# An Evaluation Model for Determining Insurance Policy Using AHP and Fuzzy Logic: Case Studies of Life and Annuity Insurances

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*Abstract:* -This study presents an evaluation model for purchasing life insurance and annuity insurance using analytical hierarchy process (AHP) and fuzzy logic. Four factors are considered as the inputs of the proposed model including age, annual income, educational level and risk preference. To build the model, we interviewed five experts with at least three years of working experience in an insurance company. In the proposed model, fuzzy logic is used to perform necessary mappings for inputs. The AHP is utilized to generate the weights for the evaluation model.

*Key-words:* - Life insurance; Annuity insurance; Decision making; Fuzzy logic; AHP

## 1. Introduction

Life insurance and annuity insurance are major marketing products in personal insurance. Recently, with the improvement of living standard and the increment of lifespan of humans, the demand of annuity products has been gradually noticed in an aging society. The condition for benefit payment in life insurance is determined by the life or death of the insured. Therefore, if the condition for benefit payment is met, the insurer shall provide the insured amount. For people covered by annuity plans, they will receive a specific amount of benefits in regular intervals during their life time. Since the needs for the insured under these two products are very different, the marketing strategies for these two products would vary significantly. Generally, insurance sales agents (known as life insurance consultants) tend to rely on their subjective assumption or the needs and affordability of the clients for policy proposals in marketing insurance policies, including the types of insurance and the amount insured. In fact, there is no rule of thumb for the life insurance consultants to give policy proposals to the prospects. This study therefore attempts to build a model for insurance consultants to make their decision in determining insurance policy.

The fuzzy theory was first proposed by Zadeh in 1965 and then has been widely applied to different domains such as business, engineering, science, medicine, etc [1]. Basically fuzzy logic use fuzzy variable to represent the linguistic express of human beings. Fuzzy logic can be applied to mimic the thinking process of human beings, which is called a fuzzy expert system. Fuzzy expert systems are usually used as support systems to help people make their decisions. Basically, fuzzy expert systems utilize fuzzy if-then rules to emulate the decision-making process of human beings. The fuzzy variables are expressed in membership functions to describe the matching degrees ($\mu$) of how data fit particular fuzzy sets. Two types of membership functions are often used in fuzzy systems: triangular and trapezoidal functions. Fig. 1 shows a trapezoidal member function for a fuzzy set of "*The weather is moderate*". Fuzzy theory has been widely applied to build applications in many fields. We briefly discuss below.

Based on probability theory, Wang has presented a fuzzy decision system for supply chain management. A general discussion on the applications of insurance using fuzzy logic has been presented in [3]. A fuzzy regression model to calculate insurance claim reserves has been proposed to process the mutant and uncertainty of insurance environments and to evaluate the financial performance for insurance companies [4]. A schema of determining basketball zone defense patterns using fuzzy expert systems has been proposed where the authors presented fuzzy variables for generating the fuzzy expert systems [5,6]. A fuzzy approach has been used to evaluate pure premiums of automobile bodily injury liability [7]. A fine-tune fuzzy logic model has been proposed to change insurance rates using group health insurance data. [8].

Analytical Hierarchy Process (AHP) was developed by Thomas L. Saaty in 1971 for decision-making [9]. This method is mostly applied to make decisions under the situation of uncertainty and with a number of factors to be considered for decision-making. This is also a method combining quantitative and qualitative analyses. The subjective judgment of people is expressed and processed by quantitative analysis. Ratio scales are used to measure the importance and make all possible pair-wise comparisons among the factor utilized in an AHP model. These comparisons are implemented by specially designed questionnaires. The comparison ratios are then used to build a matrix. This matrix is a symmetrical reciprocal matrix due to pair-wise comparisons. Based on the maximum eigenvalue of the reciprocal matrix, people perform consistency tests to test the consistency among these factors and then generate weighs for building evaluation models for decision-making.

A portfolio decision model between goals and available resources using AHP has been proposed for evaluating new products and assessing marketing [9]. A decision support system for selecting life insurance contracts has been presented, in which the authors utilized AHP to assist prospects to make their optimal choices based on their preferences [10] A route selection model for transportation systems has been presented in [11] where the authors proposed a hybrid framework consisting of fuzzy logic and AHP.

This study proposed an evaluation model to determine insurance policy using AHP and fuzzy logic. The conceptual diagram of the proposed model is shown in Fig. 2. Two main insurances are selected to build the model: life insurance and annuity insurance. There are four factors utilized as inputs in the proposed model including age, annual income, educational level, and risk preference. We use four fuzzy variables to express the four factors. Trapezoidal functions were utilized to measure the marching degrees of the inputs for these fuzzy variables. We then applied the AHP model to generate the evaluation criteria for the two insurances life and annuity insurances. To build the AHP model, we first interview domain experts to determine the factors and rules for evaluating insurance. We designed a questionnaire and collected the opinions from the experts. We then perform consistency tests to determine the consistency of the questionnaires and finally presented the evaluation formulas for the two insurances based on the results of consistency tests.

