The measurement of all the important 3-d parameters of a spinning football by optoelectronic methods

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Abstract: - The paper describes some initial experiments performed recently to investigate the limits of performance of an optoelectronic motion analysis system for measuring the flight characteristics of a spinning football. The football was projected with a given speed and spin using a special ball projection machine. Six retroreflective patches applied to the surface of the ball were used to reflect light back to the twelve digital cameras employed in this study. The measurement yielded time histories for all the following parameters, the spin angular momentum vector, the ball velocity vector and the displacement vector referred to rectangular x,y,z axes and at a sampling rate of 500Hz. The ball radius was computed continuously throughout the trials as a check and yielded a mean value of 106.7mm with a standard deviation of 0.9mm.

Key-Words: - football, tracking, three dimensional, spin, velocity, drag, trajectory

1 Introduction

Newton (1642-1727) realised that a spinning tennis ball deviates in flight and suggested that the surrounding air was responsible for the effect. Robins (1707-1751) explained the deflection of musket balls in terms of their spin. [1] Magnus (1802-70) showed that a rotating cylinder experienced a sideways force when mounted perpendicularly to a flow of air and the effect is now called the Magnus effect or sometimes the Magnus-Robins effect. [2] Prior to the 20th century, the explanation for the effect was that spinning balls carried some air around with them in the direction of spin. This means that the flow velocity on the side of the ball moving against (with) the airflow is decreased (increased) and Bernoulli's principle indicates that the pressure on that side of the ball would be increased (decreased). This pressure imbalance would lead to a force at right angles to both the spin axis and the velocity of the ball.

Now, however, it is understood that there is a thin boundary layer around the surface of the ball. At the surface of the ball, the molecules of air are held stationary so that the flow velocity relative to the surface is zero. Further away from the ball, the air flow is faster and so there is a viscous force caused by the velocity gradient existing in the air layers surrounding the ball. This viscous region contracts towards the ball itself as the velocity of the ball through the air is increased and outside this layer viscosity can be neglected. The drag on the football is determined by the behaviour of this boundary layer. As the air flows around the ball, it must experience an acceleration as it diverts around the ball and then decelerates again as the air departs at the rear of the ball. The viscosity in the boundary layer slows the air down and, eventually, at some point towards the rear of the ball, the flow separates from the surface. This means that an eddy can be created in the air flow at the rear of the ball. The air flow beyond the separation is irregular. Turbulent eddies form in a wake behind the ball and the kinetic energy in these eddies is derived from the slowing of the ball.

With a spinning football, on the side of the ball moving with the flow the viscous force carries the air farther around the ball before the flow separates. On the side of the ball moving against the flow the air is slowed more quickly and separation occurs closer to the front of the ball. The result of this is that the air leaving the ball is deflected sideways as shown in Figure 1.

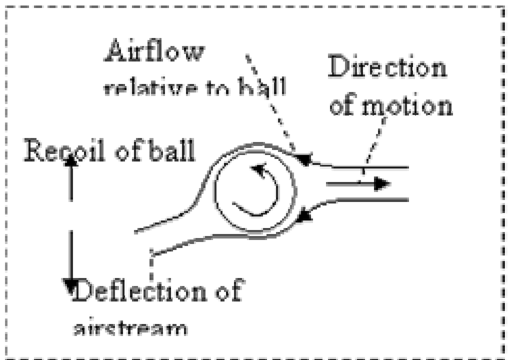

Figure 1. The different separation points on the two sides of a spinning football lead to a deflected airstream.

The sideways component of the airflow carries momentum in that direction, and since momentum is conserved, the ball must recoil with an equal but opposite momentum. This is the Magnus effect. The magnitude of the Magnus force can be written as

$$F_M = \tfrac{1}{2} C_s \rho A a \omega v \qquad (1)$$

where ρ is the density of air, A is the cross-sectional area of the ball, ω is the angular velocity, v is the velocity of the ball, a is its radius and C_s is an experimentally determined coefficient. Alternatively, the Magnus force can be written in terms of a lift coefficient C_L as

$$F_M = \tfrac{1}{2} C_L \rho A v^2 . \qquad (2)$$

This implies that the coefficients as defined here are related by

$$C_L = \frac{\omega a}{v} C_s . \qquad (3)$$

The spin on a football can be used in a direct free kick to accomplish two objectives:

(a) to swerve the ball around a defensive wall of opposing players and

(b) to deceive the opposing players, particularly the goalkeeper, that the ball is directed away from the goal when in fact it will curve in flight and finish up on target.

