# **Nondominated Archiving Genetic Algorithm for Multi-objective Optimization of Time-Cost Trade-off**

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*Abstract: -* Construction planners are encountered with challenge of optimum resource utilization to compromise between different and usually conflicting aspects of projects. Completion date and cost if project are among the crucial aspects of each project. Such problems are difficult to solve because of not having unique solutions. For time-cost optimization problems, as combinatorial optimization problems one can apply heuristics or mathematical programming. In this paper, a new improved multi-objective Genetic algorithm is used to solve the time-cost trade-off optimization problem. An 18 activity example is analyzed to illustrate its capabilities in generating optimal/near optimal solutions.

*Key-Words: -* Multi-objective Optimization, Genetic Algorithm, Time-Cost Trade-off, Pareto Front

## **1 Introduction**

One of the crucial fields of construction project planning is the appropriate trade-off between time and cost. Construction planners are encountered with challenge of optimum resource utilization to compromise between aspects of project especially time and cost which are important facets of every project. The decisions made by planners is function of allocated resources including crew sizes, equipment and materials which can influence the time and cost of implementing project activities. Considering indirect costs which independent of direct costs increase as time goes by, the relation between time and cost will be more complicated. If more productive equipment or workers are used the duration of the activity will decrease and naturally, if durations of activities are compressed the cost of them will increase due to more resources allocated for their rapid accomplishment. Obviously some activities can be expedited at lower costs than others, therefore, when a choice is presented for expedition, it is more efficient to compress the cheaper ones prior to those with expensive costs.

This important trade-off is of interest for researchers and industry practitioners in academic and real field problems. Developing efficient and robust algorithms to solve highly complex time-costtradeoff problems is still a challenging job. On the other hand, practitioners and decision makers are willing to have a reliable trade off function between the time and cost of alternative methods of performing the project. Since time-cost optimization is a combinatorial optimization problem involving a finite number of feasible solutions, in principle, the optimal solution can be found by enumeration. However, as the major construction projects often involve numerous activities, it is almost impossible to evaluate all possible combinations within a short period of time and at a reasonable cost (Ng et al. 2000). A novel searching tool would then be worthwhile for comprehensive yet efficient TCO problem.

The existing techniques for the Time-Cost Trade-off Problem (TCTP) can be categorized as two distinct classes: heuristic methods, and mathematical programming approaches (Feng et al. 1997 and Li and Love 1997). The weaknesses of the heuristics and mathematical methods are widely documented in the literature (e.g. Zheng et al. 2002), but the major deficiency with most of the mathematical models is their inability to handle more than one objective. In addition, these methods often employ the hill climbing algorithms, which has only one randomly generated solution exposed to some kind of variation to create a better solution.

Efficient approach to a TCO problem requires a multi-objective optimization algorithm which allows for greater freedom in exploring possible solutions to reduce the likelihood of being trapped in local optima (Knowles et al. 2001). A time-cost trade-off problem is in fact a multi-objective optimization problem which selecting appropriate options for every activity to obtain the objective of time and cost is the goal. In this paper an improved multiobjective genetic algorithm will be used to solve the time-cost trade-off optimization problem.

## **2 Multi-Objective Genetic Algorithms (MOGAs)**

### **2.1 GAs**

The usual form of genetic algorithm was described by Goldberg (1989). A genetic algorithm is a search technique used in computing to find optimal or near optimal solutions to optimization and search problems, and is often abbreviated as GA. Genetic algorithms are classified as global search heuristics and particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover.

Genetic algorithms are implemented as a heuristic computer simulation approach in which a population of abstract chromosomes of candidate solutions to an optimization problem evolves toward better solutions. The evolution usually commences from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population based on their fitness, and modified by genetic operators to form a new population. The new population is then used in the next iteration of the algorithm.

