Website Structures Ranking: Applying Extended ELECTRE III Method Based on Fuzzy Notions

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Abstract: - Due to grand size of some websites and their complicated structures, prioritizing the various structures turns to be a decision making problem incorporating large uncertainty in judgment. Finding a response to such a need, the ordinary ELECTRE III method could heal some of shortcomings of uncertainties in decision making. However there are still lots of occasions in which ordinary ELECTRE III method can not reach a full ranking among the website structures. As a sensible response to this drawback, in this paper, we utilize a brand-new extension of ELECTRE III method with ability to reach a full ranking in contrast to the ordinary ELECTRE III.

Key-Words: - Website Structure, Ranking, Multi Attribute Decision Making (MADM), ELECTRE, Fuzzy Sets Theory,

1 Introduction

Most of the real-world decision problems take place in a complex environment where conflicting systems of logic, uncertain and imprecise knowledge have to be considered. To face such complexity, preference modeling requires the use of specific tools, techniques and concepts which allow reflecting the available information with the appropriate granularity [1]. For an enterprise, its website is usually the front door of its advertisements. Various important factors for web designers when considering the design of a new website include the attractiveness of the design, an effective structure to the web page to deliver information quickly, and user satisfaction among a growing and diverse set of users faced with ever increasing web contents. With the development of more and more web-based technologies and the growth in web content, the structure of a website becomes more complex and web navigation becomes a critical issue to both web designers and users, complicating the evaluation and decision process of an appropriate website design [2].

Moreover, better structure of web links takes visitors easier and sooner to their targets in a website and also it enhances website navigation.

Developing Information Technology (IT) in recent years, most of state-run organizations in Iran have focused mostly on their websites. In the current environment, the level of quality of web design and web services is a comparison factor between IT departments of different state-run organizations and firms. Chief Information Officers (CIOs) of these organizations care about website of others' so they spend considerable amount of their budget and time to enhance their own website's appearance and structure. In other words, nowadays website architecture and its content play the role of performance measurement criteria of an organization [3].

Having the hits rate of more than 100-150 person/day, Iranian CAO website is playing an integral role of a portal for the Iranian aviation industry presenting number of static and dynamic services to various users and clients. Thus, huge number of new web pages are designed and added to/substituted by the old web pages in the web site structure on a monthly basis aiming to simplify the users' access to the intended information as much as possible. Having such a gigantic size, providing an effective way of navigation for such a link structure is a real woe for the team of web designers these days. During previous months, from February 2006 to October 2006, the website of Iranian Civil Aviation Organization (CAO) (http://*www.cao.ir*) has been frequently changed to have better face to users. During this period of time, different homepages were developed. The homepage and link structure were dramatically changed again and again and imposed a considerable amount of cost on the department monetarily and time wise. Being faced

with various developed structures for CAO website a ranking and evaluation system was needed to rank the various structures and to enable the web designers to choose the best structure for online presentation [3, 4]. On the other hand, multi criteria decision making (MCDM) was introduced as a promising and important field of study in the early 1970's. Since then the number of contributions to theories and models, which could be used as a basis for more systematic and rational decision making with multiple criteria, has continued to grow at a steady rate [5]. The most outstanding MCDM methods which strictly apply this definition of outranking relation are the ELECTRE, ELimination and Choice Expressing the Reality, and PROMETHEE methods which are very important in many respects, not least historically, since ELECTRE I was the first outranking method. These methods provide the decision maker with the ability to do pair-wise comparisons and rankings among the alternatives reaching a final thorough ranking among the various alternatives [6].
Considering such a complex decision making problem in evaluating the website designs, taking in to account the existing uncertainty in judging and ranking the various structures developed for CAO website, this paper will introduce a brand new extension of ELECTRE III method, an outstanding outranking-based Multi Criteria Decision Making (MCDM) method, aiming at improving the modeling of existing uncertainties in decision making to some extent. In this regards, next section of this paper states the problem then in the third section the main contribution of our study is presented discussing the Fuzzy ELECTRE III method, Section 4 practice and subsequently compare the ordinary ELECTRE III method with its extended one and finally last section concludes the whole discussion.