The evaluation model proposed by this study could be served as a tool to eliminate subjective assumptions for insurance consultants when they give policy proposals to their prospects. This evaluation model can also help the consultants make the best choices for their customers.

## 2 Preliminaries

Basically, an AHP model is a hierarchic structure with a couple of layers in the structure. Fig. 3 shows a simple three-layer AHP. Each layer consists of several nodes associated with the factors or dimensions. Weights are used to link two nodes in the adjacent layer and perform desired mappings from inputs to final outputs (decisions). Each node is mapped by the following equation:

$$y_j = \sum_{i=1}^{n} w_{ij} x_i \quad (1)$$

where

$y_j$ = the output for node $j$ in a particular layer.

$x_i$ = the output of node $i$ in the previous layer. of this particular layer,

$w_{ij}$ = the weight linking node $j$ in the particular layer and the node $i$ in the previous layer.

$n$ = the number of node in the previous layer.

The procedure of AHP model is described as follows:

**<u>Step 1</u>:** Define problems and select factors (dimensions) for the decision model. This can be done by interviewing domain experts.

**<u>Step 2</u>:** Design questionnaire: The questionnaire is designed by making any possible pair-wise comparisons among these factors (dimension) selected in Step 1. A nine-point scale is usually utilized for AHP. Table 1 shows a typical nine-point scale for an AHP questionnaire. To make pair-wise comparisons, the questionnaire is designed to measure all importance ratios for all possible pair of factors. Table 2 shows a simple example of the questionnaire where three factors are selected including factors A, B, and C. In Table 2, if factor A is twice importance than factor B, the ratio of factor A and factor B is 2:1. In the row of pair-wise comparison of factor A and factor B, we mark " ˅ " in the cell associated with a value of 2 (closed to the factor of A), as indicated in Table 2. Similarly, the importance ratio for factors A and C is 3: 1, and the importance ratio for factors B and C is 1:5. These are all shown in Table 2.

**Step 3:** Use the questionnaire to collect the experts; opinions of the importance ratios among the factors and generate a matrix of importance ratios. We give a simple example of getting a matrix of importance ratios. Consider a questionnaire filled by an expert with three pair-wise comparisons among factors A, B, and C, as indicated in Table 2. The associated matrix of importance ratios is given by

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 1/5 \\ 1/3 & 5 & 1 \end{bmatrix} \end{array} \quad (2)$$

It is important to note that the matrix of importance ratios is a symmetrically reciprocal matrix with a value of 1 in its diagonal items.

**Step 4 :**Test the consistency:

The Consistency Index (CI) is utilized to express the degree of consistency, given by

$$CI=\frac{\lambda_{\max}-n}{n-1} \quad (3)$$

where n is the number of factors, $\lambda_{\max}$ is the maximum eigenvalue of the importance ratio matrix.

The Constituency Ratio (CR) is calculated by

$$CR=\frac{CI}{RI} \quad (4).$$

where RI (Random Index) is given by Table 3. If CR is less than or equal to 0.1, the weights for the factors in an AHP model are the elements in the normalized eigenvector associated with the maximum eigenvalue ($\lambda_{max}$). If CR is greater than 0.1, the questionnaire fails.

# 3 Experimental Setup and Results

Five experts from an insurance company in Taiwan were selected. All of the five experts are with at least three years of insurance consulting experience. There are four factors were suggested to evaluate the purchases of life insurance and annuity insurance including age, annual income, educational level and risk preference. Trapezoidal functions are suggested by the domain experts to evaluate the matching of the four fuzzy variables since they are easy to use and appropriately express the gradual transitions of the four variables.

Member functions to express the fuzzy variables of the four factors are described as follows:

1. Age:
   People at the age of 25 and under are defined as young age. People at the age of 25~45 are medium age, and people at the age of 45 and higher are old age (see Fig. 4)
2. Annual income:
   People with an annual income less than 500,000 NTDs (New Taiwan Dollars) are classified as low income. People with annual income between 500,000 and 1,250,000 NTDs are classified as medium income. People with annual income more than 1,250,000 NTDs are classified as high income (see Fig. 5). The exchange rate for NTD to USD (United States Dollar) is approximately 33: 1 and the per capita GNP (Gross National Product) in Taiwan is 523,799 NTDs in 2006 [12].
3. Educational level:
   Four educational levels are used in the proposed model ranging from 1(lowest level) to 4 (highest level), as shown in Fig. 6.
4. Risk preference:
   Risk preference is defined by the score of 1-10 based on the attributes of the insurance buyers evaluated by the experts, as shown in Fig. 7.