#### **2.2 Multi-objective Optimization**

Many problems of the real-world are optimization of more than one objective function at the same time. The fact of optimizing several objectives simultaneously has made the problem solving more complicated in multi-objective optimization. The existence of many multi-objective problems in the real-world, their intrinsic complexity and the advantages of metaheuristic procedures to deal with them has strongly developed this research area in the last few years (Gandiblex et al, 2004; Goldberg 1987).

According to Zitzler et al. After the first studies on multi-objective optimization appeared in the mideighties (Schaffer,1984,1985; Fourman, 1985) several different evolutionary algorithm implementations were proposed in the years 1991- 1994(Kursawe,1991; Hajela and Lin,1992; Fonseca and Fleming 1993; Horn et al., 1994; Srinvas and Deb, 1994). Later, these approaches were successfully applied to various multi-objective optimization problems. In recent years, some researchers have investigated particular topics of evolutionary multi-objective search, while others have concentrated on developing new evolutionary techniques.

In the presence of various techniques for multiobjective optimization problem there has been little or no exhaustive comparison between these methods until 1999, 2000 when Zitzler et al. compared them systematically using six chosen test functions. As a result of this research it can be concluded that there is a clear performance gap between SPEA and NSGA and remaining algorithms. Elitism is an important factor which is used in mentioned highperformance algorithms. Herein an approach based on multi-objective genetic algorithm together with innovative genetic operators and elitism factor will be applied to time-cost trade-off problem.

#### **2.2.1 Pareto front**

The goal of multi-objective optimization problems is to find the best compromise between multiple and conflicting objectives. Usually there is more than one solution which optimizes simultaneously all the objectives and there is no distinct superiority between these solutions. Therefore we face with a set of nondominated solutions in these problems that is called Pareto optimal. Among the feasible solutions, a solution is identified as dominant if it is better than all other solutions in all of the considered objectives simultaneously. Among the feasible solutions, those belonging to Pareto front are known as nondominated solutions, while the remainder solutions are known as dominated. Since none of the Pareto set solutions is absolutely better than the other nondominated solutions, all of them are equally acceptable as regards the satisfaction of all the objectives. For a set of possible solutions Fig. 1 demonstrates the dominated and nondominated (Pareto front) solutions.



**Fig. 1.** Pareto Front and ranked solutions for a sample set of possible solutions

### **3 Problem Description and formulation**

Reaching the best set of nondominated solutions forms the goal of time-cost optimization problem. Strategies of performing activities determine the total time and cost of project. These strategies for each activity differ in resources utilization and they have an expected daily production and cost rate, as such we have few feasible strategies for each activity. Each combination will result in a specific finish date and total cost. As a result of numerous possible combinations of these allocated strategies there will be a large searching space. Project completion date is determined by scheduling the activities using the assigned activity durations. The project cost is equal to the sum of the cost of all the individual activities that make up the project. Other costs can easily be included in the project cost to make it more comprehensive if desired. For example, indirect and overhead costs can be expressed as a function of the project duration.

Number of the possible combinations in a project with *i* activities and *n* resource utilization options for each activity will be equal to  $n \times i$ . As a case in point, the total number of alternative combinations of time and cost for a project with only 18 activities and 4 possible resource utilization options for each activity will exceed 6 billion. Here arises the major challenge of construction planners; large search space and embedded optimal solution in the heart of this complexity. The present approach with a robust searching tool can be applied for such problems.

Multi-objective time-cost trade-off problem can be solved by three different methods. One seeks the satisfactory solution from the non-inferior solutions depending on the experiences and knowledge of decision-makers, whereas the determination of the non-inferior solutions is very difficult and complicated. The second converts the multiobjective problem to a single-objective problem, and then utilizing a single-objective optimization approach to find the satisfactory solution. The final one utilizes a multi-objective optimization approach to find the satisfactory solution. The approach proposed in this paper belongs to the latter one, for which not only provides the satisfactory solution, but also determines the nondominated set that is beneficial for the further decision-making process.

