Mode I fatigue crack growth evaluation at notches

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Abstract: - An analytical elastic-plastic model describing the fatigue life of components with elliptical notches under constant amplitude loading has been proposed. The calculation occurs by integrating a crack growth law from a starting crack size of micro-structural dimension till up to the total fracture of the component. Plasticity induced crack opening and closure effects are explicitly taken into account. Thereby, calculated opening load levels for cracks growing in notch affected areas have been found out to be in good agreement with corresponding experimental values determined from notched specimens made of two different metallic materials. Furthermore, the comparison of experimentally determined and calculated crack growth curves for specimens with central notches confirm the calculation accuracy of the model.

Key-Words: - Fatigue, Fracture Mechanics, Notches, Life prediction

1 Introduction

Stress concentrations such as notches are failure-critical locations that may significantly reduce the lifetime of components due to the cyclic plastic deformation that may arise in the notch area under service loading conditions. Thereby, short fatigue cracks initiated in the notch root propagate till to the total fracture of the component. Fracture Mechanics offers an efficient tool to describe the fatigue behaviour and predict the lifetime of engineering components under cyclic loading.

Based on fracture mechanics, Vormwald [1, 2] proposed an analytical elastic-plastic model to calculate the lifetime of notched specimens till up to the initiation of a crack at the specimen surface with length of approximately 1mm under uniaxial fatigue loading using the cyclic J-integral as a crack driving force parameter. Savaidis et al. [3, 4] extended this procedure to multiaxial-proportional loading cases. Further activities to extend the procedure to non-proportional loading cases are in progress. Dankert et al. [5, 6, 7] developed J-integrals for cracks in elliptically shaped notches and proposed a calculation procedure to describe crack growth behaviour in such notches. Recently, Brüning et al. [8] introduced some modifications into Dankert's et al. model enabling a more precise calculation of the effective ranges of the crack driving force, ΔJ_{eff}, as well as the applicability of the model to arbitrary notch situations.

In the following, details and results of a model are presented within which the tools of the elastic-plastic fracture mechanics are applied for describing the crack growth process. Special emphasis is given in the description of the opening and closure behaviour of cracks when growing in non-uniformly stressed areas affected by notches. The analytical procedure proposed is based on Newman's well-known relationships [9]. Therewith, effective ranges of stresses and strains can be determined and used to describe the effective crack driving force, i.e. the range of the J-integral, ΔJ_{eff}. The accuracy of the crack growth calculation procedure is demonstrated by comparing calculation with corresponding experimentally determined results of uniaxially loaded notched plates.

2 Fatigue life evaluation procedure

The origin of the fatigue life evaluation procedure described in this section goes back to the works of Vormwald [1], Savaidis et al. [2] and Dankert et al. [7]. According to it, the fatigue lifetime of a notched component can be evaluated as mode I growth of a crack with dimensions corresponding to the size of the microstructure of the material lying in the very notch root of the component. The fatigue lifetime till to the initiation of this micro-crack is assumed to be very small. Its dimensions can be easily calculated from the material's Woehler curve.

The effective range of the J-integral, ΔJ_{eff}, is being used as an appropriate parameter describing and controlling the crack propagation behaviour. The analytical relationships used to approximate the value of ΔJ are discussed in subsection 2.1. The evaluation of its effective range, ΔJ_{eff}, is performed taking into account the crack opening and closure behaviour during cyclic loading. Details to this topic are given in subsection 2.2

2.1 Crack driving force parameter

To describe the driving force of a fatigue crack growing in elastic-plastic deformed material, the effective range of the J-integral, ΔJ_{eff}, is applied. The general definition of ΔJ is given by

$$\Delta J = \int_G \left[\Delta W_\varepsilon dy - \Delta t_i \frac{\partial(\Delta u_i)}{\partial x} ds \right], \tag{1}$$

where

$$\Delta W_\varepsilon = \int \Delta\sigma_{ij} \, d\left(\Delta\varepsilon_{ij}\right). \tag{2}$$

Here the increments "Δ" of the stress, strain, traction, and displacement quantities designate the changes in these quantities from their respective reference values, whereas ΔJ and ΔW_ε are single-valued functions of their arguments.