2 Problem Statement

Being faced with various developed structures for CAO website and in order to evaluate and rank these structures, some criteria were selected from literature, based on the job by Zhang and Dran [2]. As CAO is a state-run company, the listed criteria, easy to navigate, clear layout of info, up-to-date info, appropriateness of search tools and accuracy of info, have been selected from the various categories proposed by Zhang and Dran. The related normalized weights for these criteria based on what Zhang and Dran have suggested are: 0.27, 0.21, 0.18, 0.17, and 0.17, respectively.

Having used the criteria, six different scenarios for CAO website were evaluated. As the criteria are some how technical and need some technical information for accurate evaluation, we used 10 different web designers who were engaged with CAO website for more than 2 years as the referees to score the criteria about each developed scenario. Each criterion was scored based on a score range of 1-10, 1 presents an ultimate weak performance while 10 shows an ultimate strong performance of the structure under the related criterion. The decision matrix has been presented in table 1, based on the average score reached for each structure under each criterion.

TABLE 1
Performance of six alternative scenarios for the structure of CAO website system

Scens.	*Cri.1*	*Cri.2*	*Cri.3*	*Cri.4*	*Cri.5*
1	8	3	6	1	10
2	2	8	1	9	5
3	5	10	9	2	4
4	1	2	9	9	9
5	9	9	2	3	2
6	10	1	3	9	2

3 Fuzzy ELECTRE III: a full-ranking method

Having the concordance, $c(a,b)$, and discordance, $d_j(a,b)$, indices in hand, as are defined in ordinary ELECTRE III method [7-10], we focus on the way the credibility index is calculated. The concordance and discordance indices incorporate some senses of the membership degree as is in the fuzzy literature. In other words, the concordance and discordance indices play the role of a membership degree for various pair-wise relations of alternatives, presenting their degree of adhering to the supporting and contradicting criteria, respectively. Having approved such a definition of these indices, $\rho(a,b)$ is actually their intersection, $\mu_{a\cap b}$, presenting the index of credibility of the pair-wise relation between the alternatives. Doing so, $\rho(a,b)$ takes in to account both the concordance index, the degree of adherence of the outranking relation to the supporting criteria, and the discordance index, the degree of adherence of the outranking relation to the contradicting criteria. Having the concept of fuzziness in defining those indices, their product as the relation for calculating their intersection, is one of the common ways of reaching the intersection in fuzzy literature. However, there are some other ways for obtaining such output from those indices. Paraphrastically, from the fuzzy literature perspective, the intersection of concordance and discordance indices can be calculated utilizing some other ways for doing so in addition to what is suggested in ordinary ELECTRE III method. Based on fuzzy literature, the most common ways of reaching such intersection can be as follows [11, 12]:

1. Natural Definition ($A \bigcap B$):

$$\mu_{a \cap b} = \min(\mu_a, \mu_b)$$

2. Algebraic Product ($A \bullet B$):

$$\mu_{a \bullet b} = \mu_a \times \mu_b$$

(as has been used in ordinary ELECTRE III for calculating the $\rho(a,b)$, as the intersection of two membership degrees of *c(a,b)* and $\frac{1-d_j(a,b)}{1-c(a,b)}$ for each criterion of *j*).

3. Bounded Subtraction ($A \Theta B$):

$$\mu_{a \Theta b} = \max(0, \mu_a + \mu_b - 1)$$

Three aforementioned ways of calculating the intersection of two sets are the common ones presented using the fuzzy environment. However, any function having the conditions of a t-norm can act as an intersection of two various sets [11, 12]. In this paper, we consider these three various ways for calculating the intersection and prove the following lemma for them:

Lemma 1: For all sets of A and B, we invariably have:

$$\mu_{A \Theta B} \le \mu_{A \bullet B} \le \mu_{A \cap B}$$

Proof: Due to lack of enough space, we accept the lemma with out proof, however it can be proved simply.