As mentioned, TCO problem mainly concentrates on selecting appropriate options for every activity to obtain the objective of time and cost of a project. The objective of time may be presented as:

$$
T = \max_{L_k \in L} \left[ \sum_{i \in L_k} t_i^{(k)} x_i^{(k)} \right] \tag{1}
$$

Where  $t_i^{(k)}$  represents the duration of activity i when performing the kth option; and  $x_i^{(k)}$  stands for the index variable of activity i when performing the kth

option. If  $x_i^{(k)} = 1$  then the activity i perform the kth option, while  $x_i^{(k)} = 0$  means not. The sum of index variables of all options should be equal to 1.  $L<sub>k</sub>$  means the activity sequence on the kth path, and  $L_k = \{i1k, i2k, ..., ink\}$  where ijk represents the sequence number of activity j on the kth path. *L* stands for the set of all paths of a network, and  $L = \{ L_k \mid k=1, 2, \ldots, m \}$ , where m symbolizes the number of all paths of a network.

The total cost of a project consists of two parts: direct cost and indirect cost. Direct cost is determined as the sum of direct cost of all activities within a project network. On the other hand, indirect cost is composed of the expenditure on management during project implementation, which depends heavily upon the project duration, i.e. the longer the duration, the higher the indirect cost.

In a real construction project, it is feasible to evaluate indirect cost per time unit to calculate the total cost. Subsequently, equation 2 can be forwarded to compute the total cost of a project.

$$
C = \sum_{i \in A} d c_i^{(k)} x_i^{(k)} + T \times i c_i^{(k)}
$$
 (2)

Where  $dc_i^{(k)}$  direct cost of activity i under the kth option, which equals to the quantities of the activity multiplied by its price;  $ic_i^{(k)}$  = indirect cost per time unit of activity under the kth option, which can be generated by experts through estimation or derived from division of the indirect cost of budget report according to contractual duration; and  $A =$  set of activities in a network.

## **4 Proposed Multi-Objective Genetic Algorithm**

As mentioned the firs step in the procedure of modeling the problem is formulation of the Gene and chromosome of the genetic algorithm. Fig. 2 illustrates the structure of the proposed gene and chromosome for a problem with N activities and k possible resource utilization option for each activity.



In fact each chromosome is a possible solution for the problem in which genes are representative of resource utilization options. The present model is implemented in the following steps:

Step 1: Determine problem parameters such as number of activities, precedence relationship of activities, available construction methods for each activity and their corresponding time and cost.

Step 2: Determine model parameters such as population size, number of generations, selection strategy, crossover rate and method, mutation rate and method.

Step 3: Generate initial solutions to establish the first population of the model. As mentioned each solution represents a possible combination of selected construction method for each activity.

Step 4: Calculate objective functions of project; total duration and total cost. Total duration will be calculated using Eq.1 and total cost of project will be calculated based on Eq.2.

Step 5: Calculate Pareto optimal rank for each solution in the set of generated solutions and transfer Pareto optimal (Rank 1) to the archive.

This approach classifies existing solutions to nondominated fronts based on dominance capability of answers. As such the population is ranked according to a dominance rule, and then each solution is assigned a fitness value based on its rank in the population. It is the intent of the model to minimize the time and cost of the project and therefore a lower rank corresponds to a better solution here. Fig. 1 illustrates the approach for a sample set of solutions. Thereafter set of solutions which is categorized as rank 1 will be transferred to an archive set.

In fact this is a vehicle for elitism approach. In single-objective genetic algorithms elitism is implemented surviving the best solution found so far to the next generation. Therefore in multi-objective problems all the nondominated answers in the archive are elite solutions. Using this point of view these solutions are stored and reintroduced to the population.

Step 6: select and reproduce based on the ranked solutions.

For this purpose following equations will be used to favorite answers with lower optimal ranks:

$$
p_i = f_i / \sum f_i
$$
 (3)

 $f_i = 1.3^{(Rank_{\max}-Rank_i)}$ 

Where  $P_i$  is the probability of selection and  $f_i$  is the fitness value calculated for each solution based on its rank (*Rank<sub>i</sub>*) and maximum found rank (*Rank<sub>max</sub>*) in the population.