The effective value of ΔJ is calculated taking into account the effective stress-strain state, i.e. the stress-strain values at which the crack opens and closes under cyclic loading.

Within the framework of a crack growth calculation it is essential to use approximation formulas for the computation of the J-integral. Thereby, the widely accepted approach of Shih et al. [10] is applied where the total value is composed of an elastic and a plastic part. Formulated in effective ranges, this approximation reads

$$\Delta J_{eff} = \Delta J_{eff,el} + \Delta J_{eff,pl} \tag{3}$$

For the elastic part there is

$$\Delta J_{eff,el} = \Delta K_{eff}^2 / E' \tag{4}$$

with the modulus $E' = E/(1-\nu^2)$ (E is Young's modulus, ν is the Poisson's ratio) assuming plane strain condition at the crack front. The stress intensity factor may be written as

$$\Delta K_{eff} = \Delta S_{eff} \sqrt{\pi \cdot a} \cdot Y_{el}, \tag{5}$$

where ΔS_{eff} is the effective range of the gross section nominal stress. The formulation of the geometry correction function, Y_{el}, follows the suggestion of Raju, Atluri and Newman [11] for (semi-)circular notches. The equations (6), (7) and (8) present Y_{el} for the semi-elliptical surface crack,

$$Y_{el} = \left[M_1 + M_2 \cdot (2c/t)^2 + M_3 \cdot (2c/t)^4 \right] \cdot x_1 \cdot x_2 \cdot F_{notch} \cdot f_{shah} \cdot f_j \cdot f_w \cdot \sqrt{c/a} \cdot 1/\sqrt{Q}, \tag{6}$$

for the quarter-elliptical surface crack,

$$Y_{el} = \left[M_1 + M_2 \cdot (c/t)^2 + M_3 \cdot (c/t)^4 \right] \cdot \xi_1 \cdot \xi_2 \cdot \xi_3 \cdot F_{notch} / q_{cc} \cdot f_{shah} \cdot f_j \cdot f_w \cdot \sqrt{c/a} \cdot 1/\sqrt{Q}, \tag{7}$$

for the through-thickness crack,

$$Y_{el} = F_{notch} \cdot f_{shah} \cdot f_w. \tag{8}$$

For the symmetric cases studied here $f_{shah} = 1$ holds. The finite boundary correction is taken as

$$f_w = \left[1 - 0.025 \cdot \left((\bar{a}+a)/w\right)^2 + 0.06\left((\bar{a}+a)/w\right)^4 \right] \Big/ \sqrt{\cos\left[(p/2) \cdot \left((\bar{a}+a)/w\right)\right]}. \tag{9}$$

First solutions of the notch crack problem have been published by Neuber [12]. These solutions have been generalized [7] giving the influence function for the consideration of the two-dimensional notch effect as

$$F_{notch} = \left(1 + \frac{0.1215}{(1+l)^d} \right) \cdot \left(1 + \left((K_{t,\infty} - 1)^{-2.2} + \left(\sqrt{C_0/l + 1} - 1 \right)^{-2.2} \right)^{-\frac{5}{11}} \right), \tag{10}$$

where $\lambda = a/\rho$ is the ratio of the crack depth, a, to the notch root radius, ρ, and the remaining functions are as follows:

$$K_{t,\infty} = 1 + 2\bar{a}/\bar{b}, \tag{11}$$

$$C_0 = \left(\frac{\bar{a}^2}{\bar{b}^2} \right) \Big/ \left(1 + 0.122 \cdot \left(1/\left(1 + \bar{b}/\bar{a}\right) \right)^{5/2} \right)^2, \tag{12}$$

$$\delta = \left(3.21 - 2.32 \cdot 0.5^{a/\rho} \right) / \left(K_{t,\infty} - 1 \right). \tag{13}$$

The functions M_1, M_2, M_3, Q, and f_φ are identical to and can therefore be taken from the solution of Raju, Atluri and Newman [11] for cracks in circular notches. The quarter-elliptical surface crack requires an additional function

$$q_{cc} = 1 + 0.05/(1+\lambda)^2. \tag{14}$$

Special modifications for considering non-symmetric cases as well as edge-notched instead of centre-notched cases can be found in [7].