Wesson has produced a good account of the science of soccer, which includes a treatment of the banana kick, the Magnus effect and producing targeted flight with spin. [6] Assuming that the lateral force on the ball is constant, the ball's lateral deflection D at time t is proportional to t^2. Since the forward distance travelled by the ball is $x = vt$, the lateral displacement is proportional to x^2 and the overall trajectory is of the form of a parabola. Wesson has proposed that the deflection D after a flight of length L is given by

$$\frac{D}{L} = C_s \frac{N_{rots}}{52} \qquad (4)$$

assuming that the spin rate of the ball is constant, where N_{rots} is the number of rotations completed by the ball during its flight. [5] In the absence of any data taken on footballs, Wesson estimates the approximate value of the constant C_s to be about 0.5.

The influence of spin on a football has not received a great deal of experimental investigation until recently. Bray and Kerwin have used two digital video cameras and a DLT procedure to model the flight of a football in a realistic free kick situation. [3] Drag coefficients were measured as 0.25 to 0.30 and lift coefficients 0.23 to 0.29 with a mean ball velocity of 22.3 m.s^{-1}, although in this study neither the spin rate or the orientation of the spin axis was measured. Carré et al have obtained drag and lift coefficients for a football projected with a kicking machine such that the spin axis was horizontal.[4] It was found that the amount of drag and lift increased with imparted spin for tests carried out with the same launch velocity. For balls projected at 18 m.s^{-1}, the lift coefficient increased from 0.07 to 0.20 approximately as the imparted spin rose from 5 to 50 rad.s^{-1}. Carré et al have also shown that the seams on a football are of importance in determining the aerodynamic forces.[5] Recently, some work has been done on an image recognition system for measuring soccer ball spin characteristics. [7]

2 Problem Formulation

The motion analysis system (Vicon MX, Vicon, Oxford, UK) is able to record the positions of a reflective marker at intervals of $^1/_{500}$ second. By means of 12 cameras, it is possible to keep a marker in view for most of the time. The object of the investigation is to determine the position and velocity of the ball, together with its spin, in magnitude and direction, and follow their variation with time. To avoid interfering with the construction or the dynamics of the ball, the markers will necessarily follow the curvature of the surface. If there are n markers, then the data recorded will consist of $3n$ coordinates:

$x_1(t), y_1(t), z_1(t), \ldots x_n(t), y_n(t), z_n(t),$

for discrete values of t, separated by $\delta t = {}^1/_{500}$ s.

A symmetrical arrangement of markers should ensure that the centre of the ball has the coordinates:

$$x_c(t) = \left\{\sum x_i(t)\right\}/n, \quad y_c(t) = \left\{\sum y_i(t)\right\}/n, \quad z_c(t) = \left\{\sum z_i(t)\right\}/n \qquad .(5)$$

From these quantities, the velocity and acceleration of the ball can be determined by suitable polynomial fitting to small groups of points.

The position of the marker i relative to the centre of the ball is given by a vector $\mathbf{r}_i(t)$ with components

$$X_i(t) = x_i(t) - x_c(t), \quad Y_i(t) = y_i(t) - y_c(t), \quad Z_i(t) = z_i(t) - z_c(t). \qquad (6)$$

Squaring and summing these should give the square of the radius of the ball, R. The markers are of finite size and curvature, and the analysis system is designed to report coordinate positions at the centre of spherical markers. Consequently the recorded positions are some distance inside the ball, and the radius calculated in this way will be less than the true radius by the order of 1 – 2 cm. The calculated radius is useful, however, because its constancy can be used as a check on the quality of the data. The velocity of marker i relative to the centre is the vector $\dot{\mathbf{r}}_i(t)$, with components:

$$u_i(t) = \mathrm{d}X_i(t)/\mathrm{d}t, \quad v_i(t) = \mathrm{d}Y_i(t)/\mathrm{d}t, \quad w_i(t) = \mathrm{d}Z_i(t)/\mathrm{d}t, \qquad (7)$$

these quantities being evaluated by differentiating polynomial fits to the data.