We can define the credibility index, $\rho(a,b)$, as a fuzzy triangular number as follows:

$$\rho(a,b) = \begin{cases} [C \Theta D, C \bullet D, C \bigcap D] \\ \quad \text{if } V = \{j \in J : d_j(a,b) > c(a,b)\} \neq \varnothing \\ [C, C, C] \\ \quad \text{if } V = \{j \in J : d_j(a,b) > c(a,b)\} = \varnothing \end{cases}$$

When the set of *V* is empty, meaning that there is no *d$_j$(a,b)* greater than *c(a,b)*, then the credibility index, $\rho(a,b)$, is actually a constant value presented in the form of a triangular fuzzy number in relation 16.

Nevertheless, when the set of *V* is not empty, meaning that there is at least one criterion under which *d$_j$(a,b)* is greater than *c(a,b)*, then the credibility index, $\rho(a,b)$, is a fuzzy triangular number as presented in relation 16, having the $\mu_{a \Theta b}$ and $\mu_{a \cap b}$ as the lower and upper boundaries, respectively and the $\mu_{a \bullet b}$ as its medium value.

Having obtained the credibility index, $\rho(a,b)$, in fuzzy format, modeling much more uncertainty in doing the subsequent calculations, the final full ranking of the alternatives is reached using the following steps:

- As we also have in the ordinary ELECTRE [7-9], each alternative is related to different sets of credibility indices. The first set of indices includes the ones presenting the outranking strength under which the alternative outranks the other alternatives in the set *A*. The second set consists of the indices presenting the strength under which the former alternative has been outranked by the other alternatives in the set *A*. These two different sets are presented for the alternative a_i as below:

1. $\phi_1 = \{\rho(a_i, a_j); a_i \succ a_j \ for\ j = 1, \cdots, m\ \&\ j \neq i\}$

2. $\phi_2 = \{\rho(a_j, a_i); a_j \succ a_i \ for\ j = 1, \cdots, m\ \&\ j \neq i\}$

It is noticeable that ϕ_1 and ϕ_2 are two fuzzy sets as their members, $\rho(a,b)$ s, are some triangular fuzzy numbers.

- Having approved the fact that two aforementioned sets, ϕ_1 and ϕ_2, show the strength of each alternative in outranking the other alternatives in the set *A* and being outranked by the other ones in the set *A*, in order to define proper representatives for these two sets enabling us in doing following calculations, we can define two various triangular fuzzy numbers (Index$_1$ and Index$_2$) as their representatives as below:

Index1= [*yager* (*min* $\rho(a_i, a_j)$), *yager* (*average* $\rho(a_i, a_j)$),
yager (*max* $\rho(a_i, a_j)$)]; $\rho(a_i, a_j) \in \phi_1$

Index2= [*yager* (*min* $\rho(a_j, a_i)$), *yager* (*average* $\rho(a_j, a_i)$),
yager (*max* $\rho(a_j, a_i)$)]; $\rho(a_j, a_i) \in \phi_2$

where *yager (min* $\rho(a,b)$*)* and *yager* (*max* $\rho(a,b)$) are the lower and upper boundaries of the fuzzy triangular number, respectively and *yager* (*average* $\rho(a,b)$) is its medium value.

As $\rho(a,b)$ is a fuzzy triangular number, reaching a crisp value for this fuzzy number, for constructing the fuzzy numbers of Index1 and Index2, we utilize the yager index as proposed by [13]. As an instance, considering the membership functions of the fuzzy numbers as some linear functions, the yager index of the fuzzy number of $\rho(a,b) = [C\Theta D, C \bullet D, C \cap D]$ will be a crisp value as follows [13]:

Yager ($\rho(a,b)$) =

$$((3 * C \bullet D) - (C \bullet D - C\Theta D) + (C \cap D - C \bullet D))$$

As stated above, Index1 and Index2 are the representatives for the sets of ϕ_1 and ϕ_2, presenting the strength under which an alternative outranks the other alternatives in the set *A* and also been outranked by the other alternatives in the set *A*.