The plastic part of the effective J-integral is approximated by

$$\Delta J_{eff,pl} = \Delta J_{eff,el} \cdot Y_{pl} = \Delta J_{eff,el} \cdot \zeta \cdot \left(\Delta S_{eff} / \Delta S_{ref} \right)^{1/n'-1}, \tag{15}$$

where ζ and ΔS_{ref} have been calibrated such that the results of an extensive finite element (FE) investigation are successfully reflected [6, 7]. The equations (6), (7) and (8) present the ζ-function for the semi-elliptical surface crack,

$$\zeta = 1 \text{ for } \varphi = 0; \; \zeta = a/(c \cdot 0.85) \text{ for } \varphi = \pi/2, \tag{16}$$

for the quarter-elliptical surface crack,

$$\zeta = 0.51 \cdot (a/c)^{0.073-0.916(a/c)} \text{ for } \varphi = 0;$$
$$\zeta = 1.05(a/c) \cdot 0.51(a/c)^{0.073-0.916(a/c)} \text{ for } \varphi = \pi/2 \quad (17)$$

for the through-thickness crack, $\zeta = 1$, (18)

$$\Delta S_{ref} = 0.0008^{n'} \cdot 2K' \cdot \left(\left(w - \left(\bar{a} + a\right)\right)/w\right)$$
$$= 0.0008^{n'} \cdot 2K' \cdot f_n \quad (19)$$

In equations (15) and (19) the material parameters n' and K' of the Ramberg-Osgood-type description of a hysteresis loop branch are used

$$\Delta\varepsilon/2 = \Delta\sigma/(2E) + (\Delta\sigma/2K')^{1/n'}. \quad (20)$$

This means that the material is assumed to be in the cyclically stabilized condition and that it obeys to the so-called Masing behavior, i.e. that the branches of the hysteresis loops can be constructed by doubling the cyclic stress strain curve with its parameters n' and K'.

2.2 Crack opening and closure

Usually available approximation formulae for the estimation of crack opening and closure levels are restricted to cracks growing in homogeneous (in the uncracked situation) stress and strain fields. The most prominent of these formulae going back to Newman [7] has been modified by Savaidis et al [13] in order to take into account the influence of the notch field. The algorithm has been used here without modifications.

The starting point is an approximate notch stress distribution in the uncracked situation in the crack line according to the Theory of Elasticity,

$$\Delta\sigma_{el}(x) = \Delta S/(f_n \cdot Z)\left(A \cdot s^6 + B \cdot s^4 + C \cdot s^2 + D\right)$$
$$= \Delta S/f_n \cdot K_t(x) \quad (21)$$

with x as the distance from the notch root and

$$A = -K_t^3 + 9K_t^2 - 19K_t + 3, \quad (22)$$

$$B = +K_t^3 + 5K_t^2 - 29K_t + 9, \quad (23)$$

$$C = +K_t^3 - K_t^2 - 13K_t - 11, \quad (24)$$

$$D = -(K_t + 1)^3, \quad (25)$$

$$Z = \left[\left(K_t - 3\right)s^2 + \left(K_t + 1\right)\right]^3, \quad (26)$$

$$s = (q - \sqrt{q^2 + (K_t + 1)(K_t - 3)})/(K_t - 3),\ K_t \neq 3;$$
$$s = 1/(x/\rho + 1) \text{ for } K_t = 3 \quad (27)$$

$$q = \left(4x/\rho + \left(K_t - 1\right)^2\right)/\left(K_t - 1\right), \quad (28)$$