Consider the vector product

$$\mathbf{\Omega}_i(t) = \mathbf{r}_i(t) \times \dot{\mathbf{r}}_i(t) . \qquad (8)$$

If $\mathbf{r}_i$ is divided into two components, a_i parallel with the spin axis and b_i perpendicular to the axis, then both of these components are perpendicular to $\dot{\mathbf{r}}_i(t)$ so that $\mathbf{\Omega}_i$ will have a component $a_i(t)\, \dot{\mathbf{r}}_i(t)$ perpendicular to the axis, and $b_i(t)\,\dot{\mathbf{r}}_i(t)$ parallel with it. The parallel component is equal to $b_i^2\omega$, where ω is the spin. In practice, resolving **r** along the axis is not possible, since the direction of the axis has not yet been found. If a symmetrical distribution of markers has been used, however, then in the vector sum $\sum\mathbf{\Omega}_i$, the perpendicular components will cancel out, leaving only the parallel components. The value of the sum is then

$$\sum\mathbf{\Omega}_i = \omega\sum b_i^2 = I\omega\,, \qquad (9)$$

where I is the moment of inertia of the set of markers about the axis (strictly, the moment of inertia of a set of unit masses placed at these positions).

For this to be a practical method of determining the spin, the arrangement of markers must have the property of possessing the same moment of inertia about any axis. This leads to the mathematical question of what arrangement of points will have this property. Clearly, it is not possible to arrange only one or two points in this way. Three points placed 90° apart would have this property, but do not have sufficient symmetry to allow the centre of the ball to be computed from equation (5). Four points placed at the corners of a regular tetrahedron, however, should have sufficient symmetry. Checking this arrangement carefully reveals that

$$I = {}^{8}/_{3}R^2, \qquad (10)$$

when evaluated about *any* axis.

A tetrahedral arrangement is therefore the minimum configuration. The design of a football makes it relatively easy to locate a tetrahedral set of points. In the design consisting of 12 pentagons and 20 hexagons, markers may be placed at the centres of four of the hexagons. In the older 18-panel design with cubic symmetry, four of the corners of the cube will have the correct relationship.

There may be advantages in acquiring redundant information, in case some data points are lost. Calculations show that five points cannot be arranged with the desired properties. Six points can be placed at points 90° apart (e.g. on the $\pm x$, $\pm y$, $\pm z$ axes). Eight points can be placed on the corners of a cube, although this arrangement is not unique, since two sets of four points could be placed on independently orientated tetrahedra (plus an infinite number of other arrangements). There are also an infinite number of configurations for seven points, but these do not possess a simple symmetry and so would be difficult to set up. An infinite number of different arrangements are possible for nine points, and for all larger numbers and, for some of these (12, 20, 30, for example), well-known symmetry patterns can be used. For any symmetric arrangement of n points, the moment of inertia is $I = {}^{2n}/_{3}R^2$.

The motion analysis system was used to track the position of a 32 panel, generic football (an Umbro ball with the standard hex-pent pattern) using six markers at orthogonal positions (disks of retroreflective tape). These are 2.5 cm in diameter and are arranged in orthogonal positions on the ball as shown in Figure 2. The tape was extremely thin and did not alter the weight of the ball or the air flow over its surface. This work builds upon previous work in which four ball markers were used. [8]

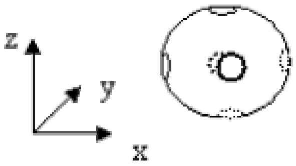

Figure 2. A football with six surface markers in an orthogonal configuration. The position vectors of the markers are measured with reference to an origin and axes fixed in the laboratory.

In the present investigation, it was proposed to investigate the accuracy with which a modern motion analysis system could obtain a complete description of the ball parameters, i.e. the ball's trajectory in three dimensional space, its spin vector also in three dimensions and its instantaneous velocity along its path.

3 Problem Solution

In the investigation reported here, a ball projection machine (Mechanical Engineering Department, Sheffield University) projected the football towards a net placed at one end of a motion analysis laboratory. The ball passed through four spinning rollers within the barrel of the machine and emerged with a given spin and speed. The ball passed through a capture volume approximately 5m in length. The experimental layout in the laboratory is illustrated in Figure 3.

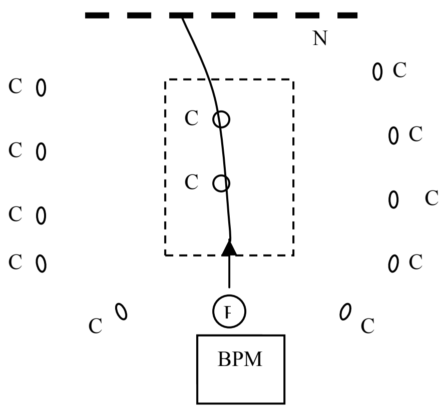

Figure 3. Experimental layout of laboratory in horizontal plane (not to scale). One possible ball trajectory is represented by the curved line on the figure. The origin of the motion analysis system lies at the centre of the capture volume. The z axis points vertically up and is at right angles to the plane of the page. C- camera (two overhead),

BPM – ball projection machine, B – ball, N – net. Dashed rectangle – capture volume.

