fracture of material. If the material properties are known, by numerical simulation an analysis of the conditions that lead to fracture material can be carried out. Starting under these circumstances this work presents a numerical study of material fracture in the case of an elbow under internal pressure. By this way, an analysis of loading capacity of such a structure can be made. So, the experiments are not eliminated but they can be better prepared and the results can be more significant. The analysis made by the authors praises the influence of different factors upon the material fracture. The conclusions could be useful in engineering practice.

Key-Words: stress, strain, strain rate, material model, material fracture, elbow.

1. Introduction

The paper present a numerical study regarding to the behavior of an elbow under internal pressure. The elbow is modeled by finite elements of different types: SHELL and SOLID. The study started with an static analysis, the material being considered an elastic and isotropic one. Then, special material models were used, dynamic loading was considered. In the same conditions, an analysis of geometric parameters influence was performed.

2. Static and linear-elastic analysis

The static analysis was carried out for some geometric characteristics of an elbow, under linear-elastic behavior of the material. Different ratio of the curvature radius and internal diameter (R/D) were considered and also different ratio of the curvature radius and the thickness (R/T) were considered too. One of the finite element model is presented in the figure 1, where the ratio R/D=1.0 and the ratio R/T=10.0 where only finite elements SHELL were used. This type of finite element was used in two variants of node number: finite element with 4 nodes and with 8 nodes with 6 degree of freedom (DOF) per node.

An other finite element model is presented in the figure 2 where SOLID elements were used under the same conditions. This finite element could have 8, 12 or 20 nodes, each node having 3 degree of freedom per node.

In all the cases the results were about the same but the computer time for solving the problem was different, the shell elements being better from this point of view.

The models presented in this paper don't represent the best; these has had a much more nodes and elements; its size was about 1 mm, but such a model would have been unclear.

As we can see in the figures 1 and 2, only a half of the structure was considered because a symmetry plan exists. For those two finite element models, the maximum equivalent stress (von Mises) occurred just in the curvature on the inner part of this. Figures 3 and 4 present this aspect.

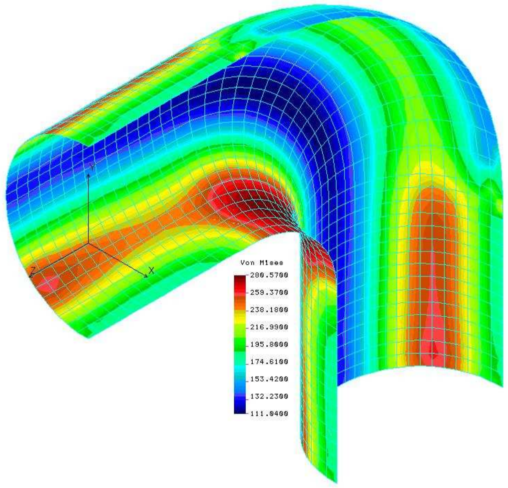

Fig. 3 Von Mises stresses in SHELL element model

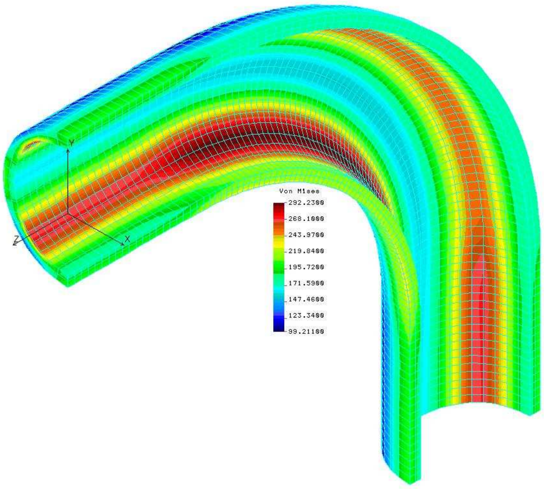

Fig. 4 Von Mises stresses in SOLID element model

In the table 1 the stresses comparatively are presented, corresponding to those two models, having R=20 mm, D=20 mm and thickness (T) of 2 mm..

Table 1 Stresses corresponding to those two FE models

	$\sigma_{r_{R1}}$	$\sigma_{t_{R1}}$	$\sigma_{r_{R2}}$	$\sigma_{t_{R2}}$	$\sigma_{ech_{III\max}}$	$\sigma_{ech_{V\max}}$
	N/mm²					
Shell	40.0	274.94	0.0	274.94	314.94	280.56
Solid	40.0	295.35	0.0	265.76	335.35	292.23
Error [%]	***0.0***	***6.9***	***0.0***	***3.4***	***6.1***	***3.9***

The error values for the models presented in the figure 1 and 2 (not being the best) recommend these models to be available for such analysis. SOLID elements allow getting the stress variation along the thickness of the wall.

3. Dynamic and linear-elastic analysis

This kind of analysis was performed in the same conditions of loading and geometrical parameters. The results seem to give a surprise, because the differences between stresses under dynamic loading are not so far to those under static loading . The table no. 2 presents these aspects. The analysis time for variation of the pressure from zero to nominal value (40 MPa) was 40 µs. Next to the linear-elastic behavior of the material, small displacements and small strains were considered. The aspect of the stress field is the same corresponding to the static analysis.