⌖ Having obtained Index1 and Index2, their subtraction value, *Q(a)*, is a sensible representative, showing the net proportional strength of an alternative in comparison to other ones considering the fuzzy value of the strength under which the alternative outranks the other alternatives in the set *A* and also the fuzzy value of the strength under which the alternative is outranked by the other ones in the set *A*.

Q(a)= Index1 – Index2

⌖ Finally, reaching an appropriate value for doing the ranking, the yager index of the *Q(a)*, as defined in equation 20, is obtained.

Existence of some relations as *aIb* and specially *aRb* which declare the indifference and incomparability of the alternatives of *a* and *b* respectively, in final results of the ordinary ELECTRE is the main reason that the final obtained pre-order turns to be partial instead of full (complete) ranking [7-9, 14]. Under such condition, we can not be sure from the first that all the alternatives of our problem can be ranked in relation with each other as we may have some situation in which some alternatives are found to be indifferent and/or incomparable. Utilizing our extended method in ranking the various alternatives in the set *A*, we can not only reach a full ranking of the alternatives in contrast to partial ranking of ELECTRE III by eliminating the chance of happening the state of incomparability in the relation between the alternatives, but also model much more uncertainty in comparing and ranking the alternatives, attenuating the chance of happening the state of indifference in the relation between the alternatives which is so common in ordinary ELECTRE III method, making the whole ranking of the alternatives vague.

4 Ranking the Scenarios

4.1 Utilizing ordinary ELECTRE III

Utilizing ordinary ELECTRE III method, the following ranking of six scenarios (Table 2) was reached based on the crisp data presented in table 1. The ranking of scenarios based on their performances evaluated by the designers, presented in table 1, is reached as: $1 \rightarrow 6 \rightarrow \begin{Bmatrix} 3 \\ 5 \end{Bmatrix} \rightarrow 4 \rightarrow 2$, (in which the scenarios 3 and 5 are identified as identical), utilizing “Electre III-IV” software [15].

4.2 Utilizing Fuzzy ELECTRE III for ranking the scenarios

TABLE 2
Outranking relation of six CAO website scenarios using ordinary ELECTRE III

	A0001	A0002	A0003	A0004	A0005	A0006
A0001	**I**	**P**	**P**	**P**	**P**	**P**
A0002	**P⁻**	**I**	**P⁻**	**P⁻**	**P⁻**	**P⁻**
A0003	**P⁻**	**P**	**I**	**P**	**I**	**P⁻**
A0004	**P⁻**	**P**	**P⁻**	**I**	**P⁻**	**P⁻**
A0005	**P⁻**	**P**	**I**	**P**	**I**	**P⁻**
A0006	**P⁻**	**P**	**P**	**P**	**P**	**I**

Based on the six alternatives’ performances in table 1, the scenarios are ranked utilizing fuzzy ELECTRE III as described above. Doing so, we need to calculate the concordance matrix along with the discordance matrices. Table 3 presents the matrix of *c(a,b)* for six website structure alternatives.

Having calculated the concordance and discordance matrices, we obtain the fuzzy credibility indices, $\rho(a,b)$, for all pair-wise relations of alternatives. Based on the calculated fuzzy credibility matrix, each alternative has two various fuzzy sets of

TABLE 3
The concordance matrix for six CAO website structures alternatives

	1	2	3	4	5	6
1	1.00	0.62	0.73	0.77	0.79	0.83
2	0.38	1.00	0.73	0.707	0.56	0.56
3	0.74	0.83	1.00	0.66	0.82	0.56
4	0.73	0.79	0.61	1.00	0.52	0.73
5	0.71	0.773	0.82	0.48	1.00	0.83
6	0.77	0.733	0.61	0.65	0.79	1.00

TABLE 4

FUZZY OUTRANKING INDICES AND THEIR CRISP INDICES OF YAGER (INDEX1S)

