Power-Transformed-Measure and its Choquet Integral Regression Model

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Abstract: - Both the well known fuzzy measures, λ-measure and P-measure, have only one solution of measure function with no more choice. In this study, we propose the power-transformed-measures for any given fuzzy measure, those new measures with infinitely many solution of measure function can be chosen the best one to apply for improving the forecasting performances. A real data experiment by using a 5-fold cross-validation mean square error (MSE) is conducted. The performances of four Choquet integral regression models based onλ-measure, P-measure, power-transformed-measures of λ-measure, and power-transformed-measures of P-measure, respectively, ridge regression model, and the traditional multiple linear regression model are compared. Experimental results show that the performances of Choquet integral regression model based on the proposed power-transformed-measures outperform the performances of other models.

Key-Words: - λ -measure, P-measure, power-transformed-measure, Choquet integral, ridge regression, Choquet integral regression

1 Introduction

When interactions among independent variables exist in forecasting problems, the performance of the multiple linear regression models is poor. The traditional improved methods exploited the ridge regression models [1], we suggest using fuzzy integral regression models based on some fuzzy measures. The well known λ -measure [2], and P-measure [3] have only one solution of measure function with no more choice. In this study, we propose the power transformation fuzzy measures for any given fuzzy measure, those fuzzy measures with infinitely many solution of measure function can be chosen the best one to apply for improving the forecasting performances.

This paper was organized as followings. The multiple linear ridge regression was introduced in section 2. In section 3, fuzzy measures were reviewed. Our new fuzzy measures, the power transformation fuzzy measures for any given fuzzy measure, were described in section 4. Choquet integral regression model based a fuzzy measure was described in section 5. Experiment and result were described in section 6 and final section is for conclusions.

2 Multiple Linear Ridge Regression

Let $\underline{Y} = X\underline{\beta} + \underline{\varepsilon}$, $\underline{\varepsilon} \sim \mathrm{N}\left(\underline{0}, \sigma^2 I_n\right)$ be a multiple linear model, $\hat{\underline{\beta}} = (X'X)^{-1} X'Y$ be the estimated regression coefficient vector, and $\hat{\underline{\beta}}_k = (X'X + kI_n)^{-1} X'Y$ be the estimated ridge regression coefficient vector, Hoerl, Kenard and Baldwin [1] suggested

$$\hat{k} = \frac{n\hat{\sigma}^2}{\hat{\underline{\beta}}'\hat{\underline{\beta}}} \tag{1}$$

3 Fuzzy Measures

The well known fuzzy measures, λ-measure and P-measure, are reviewed as follows.

3.1 Fuzzy Measure [2, 4, 5]

A fuzzy measure μ on a finite set X is a set function $g_\mu : 2^X \to [0,1]$ satisfying the following axioms:

(i) $g_\mu(\phi) = 0, g_\mu(X) = 1$ (boundary conditions) (2)

(ii) $A \subseteq B \Rightarrow g_\mu(A) \le g_\mu(B)$ (monotonicity) (3)

3.2 Singleton Measure [6]

The singleton measure s of a fuzzy measure μ on a finite set X is a function $s : X \to [0,1]$ satisfying

$$s(x) = \mu(\{x\}), \forall x \in X. \tag{4}$$

3.3 λ- measure [5]

For given singleton measure s, let $\lambda \in (-1, \infty)$, a λ-measure, g_λ, is a fuzzy measure on a finite set X, $|X| = n$, satisfying:

(i) $A, B \in 2^X, A \cap B = \phi, A \cup B \ne X$
$$\Rightarrow g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B) \tag{5}$$

(ii) $$\prod_{i=1}^{n} \left[1 + \lambda s(x_i)\right] = \lambda + 1 > 0 \tag{6}$$

Note: Since $\lambda \in (-1, \infty)$, the above equation of variable λ with degree n has only one solution, and the solution is always not a closed form.

3.4 P- measure [6]

For given singleton measure s, a P-measure, g_P, is a fuzzy measure on a finite set X, satisfying:

(i) $g_P(\phi) = 0, g_P(X) = 1$ (7)

(ii) $\forall A \subset X, A \ne X, \ni g_P(A) = \max_{x \in A} s(x)$ (8)

Note: P-measure has only one solution, and the solution is a closed form, but it is not sensitive, since it only considers the maximum of singleton measures in event set.

4 Power Transformed Fuzzy Measure

The well known λ-measure and P-measure have only one solution of measure function with no more choice. Here, we propose the power transformation fuzzy measures for any given fuzzy measure, those fuzzy measures with infinitely many solution of measure function can be chosen the best one to apply for improving the forecasting performances.

4.1 Power-transformed Measure

For any $m \in (0,1]$, μ is a fuzzy measure on a finite set X, a power transformed measure μ^m on a finite set X is a set function $g_{\mu^m} : 2^X \to [0,1]$ satisfying:

$$g_{\mu^m}(A) = \left[g_\mu(A)\right]^m, \forall A \in X \tag{9}$$

4.2 Properties of Power–transformed-measure

[property 1]

For any $m \in (0,1]$, if μ is a fuzzy measure on a finite set X, then power-transformed- measure μ^m on a finite set X is also a fuzzy measure.

Note : A power-transformed- measure is also called **A** power-transformed fuzzy measure.

[property 2]

Let $m \in (0,1]$, μ is a fuzzy measure on a finite set X, if m=1, then power-transformed-measure μ^m on a finite set X is just the fuzzy measure μ.

In other word, any fuzzy measure is a special case of its power-transformed fuzzy measure.

[property 3]

If g_{μ^m} is the measure mapping of power-transformed-measure, μ^m, then g_{μ^m} is a decreasing function of m.