Step 7: Create new child population using selected solutions and crossover operator as well as mutation.

Step 8: Transfer archived solutions to the new population.

In this step archived elite solutions which mentioned in step 4 will be reintroduced to the population.

Step 9: verify if the end condition is met or not. If not the same steps from step 4 will be performed for new generation.

Fig. 3. represents a sample flowchart of proposed Improved MOGA for time-cost optimization.



**Fig. 3**. Flowchart of Proposed Algorithm

## **5 Case Study**

In order to illustrate the concept and performance of the proposed algorithm, an 18 activity network configuration is used as a case study (Fig. 3). The example was originally introduced by Feng et al. (1997) to illustrate construction time-cost trade-off analysis. The total direct cost of the project is \$99,740 for project duration of 169 days. For this simple example, there is an average of 3.4 units of resource utilization options to construct each of the

18 activities, which produces more than 3.6 billion  $(i.e., 3.4<sup>18</sup>)$  possible combinations for delivering the entire project (Khaled El-Rayes, and Amr Kandil, 2005). Each of these possible combinations leads to a unique impact on project performance, and the main challenge here is to search this large solution space to find solutions that establish an optimal/near optimal and delicate balance among construction time and cost.



**Fig. 4**. Network Diagram of 18 Activity Case Example

Running the model using the mentioned data set resulted in selection of four nondominated solutions. To verify the capability and efficiency of proposed algorithm, it is compared with that of weighted method which was modeled by Zheng et al. (2005) for the same example. Zheng basically has used adaptive weighted method approach in his work. Moreover three modules which differ in selection and mutation methods are developed. The first one named module 1 is modeled with fixed mutation rate and Roulette wheel as a selection approach. Module 2 is designed with adaptive Mutation rate which is based on a formula that reduces its rate as generations go on and Pareto ranking for selection. Module 3 is implemented using adaptive mutation rate and Pareto ranking as selection strategy, furthermore niche formation is applied in this module. Niche formation by fitness sharing was first introduced by Goldberg (1989) to promote uniform sampling and maintain population diversity. Results reported by Zheng for all three mentioned modules are compared in Table. 1 with present proposed algorithm. It can be seen that the present model yields four nondominated solutions in early generation numbers as well as dominating all Nondominated solutions derived from three modules of weighted method(Table 1).

Furthermore the same example was analyzed without considering indirect cost which resulted in selection of 44 nondominated solutions as shown in Fig. 4.

		Module 1		Module 2		Module 3		Improved MOGA	
Generation No.		Time (days)	Cost $(\$)$	Time (days)	Cost (3)	Time (days)	Cost $(\$)$	Time (days)	Cost ( \$ )
100	$\mathbf{1}$	108	300820	106	283508	100	293720	100	283,320
	2	11	298120	112	276708	101	290520	101	279,820
	3	117	282420	<b>NA</b>	<b>NA</b>	110	280320	104	276,320
								110	271270
200	1	108	300820	105	288208	100	293720	100	283,320
	2	111	298120	106	281708	101	290220	101	279,820
	3	117	282420	112	276708	110	280320	104	276,320
	4	<b>NA</b>	<b>NA</b>	<b>NA</b>	<b>NA</b>	<b>NA</b>	<b>NA</b>	110	271270
300	1	108	300820	103	292308	100	293720	100	283,320
	2	111	298120	105	288208	101	290220	101	279,820
	3	117	282420	106	281708	104	286720	104	276,320
	4	<b>NA</b>	<b>NA</b>	112	276708	110	275720	110	271270
400	1	108	300820	103	287708	100	287720	100	283,320
	2	111	298120	106	281708	101	284020	101	279,820
	3	117	282120	112	276708	104	281520	104	276,320
	4	<b>NA</b>	<b>NA</b>	<b>NA</b>	<b>NA</b>	110	287720	110	271270
500	1	108	300820	102	290870	100	287720	100	283,320
	2	111	298120	103	286070	101	284020	101	279,820
	3	117	282420	106	281708	104	280020	104	276,320
	4	<b>NA</b>	<b>NA</b>	112	276708	110	273720	110	271270