The machine was set up to aim the ball at the net with various combinations of ball spin and speed. The capture volume was arranged to be about two metres tall, 5.1 m long and 1.9 m wide and was about 1 m above the floor of the laboratory so that the ball passed through the volume at a height of approximately 2.5–3 m above the floor of the laboratory en route from the origin to the target.

3.1 Trajectory in x-y plane

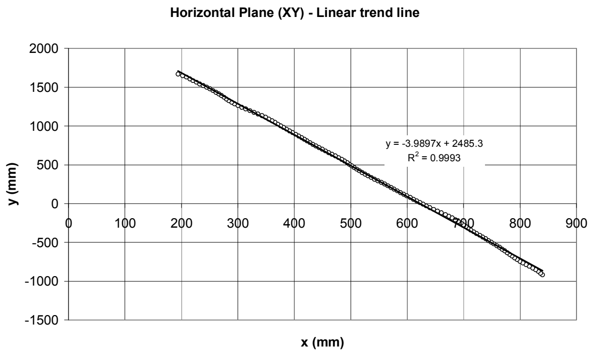

Figure 4. Experimental measurement of the movement of the ball in a horizontal plane with linear trend line fit.

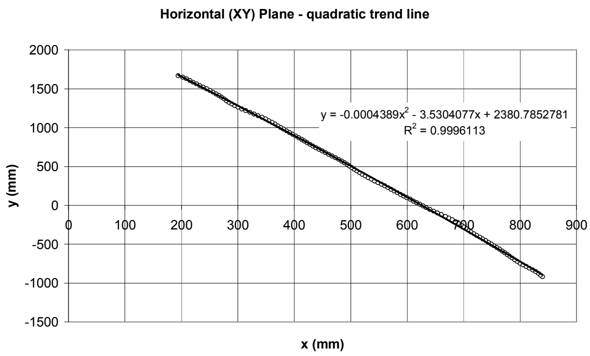

Figure 5. Experimental measurement of the movement of the ball in a horizontal plane with a quadratic trend line fit

3.2 Rates of spin along x, y and z axes

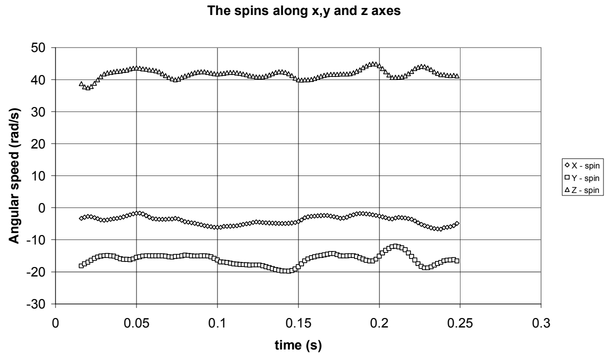

Figure 6. Experimental results for the spin angular velocity components along the x, y and z axes.