Outranking Strength		*Min*			*Average*			*Max*		
		a	*m*	*b*	*a*	*m*	*b*	*a*	*m*	*b*
1	*Fuzzy Num.*	0.370	0.408	0.620	0.748	0.755	0.790	1.0	1.0	1.0
	Yager Index		0.466			0.764			1.0	
2	*Fuzzy Num.*	0.310	0.318	0.560	0.451	0.589	0.656	1.0	1.0	1.0
	Yager Index		0.396			0.566			1.0	
3	*Fuzzy Num.*	0.560	0.560	0.560	0.768	0.768	0.768	1.0	1.0	1.0
	Yager Index		0.560			0.768			1.0	
4	*Fuzzy Num.*	0.270	0.271	0.520	0.525	0.652	0.730	1.0	1.0	1.0
	Yager Index		0.354			0.636			1.0	
5	*Fuzzy Num.*	0	0.444	0.480	0.647	0.747	0.769	1.0	1.0	1.0
	Yager Index		0.308			0.721			1.0	
6	*Fuzzy Num.*	0	0.610	0.610	0.657	0.759	0.759	1.0	1.0	1.0
	Yager Index		0.407			0.725			1.0	

credibility indices; the set of fuzzy indices showing the strength of outranking relation under which the specified alternative outranks the other alternatives in set A, and the other one is the set of fuzzy credibility indices presenting the power of outranking relations under which the specified alternative is outranked by the other alternatives in set A. As an instance, the aforementioned fuzzy sets for the first scenario of CAO website structure are as follows:

1. $\phi_1 = \{(1.00, 1.00, 1.00), (0.37, 0.408, 0.62), (0.73, 0.73, 0.73), (0.77, 0.77, 0.77), (0.79, 0.79, 0.79), (0.83, 0.83, 0.83)\}$
2. $\phi_2 = \{(1.00, 1.00, 1.00), (0.38, 0.38, 0.38), (0.74, 0.74, 0.74), (0.73, 0.73, 0.73), (0.46, 0.612, 0.71), (0.77, 0.77, 0.77)\}$

The former fuzzy set denotes the outranking strength of the first scenario, as an alternative, and the second fuzzy set shows the power of being outranked for

TABLE 5

FUZZY INDICES, THE STRENGTH OF BEING OUTRANKED, AND THEIR CRISP YAGER INDICES

Strength of Being Outranked		*Min*			*Average*			*Max*		
		a	*m*	*b*	*a*	*m*	*b*	*a*	*m*	*b*
1	*Fuzzy Num.*	0.380	0.380	0.380	0.680	0.705	0.722	1.000	1.000	1.000
	Yager Index		0.380			0.702			1.000	
2	*Fuzzy Num.*	0.370	0.408	0.620	0.749	0.756	0.791	1.000	1.000	1.000
	Yager Index		0.466			0.765			1.000	
3	*Fuzzy Num.*	0.360	0.391	0.610	0.485	0.704	0.750	1.000	1.000	1.000
	Yager Index		0.454			0.646			1.000	
4	*Fuzzy Num.*	0.000	0.444	0.480	0.589	0.688	0.711	1.000	1.000	1.000
	Yager Index		0.308			0.663			1.000	
5	*Fuzzy Num.*	0.270	0.271	0.520	0.705	0.705	0.747	1.000	1.000	1.000
	Yager Index		0.354			0.719			1.000	
6	*Fuzzy Num.*	0.310	0.318	0.560	0.588	0.711	0.752	1.000	1.000	1.000
	Yager Index		0.396			0.684			1.000	

that scenario. Sensibly, the existing difference among these two fuzzy sets can be an appropriate measure for finding the rank of the first scenario against the other ones.

Finding the aforementioned difference between the given fuzzy sets, we define two indices as two representatives for those sets, the differences of which can appropriately play the role the ranking measure.