**Table 1** Comparision of Pareto Front Generated by three modules(Zheng,2005) and Proposed Improved MOGA

## **6 Conclusion**

Construction time-cost trade-off problems are optimization problems in which existing techniques using heuristic and mathematical programming may not evaluated as efficient and /or as accurate as needed to locate the Pareto front in a real-life construction projects. Herein An Improved Nondominated Archiving Multi-Objective Genetic Algorithm together with approach of elitism is applied to time-cost trade-off problem. The efficiency of the proposed algorithm is verified by a medium-size project example which confirms the capability of model in generating Pareto optimal. Moreover, the model was implemented to optimize time-cost trade-off for the same example and compared to the results of MAWA approach (Zheng 2005) which validated the robust capabilities of the present model. The efficiency of the new algorithm is due to searching only a small fraction of the total search space. This approach provides an attractive alternative for the solution of the construction multiobjective optimization problems in comparison with commonly practiced metaheuristic algorithms.



**Fig. 5**. Time-Cost Trade-off for the Example with no Indirect Cost

References:

- [1] Daisy X. M. Zheng, S. Thomas Ng. (2005). "Stochastic Time-Cost Optimization Model Incorporating Fuzzy Sets Theory and Nonreplaceable Front." J. Construction. Eng. Manage., 131(2), 176-186.
- [2] Daisy X. M. Zheng, S. Thomas Ng, and Mohan M. Kumaraswamy. (2005). "Applying Pareto Ranking and Niche Formation to Genetic Algorithm-Based Multiobjective Time–Cost Optimization." J. Construction. Eng. Manage., 131(1), 81-91.
- [3] D.E. Goldberg, J.Richardson, Genetic Algorithms with Sharing for Multimodel Function Optimization, Proc. Second International Conference on Genetic Algorithms(ICGA'87), Hillslade, USA, 1987, pp.218-229
- [4] Feng, C. W., Liu, L., and Burns, S. A. (1997) "Using genetic algorithms to solve construction time-cost trade-off problems." J. Comput. Civ. Eng., 11\_3\_, 184–189.
- [5] Feng, C. W., Liu, L., and Burns, S. A. (2000) "Stochastic construction time-cost trade-off analysis" J. Comput. Civ. Eng., 11\_3\_, 184–189.
- [7] Hegazy, T. (1999). "Optimization of construction time-cost trade-off analysis; using genetic algorithms" Canadian. J. Civil. Eng., 26(6), 685–697.
- [8] Khaled El-Rayes, Amr Kandil (2005), "Time-Cost-Quality Trade-off Analysis for Highway Construction" J. Construction. Eng. Manage., 131(4), 477-485
- [9] Knowles, J. D., Watson, R. A., and Corne, D. W. (2001). "Reducing local optima in singleobjective problems by multiobjection" Lecture Notes in Computer Science, Vol. 1993, E. Zitzler et al., eds., Springer, Berlin, 269–283.
- [10] Li, H., and Love, P. (1997). "Using improved genetic algorithms to facilitate time-cost optimization." J. Constr. Eng. Manage., 123(3), 233–237.
- [11] Zheng, D. X. M., Ng, T. S. T., and Kumaraswamy, M. M. (2002). "Applying genetic algorithms techniques for time-cost optimization" Proc., 18th Annual Conf. ARCOM, D.Greenwood, ed., Univ. of Northumbria, Middleborough, U.K., September 2–4, 801–810.
- [12] X. Gandibleux, M. Sevaux, K.Sorensen, V. T'Kindt(Eds), Metaheuristics for Multiobjective Optimization, Lecture Notes in Economics and Mathematical System 535, Springer Verlag, 2004.
- [13] Zitzler E., Deb K., Thiele L., Comparison of Multi-objective Evolutionary Algorithms: Empirical Results, 2000, *Evolutionary Computation*