3.3 Velocities along x, y and z axes

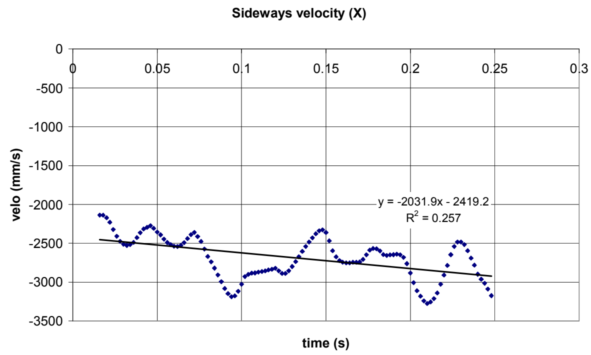

Figure 7. Experimental measurement of velocity of ball in the sideways direction

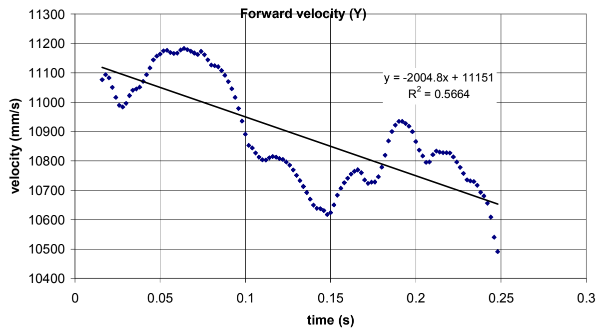

Figure 8. Experimental measurement of velocity of ball in forward direction

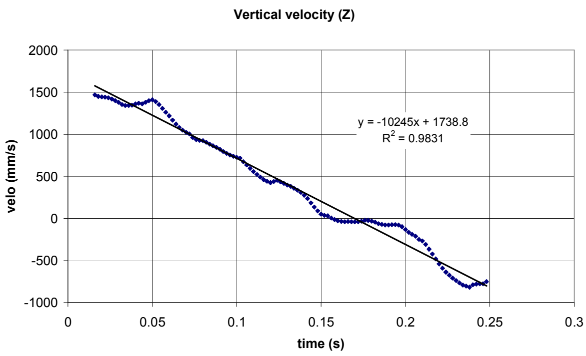

Figure 9. Experimental measurement of velocity of ball in vertical direction

3.4 The radius of the ball

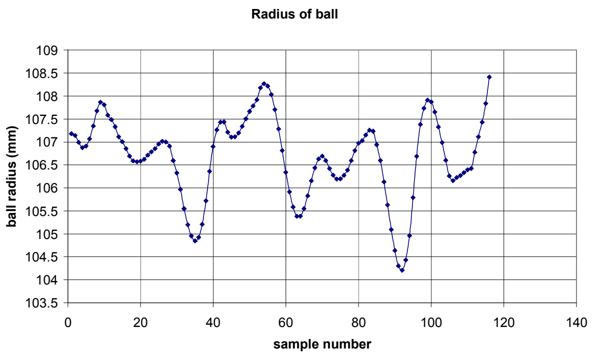

Figure 10. Experimental measurement of the radius of the ball.

3.5 The y – residuals

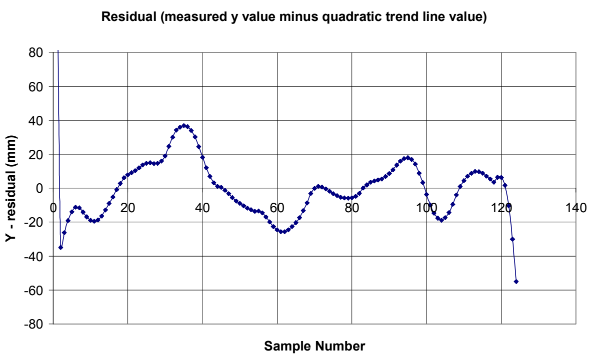

Figure11. Calculated residuals (the difference between the measured y values and the quadratic trend line y values)

3.6 Discussion

The data of section 3.4 (Figure 10) shows clearly that the radius of the ball has been found accurately by the analysis method (mean = 106.7mm, st. dev. = 0.9mm). In the absence of Magnus forces, the trajectory in the xy plane (section 3.1) would be expected to be a straight line. The scatterplots show that only a very marginal improvement was obtained by selecting a quadratic curve fit. The difference between the measured y values and those obtained from the quadratic trend line (the y residuals, section 3.5, Figure 11) are larger than expected, ranging from -25 to +36mm. It is currently not known if this due to a real fluctuating force on the ball due to air turbulence around the ball, or if it is an uncertainty in the measurement. More work is needed to answer this question. The downward acceleration obtained from section 3.3, Figure 9 is 10.245m/s^2, close to the acceleration due to gravity. The magnitude of the sideways velocity (Figure7) indicates an average acceleration of 2.03m/s^2, although the instantaneous velocity fluctuates considerably. If this force were due to the Magnus effect, the recoil force on the ball in the x-direction is approximately 0.8N at this velocity. The initial velocity of the ball was 11.39m/s.

4 Conclusion

The results show that the trajectory parameters of a football can, in principle, be measured extremely accurately by using the motion analysis system. However, there is one problem. This is that the non-spherical nature of the markers makes it more difficult so that the markers cannot be tracked continuously across the capture volume. This means that the trajectories are broken and labeling can be extremely difficult.

The preliminary data presented here is the result of software manipulation of the data after data capture. Gaps can be filled by a Woltring routine. However, this is not sufficient to recreate the ball completely. We have developed ways of recreating the missing marker coordinates even if they are not captured initially by the camera system. This is possible provided at least three markers are visible the whole time and the missing markers can be recreated by the software (Nexus 1.1, Vicon, Oxford, UK).

This research is still at an early stage. One way of obtaining better data is to increase the number of cameras to improve the coverage of the ball. Other methods are to develop software which will recognize non-spherical markers and which will use intelligent decision-making to fill gaps in ball trajectories.

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