The questionnaire for the study is shown in Table 4. The evaluation rules for the life insurance and annuity insurance suggested by the experts are shown in Table 5.After collecting questionnaires from the domain experts, we then calculated the maximum eigenvalue ($\lambda_{max}$.) of the matrix of importance ratios and applied Eqs. (3) and (4) to the consistency tests. The results of consistency tests are shown in Table 6. Only two questionnaires (expert 1 and expert 5) passed the consistency tests. We then took geometrical averages of the weights from the questionnaires which passed the consistency tests.

The evaluation model for the life insurance is given by taking the normalized eigenvector associated with the maximum eigenvalue $\lambda_{max}$.

$$y=0.46x_1+0.32x_2+0.07x_3+0.15x_4 \quad (5)$$

where

$x_1$ = the fuzzy variable of age,
$x_2$ = the fuzzy variable of annual income,
$x_3$ = the fuzzy variable of educational level, and
$x_4$ = the fuzzy variable of risk preference.

In Eq. (5), the higher value of $y$ means the stronger recommendation of purchasing life insurance.

Similarly, the evaluation model for the annuity insurance is given by

$$y=0.20x_1+0.19x_2+0.15x_3+0.46x_4 \quad (6)$$

where fuzzy variables $x_1$ to $x_4$ are the same as the variables in Eq. (5)

## 4. Conclusions

We proposed an evaluation model for purchasing life insurance and annuity insurance using AHP and fuzzy logic. Four factors were considered in this model including age, annual income, educational level and risk preference. To build the model, we interviewed five experts with at least three years of working experience in an insurance company. We used fuzzy logic to perform necessary mappings for the inputs and AHP to generate the desired weights for the evaluation model.

As for the directions of further studies, we suggest to use more attributes as the inputs of the evaluation model. Delphi technique might be considered to integrate more experts' opinions.

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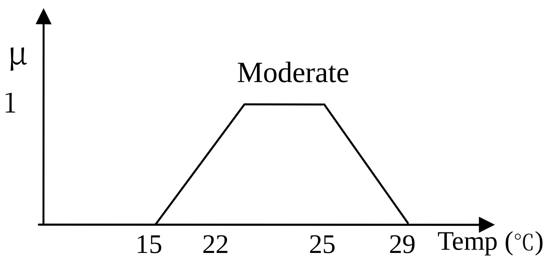
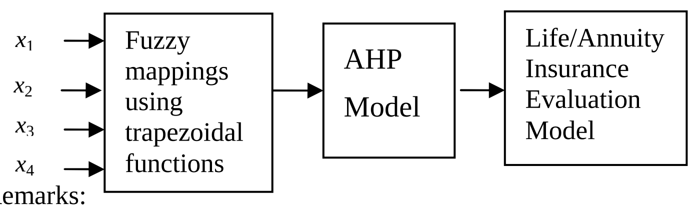



Fig. 1: An example of member function of temperature.




Remarks:

$x_1$: age; $x_2$: annual income;

$x_3$: educational level; $x_4$: risk preference.

Fig. 2: The conceptual diagram of the propose model

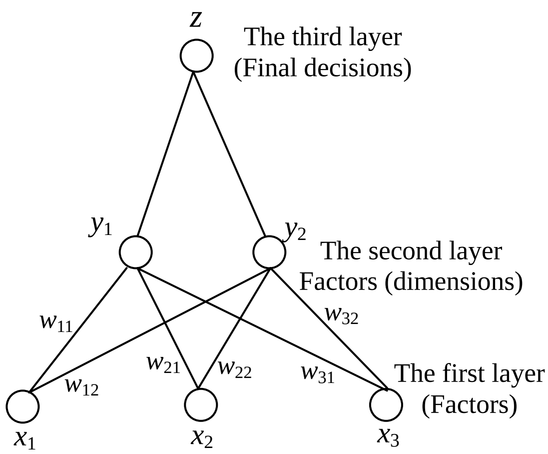



Fig. 3: A simple three-layer AHP

Table 1: The definition and explanation of AHP 9-point scale [9]

| Intensity of Importance | Definition | Explanation |
|---|---|---|
| 1 | Equal Importance | Two activities contribute equally to the objective |
| 3 | Weak importance of one over another | Experience and judgment slightly favor one activity over another |
| 5 | Essential or strong importance | Experience and judgment strongly favor one activity over another |
| 7 | Demonstrated importance | An activity is strongly favored and its dominance is demonstrated in practice |
| 9 | Absolute importance | The evidence favoring one activity over another is of the highest possible order of affirmation |
| 2, 4, 6, 8 | Intermediate values between the two adjacent judgments | When compromise is needed |