Table 2. Stresses in static and dynamic loading

Shell fe model	Maximum values of stresses [N/mm²]				
	σ_1	σ_2	σ_3	$\sigma_{ech_{III}}$	σ_{ech_V}
Static	316.09	141.25	1.15	314.94	280.56
Dynamic	324.72	140.45	1.18	323.54	286.52

4. Dynamic and nonlinear analysis

For performing of this type of analysis a biliniar plastic kinematic material model, strain rate dependent, was used. This material type is one of the most used material model, adopted for dynamic and nonlinear analysis, just in the case of impact problems. The elastic plastic with kinematic hardening model, was formulated by Krieg and Key and it is implemented in the most powerful software for structure dynamic nonlinear analysis. For an elastic-plastic material, a combination between isotropic and kinematic hardening can be obtained by varying the parameter β between 0 and 1. As a bilinear hardening plasticity model, this is characterized by the parameters σ_y (yield stress) and E_T (tangent modulus). The yield function is given by:

$$\sigma_y = \left[1+\left(\frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}}\right]\left(\sigma_0 + \beta E_p \varepsilon_p^{ef}\right) \qquad (1)$$

where σ_0 is the initial yield stress, ε_p^{ef} is the effective plastic strain, E_p is the plastic hardening modulus which is given by:

$$E_p = \frac{E_T E}{E - E_T}, \qquad (2)$$

β being the hardening parameter that can vary between 0 and 1 depending on plasticity type (0 for kinematic and 1 for isotropic respectively), C and P are strain rate parameters, known as Cowper-Symonds (from 1983 and Jones) coefficients.

Table 3 Comparative results

	Maximum values of stresses [N/mm^2]				
	σ_1	σ_2	σ_3	$\sigma_{ech_{III}}$	σ_{ech_V}
R/D=1.0	296.48	116.75	-0.43	296.91	259.92
R/D=1.5	239.46	130.57	-0.08	239.54	207.42
R/D=2.0	222.81	122.77	-0.02	222.83	194.04

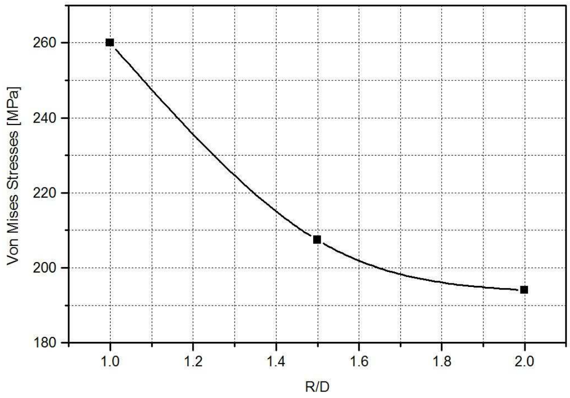

Fig. 5 Stress variation versus R/D

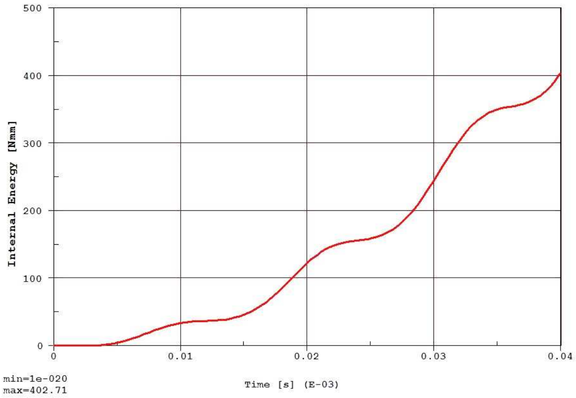

Fig. 6 Internal energy variation in time

For this model, the user has to specify the failure strain for which elements will be eliminated.

The values presented in the table 3 show the variation of the stresses versus ratio curvature radius per elbow diameter.

In the figure 5 this variation is graphically presented. So, around the value of 1.0 the variation is powerful and over R/D=1.5 this variation becomes a soft one.

Using the elastic plastic with kinematic hardening material model a lot of other information can be obtained.

For example, we can get the variation of the internal energy during the analysis time (figure 6) which praises the vibration phenomenon.

A very important aspect in such analysis is to find out the pressure which could produce a fracture of the material.

For this structure, for a ratio R/D=1.0 such an internal pressure has the value of 108 MPa. Figure 7 presents the elbow in the damage state.

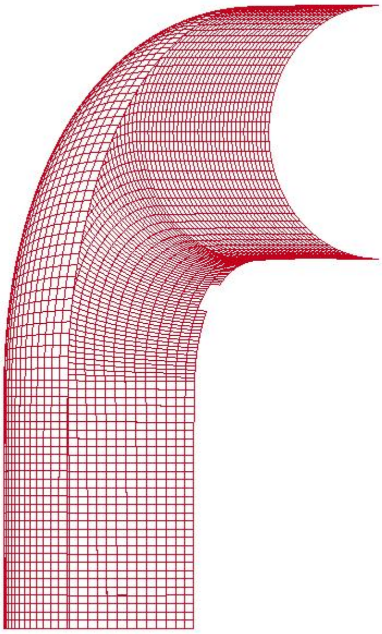

Fig. 7 The elbow in damage state

5. Conclusion

The elbow under internal pressure is a special structure often used in many industrial or manufactoring private activities. As we can see the using only the linear behavior of the material leads us to an inefficient using of the material.

A dynamic and nonlinear analysis is more fitted; if all the material characteristics are known, then we can make a good appreciation about the material and structure behavior. For this aim it is necessary to use the finite element analysis and a properly material model.

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