$$K_t = \sigma_{el}(0)/(S/f_n). \quad (29)$$

Stresses and strains in the elastic-plastic region are now estimated for any position x by applying the approximation formula proposed by Seeger and Beste [14],

$$\Delta\varepsilon(x) = \left(\Delta\sigma(x)/E\right) \cdot \left[\left(\Delta\sigma_{el}(x)/\Delta\sigma(x)\right)^2 \cdot\right.$$
$$\left(2/u^2\right) \cdot \ln\left(\left(\cos u\right)^{-1}\right) + \left(1 - \left(\Delta\sigma_{el}(x)/\Delta\sigma(x)\right)\right)] \cdot \quad (30)$$
$$\Delta e/\left(\left(\Delta S/f_n\right)/E\right)$$

$$\text{with } u = (\pi/2) \cdot \left(\left(\Delta\sigma_{el}(x)/\Delta\sigma - 1\right)/\left(K_p - 1\right)\right). \quad (31)$$

The value K_p is the ratio of the load at fully plastic collapse to the one at the beginning of yielding in the notch root. This ratio has to be calculated under the assumption of an ideally elastic-ideally plastic material. The net section average strain Δe follows from equation (20) where $\Delta\varepsilon$ has to be replaced by Δe and $\Delta\sigma$ has to be replaced by $\Delta S/f_n$, here. Moreover, the mean stresses and strains of loops (as well as the maxima $\sigma_{max}(x)$ and $\varepsilon_{max}(x)$) for the material element located at the position x are evaluated according to conventional simulation techniques of the Local Strain Approach.

The opening stress for a crack with its tip at the position x is now calculated applying Newman's [9] formulae:

$$\frac{\sigma_{op}(x)}{\sigma_{max}(x)} = \begin{cases} A_0 + A_1 R_\sigma + A_2 R_\sigma^2 + A_3 R_\sigma^3, & \text{if } R_\sigma \geq 0 \\ A_0 + A_1 R_\sigma, & \text{if } R_\sigma < 0 \end{cases}, \quad (32)$$

$$\text{with} \quad A_0 = 0.535 \cdot \cos\left(\frac{\pi}{2} \frac{\sigma_{max}}{0.002^{n'} \cdot K'}\right), \quad (33)$$

$$A_1 = 0.344 \cdot \frac{\sigma_{max}}{0.002^{n'} \cdot K'}, \quad (34)$$

$$A_2 = 1 - A_0 - A_1 - A_3, \quad (35)$$

$$A_3 = 2 \cdot A_0 + A_1 - 1. \quad (36)$$

The corresponding crack opening strain $\varepsilon_{op}(x)$, the crack closure stress $\sigma_{cl}(x)$ and the crack closure strain $\varepsilon_{cl}(x)$ can be successively evaluated considering the Masing behaviour and the condition of identical crack opening and closure strains according to [1, 13]. The effective ranges of stress and strain ranges in the descending branch are,

$$\Delta\sigma_{eff} = \sigma_{max} - \sigma_{cl}, \quad (37)$$

$$\Delta\varepsilon_{eff} = \varepsilon_{max} - \varepsilon_{cl}. \quad (38)$$

Inserting these effective ranges into equation (30) and (31) leads to the effective ranges, $\Delta\sigma_{el,eff}(x)$ and ΔS_{eff}, which themselves have to be used in the equations (5) and (15).

3 Results

In this section the reliability of the main tools of the analytical model, i.e. the evaluation of the J-integral and the crack opening behavior, as well as the calculation of the end-result, i.e. the crack growth behavior in notched components is discussed and verified. For this, analytical results are compared with corresponding numerical and experimental data. Detailed description is given in the following subsections.

3.1 Calculation of J-values

Taking a corner crack in a uniaxially loaded notched plate as an example, Figure 1 shows analytically and numerically determined J values for various nominal stress values S_{br} [7].