Table 2: A simple example of questionnaire

| Factor | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Factor |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | | | | | | | | v | | | | | | | | | | B |
| A | | | | | | | v | | | | | | | | | | | C |
| B | | | | | | | | | | | | | v | | | | | C |

Table 3 Random Index [12]

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R.I. | 0.00 | 0.00 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 | 1.56 | 1.57 | 1.59 |

Remark: *n* is the number of factors

Table 4: Questionnaire for the study

| Factor | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Factor |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | | | | | | | | | | | | | | | | | | Annual Income |
| Age | | | | | | | | | | | | | | | | | | Risk Preference |
| Age | | | | | | | | | | | | | | | | | | Educational Level |
| Annual Income | | | | | | | | | | | | | | | | | | Risk Preference |
| Annual Income | | | | | | | | | | | | | | | | | | Educational Level |
| Risk Preference | | | | | | | | | | | | | | | | | | Educational Level |

Table 5: The Evaluation rules

| Insurance | Evaluation rule |
|---|---|
| Life insurance | The prospect's age is medium<br>The prospect's annual income is medium<br>The prospect's education is high<br>The prospect's risk preference is low |
| Annuity insurance | The prospect's age is high<br>The prospect's annual income is high<br>The prospect's education is high<br>The prospect's risk preference is low |

Table 6: The results of consistency tests

| Insur. Policy / Value / Expert | Life insurance | | Annuity insurance | |
|---|---|---|---|---|
| | CI | CR | CI | CR |
| 1 | 0.0292* | 0.0324* | 0.06179* | 0.0686* |
| 2 | 0.3482 | 0.3869 | 0.1410 | 0.1567 |
| 3 | 0.3412 | 0.3791 | 0.3412 | 0.3791 |
| 4 | 1.4054 | 1.5616 | 0.2354 | 0.2616 |
| 5 | 0.0385* | 0.0428* | 0.0515* | 0.0572* |

Remarks: 1.*: pass the consistency tests. 2. RI = 0.9

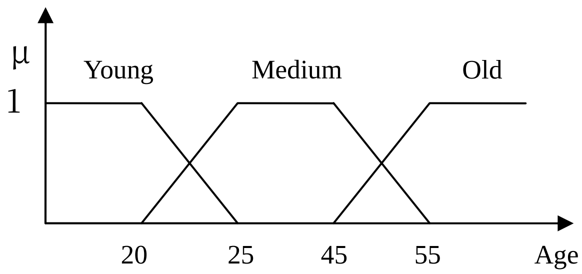



Fig. 4: Fuzzy variable of age.

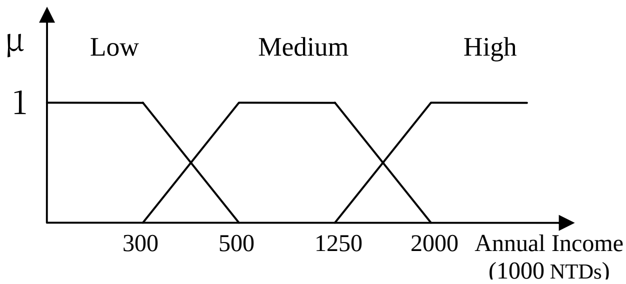
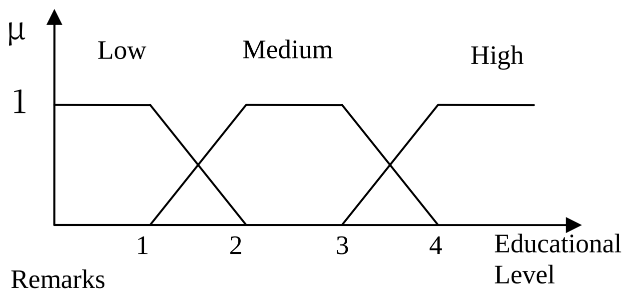



Fig. 5: Fuzzy variable of .annual income.




Remarks
1: Junior High School or below
2: Senior (Vocational) School
3: College
4: Graduate School

Fig. 6: Fuzzy variable of educational level

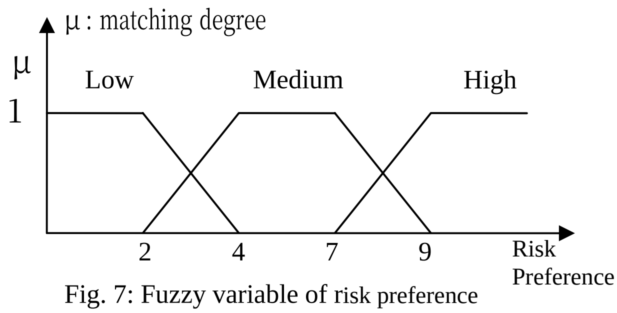



Fig. 7: Fuzzy variable of risk preference