5 Choquet Integral Regression Model

The Choquet integral and its regression model are introduced as follows.

5.1 Choquet Integral [7]

Let μ be a fuzzy measure on a finite set X. The Choquet integral of $f : X \to R_+$ with respect to μ is denoted by

$$\int_C f d\mu = \sum_{j=1}^{n} \left[f\left(x_{(j)}\right) - f\left(x_{(j-1)}\right) \right] g_\mu\left(A_{(j)}\right) \qquad (10)$$

Where $f\left(x_{(0)}\right) = 0$, $f\left(x_{(j)}\right)$ indicates that the indices have been permuted so that

$$0 \le f\left(x_{(1)}\right) \le f\left(x_{(2)}\right) \le \ldots \le f\left(x_{(n)}\right) \qquad (11)$$

$$A_{(i)} = \left\{ x_{(i)}, x_{(i+1)}, \ldots, x_{(n)} \right\} \qquad (12)$$

5.2 Choquet Integral Regression Model [6]

Let $y_1, y_2, \ldots, y_N$ be global evaluations of N objects (or by N individuals), and

$$f_1\left(x_j\right), f_2\left(x_j\right), \ldots, f_N\left(x_j\right),\ j = 1, 2, \ldots, n,$$

be their evaluations of x_j,

where $f_i : X \to R_+$, $i = 1, 2, \ldots, N$

Let μ be a fuzzy measure, $\int f_i(x)\, dg_\mu$ be a Choquet integral of $f_i : X \to R_+$ with respect to μ

$$y_i = \alpha + \beta \int f_i(x) dg_\mu + e_i\ ,\ e_i \sim N\left(0, \sigma^2\right)\ ,\ \forall i \qquad (13)$$

$$\mu^* = \arg\min_{\mu} \left[\sum_{i=1}^{N} \left(y_i - \hat{\alpha} - \hat{\beta} \int f_i(x)\, dg_\mu \right)^2 \right], \qquad (14)$$

then $\hat{y}_i = \hat{\alpha}^* + \hat{\beta}^* \int f_i(x) dg_{\mu^*}$, $i = 1, 2, \ldots, N$ is called the optimal Choquet integral regression equation of μ, where

$$\hat{\beta}^* = S_{yf} / S_{ff} \qquad (15)$$

$$\hat{\alpha}^* = \frac{1}{N} \sum_{i=1}^{N} y_i - \hat{\beta}^* \frac{1}{N} \sum_{i=1}^{N} \int f_i(x)\, dg_{\mu^*}$$

$$S_{fy} = \frac{\sum_{i=1}^{N} \left[y_i - \frac{1}{N} \sum_{i=1}^{N} y_i \right] \left[\int f_i(x)\, dg_{\mu^*} - \frac{1}{N} \sum_{k=1}^{N} \int f_k(x) dg_{\mu^*} \right]}{N-1},$$

$$S_{ff} = \frac{\sum_{i=1}^{N} \left[\int f_i(x)\, dg_{\mu^*} - \frac{1}{N} \sum_{k=1}^{N} \int f_k(x) dg_{\mu^*} \right]^2}{N-1} \qquad (16)$$

6 Experiment and Result

A real data set with 230 samples from a junior high school in Taiwan including the independent variables, examination scores of three courses, and the dependent variable, the score of the Basic Competence Test of junior high school is applied to evaluate the performances of Choquet integral regression models based on λ-measure, P-measure, power-transformed-measures of λ-measure, and power-transformed-measures of P-measure, respectively, a ridge regression model, and a multiple linear regression model by using 5-fold cross validation method to compute the mean square error (MSE) of the dependent variable. The formula of MSE is

$$MSE = \frac{1}{N} \sum_{i=1}^{N} \left(y_i - \hat{y}_i \right)^2, \qquad (17)$$

Responding the ratio of the credit hour for three courses, all of the fuzzy measures about the independent variables are assigned the same singleton measures as follows,

$$\left\{ g_\mu(\{x_1\}), g_\mu(\{x_2\}), g_\mu(\{x_3\}) \right\} = \left\{ 0.5, 0.25, 0.25, \right\}\ ,\ \mu = \lambda, P, \lambda^m, P^m, \qquad (18)$$

Once the singleton measures of any fuzzy measure μ are given, all the event measures of μ can be found, and then, the Choquet integral based on μ and the Choquet integral regression equation based on μ also can be found. Both two kinds of new fuzzy measures, λ^m, P^m have infinitely many solutions of fuzzy measure can be selecting, we choose the values as $m = 1,\ 0.1,\ 0.5,\ 0.9$, and we can exploit the one with least MSE to compare with those of the rest four forecasting models.

The experimental results of all forecasting models are listed in Table 1. In Table 1, we found the Choquet integral regression model based on the new power-transformed-measures outperforms the others,

Table 1 MSE of regression models

5 Fold MSE		Power- transformed-measure			
		1	0.1	0.5	0.9
Choquet integral regression model with fuzzy measure	P	38.51	37.19	36.66	38.16
	λ	37.64	37.59	36.61	37.40
Ridge regression model		37.90			

Multiple linear regression model	38.05

7 Conclusion

In this paper, a novel fuzzy measure, L-measure, and Choquet integral regression models with fuzzy measure are proposed. An educational data experiment is conducted for comparing the performances of a ridge regression model, a multiple linear regression model, and the proposed Choquet integral regression model with L -measure, λ -measure, and P-measure. Experimental result shows that the Choquet integral regression models based on the proposed L-measure outperforms other forecasting models.

In future, we will apply the proposed Choquet integral regression model based on fuzzy measure to develop multiple classifier system.

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