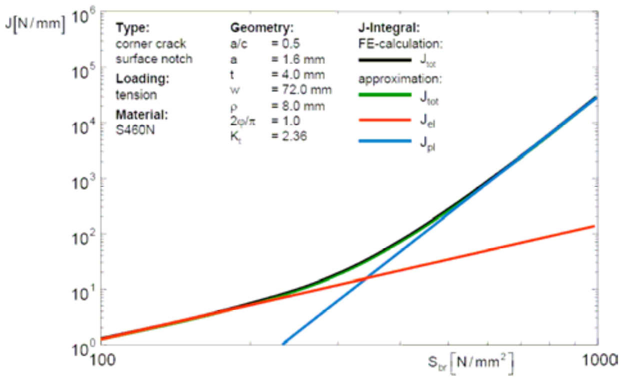

Fig. 1. Comparison of analytical and numerical determined J-values for a corner crack growing in a notched plate for various load levels

The elastic part of the J-integral, J_{el}, has been derived from the stress intensity factor, K, as proposed by Newman and Raju. The approximation of the plastic part, J_{pl}, is based on Dankert's [5, 6, 7] proposal for cracks in notches. The sum of the elastic and plastic terms yields the end-result for the approximation of the J-value. The corresponding numerical results have been obtained by finite element analysis.

An overall good agreement can be observed, confirming the accuracy of the analytical evaluation procedure. An extensive verification of the analytical calculation of J-values for various types and dimensions of cracks in notched plates has been presented by Dankert et al. [6, 7] and Brüning et al. [8].

3.2 Calculation of crack opening stresses

To explore the calculation accuracy in refer to the crack opening behavior, experimental results from thin plates made of an aluminum alloy providing a quite sharp central notch subjected to fully-reversed uniaxial loading with constant amplitudes have been exemplary used. The notch stress concentration factor amounts to K_t=3.4. The notch surfaces were mechanically polished to avoid roughness-influencing effects on the crack initiation.

Figure 2 shows the calculated and measured opening stress values S_{op} normalized by the applied load amplitude S_a versus the crack length a normalized by the notch radius ρ for two different load (nominal stress) values, S_a=61MPa and S_a=75 MPa, exemplary. The first load value, S_a=61 MPa, is slightly higher than the endurance limit of the plate. The deformations yielding in the notch area under this nominal stress are mainly elastic. On the other hand, the second load value, S_a=75 MPa, leads to a low-cycle fatigue lifetime with worth-mentioning plastic deformations in the notch area.

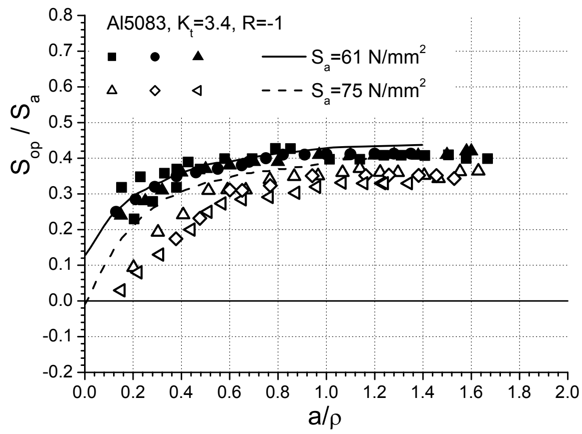

Fig. 2. Comparison of calculated and measured crack opening nominal stresses at two load levels

The experimental crack opening load values were determined measuring the local stiffness very near to the crack tip by means of small strain gages, as shown in Ref. [1, 15]. The analytical results were determined by means of the algorithms given in section 2.2

The experimental results show that the increasing of the load amplitude leads to lower Sop/Sa-values for certain crack lengths. This is due to the increase of the plastic deformation with increasing S_a. It can also be observed that the S_{op}/S_a-values increase as a/ρ increases. This is due to the descending distribution of the local stress and, therewith, the decreasing plastic deformation with increasing distance from the notch root. The crack front grows out of the highly stressed notch root into material areas with mainly elastic deformations. No significant changes of the S_{op}/S_a-values can be determined as the crack tip grows completely out of the notch area. Even at the higher load amplitude S_a=75 N/mm^2 applied here, the S_{op}/S_a-value almost stabilizes at a constant level. For a/ρ>1, the mechanisms controlling the crack propagation behavior follow well-known propagation laws of long through-thickness cracks in materials under uniform stress distribution.

The calculated results are qualitatively and quantitatively in good agreement with the measured ones, confirming the accuracy of the presented calculation procedure using Newman's equations [9].

3.3 Calculation of crack growth

Experimentally determined crack growth data from uniaxially, fully reversed loaded plates with central holes (stress concentration factor K_t=2.5) are used here exemplary to explore the calculation accuracy of the analytical model.

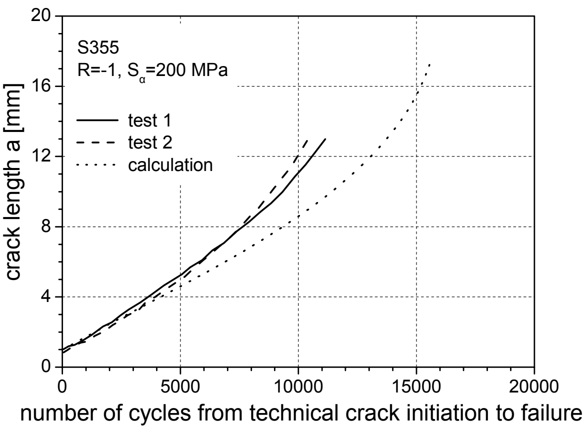

Fig. 3. Comparison of calculated with experimentally determined crack growth curves at S_a=200 MPa

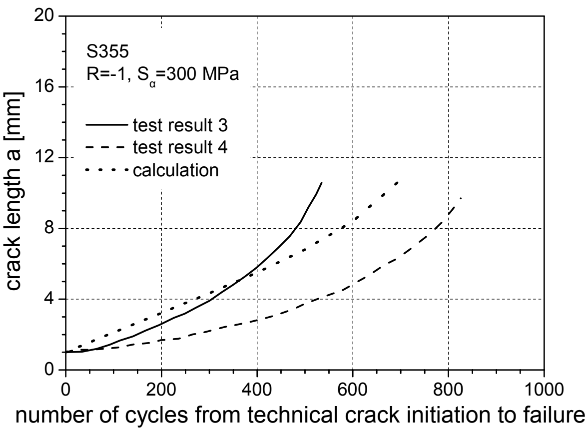

Fig. 4. Comparison of calculated with experimentally determined crack growth curves at S_a=300 MPa

Measured and calculated crack growth curves for two different nominal stress amplitudes, S_a=200 MPa and 300 MPa, are compared in Figures 3 and 4, respectively. The curves start at a crack depth of 1mm at the specimen's side from where the experimentally determined curves are reported. The number of applied cycles until this crack depth is reached is reported as crack initiation life.

The load value S_a=300 MPa yields significant plastic deformations at the very notch root, so that crack initiation and growth occur during a small number of loading cycles. The load value S_a=200 MPa leads to elastic-plastic deformation at the notch root, whereby the elastic part of the deformation is of approximately the same magnitude as the plastic one.

A satisfactory agreement between calculated and experimental results can be observed at both loading values confirming the evaluation accuracy of crack growth description of the analytical model.

4 Conclusion

An analytical crack growth evaluation model for fatigue analysis of notched components has been discussed and verified.

The model is based on elastic-plastic fracture mechanics and considers crack opening and closure behaviour explicitly. The effective range of the J-integral is used as crack tip parameter. Appropriate approximation formulae to determine the value of the J-integral for mode I cracks in notched areas have been derived from numerical investigations. The effective range is calculated using Newman's equations, which showed a satisfactory agreement with experimental crack opening stress results.

The calculation accuracy of the model has been verified based on experimental results determined from uniaxially loaded thin plates subjected to fully reversed fatigue loading with constant amplitudes. A satisfactory agreement between experimental and calculated results has been observed.

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