Being able to present the obtained indices in tables 4 and 5, we have utilized the Yager index as proposed and applied by [13], respectively. Doing so, we find two various fuzzy triangular numbers as index1 and index2 for each alternative indicating the outranking power and also the strength of be-outrankedness for each alternative, respectively. The existing difference between these two recent values can be a sensible measure for full-ranking the alternatives based upon. Table 6, indicates these indices' differences, net strengths, for various alternatives along with their final ranking.

As presented above, reaching a crisp value for ranking the alternatives based on the obtained fuzzy differences between the indices, we have utilized Yager index based on which the final ranking is obtained to be as: $3 \to 1 \to 6 \to 4 \to 2 \to 5$.

TABLE 6
The indices' fuzzy differences for six alternatives and their final ranking based on the obtained Yager index

	a	*m*	*b*	*Final Yager Index*	*Rank*
1	-0.534	0.062	0.620	0.049	*2*
2	-0.604	-0.200	0.534	-0.090	*5*
3	-0.440	0.122	0.546	0.076	*1*
4	-0.646	-0.027	0.692	0.006	*4*
5	-0.692	0.002	0.646	-0.015	*6*
6	-0.593	0.041	0.604	0.017	*3*

5 Conclusion

As the results denote, utilizing fuzzy ELECTRE III in doing ranking, empower us to not only reach a full ranking of the alternatives in contrast to partial ranking of ELECTRE III by eliminating the chance of happening the state of incomparability in the relations between the alternatives, but also model much more uncertainty in comparing and ranking the alternatives, weakening the chance of happening the state of indifference in the relations between the alternatives which is so common in ordinary ELECTRE III method. The former obtained ranking based on the utilization of ordinary ELECTRE III method, $1 \to 6 \to \begin{Bmatrix} 3 \\ 5 \end{Bmatrix} \to 4 \to 2$, shows an state of indifference between the scenarios 3 and 5. In spite of the fact that applying the proposed fuzzy ELECTRE III method presents that the obtained state of relation between these alternatives in ordinary ELECTRE III is not indifferent but also a complete preference, $3 \to 1 \to 6 \to 4 \to 2 \to 5$.

Contrasting both obtained rankings by web designers, most of them were more satisfied with the latter one.

References:

1 Fodor, J., *Preference Relations in Decision Models.* 2000.

2 Zhang, P. and G.V. Dran. *Expectations and Rankings of Website Quality Features: Results of Two Studies on User Perceptions*. in *Hawaii International Conference on Systems Science (HICSS 34)*. January 2001. Hawaii.

3 Q.Saremi, H. and Gh.A. Montazer, *Web Usability: A Fuzzy approach to the navigation structure enhancement in a website system, case of Iranian Civil Aviation Organization Website.* Transactions on Engineering, Computing and Technology, 2006. **16**: pp. 123-128.

4 Q.Saremi, H., B. Abedin, and A. M.Kermani, *Website Structure Improvement: Quadratic Assignment Problem (QAP) Approach and Ant Colony Meta-Heuristic Algorithm.* Applied Mathematics and Computation, 2007. **Article in Press**.

5 Fuller, R. and C. Carlsson, *Fuzzy multiple criteria decision making: Recent developments.* Fuzzy Sets and Systems, 1996. **78**: pp. 139-153.

6 Brans, J.-P. and B. Mareschal, *PROMETHEE Methods*, in *Multiple criteria decision analysis, state of the art surveys*, J. Figueira, S. Greco, and M. Ehrgott, Editors. 2005, Springer. pp. 163-195.

7 Figueira, J., V. Mousseau, and B. Roy, *ELECTRE Methods*, in *Multiple Criteria Decision Analysis, State of the Art Surveys*, J. Figueira, S. Greco, and M. Ehrgott, Editors. 2005, Springer. pp. 133-162.

8 Roy, B., *The outranking approach and the foundations of ELECTRE methods.* Readings in multiple criteria decision aid, 1990: pp. 155-183.

9 Tervonen, T., J.e. Figueira, R. Lahdelma, and P. Salminen, *An Inverse Approach for ELECTRE III.* 2004, Instituto de Engenharia de Sistemas e Computadores de Coimbra INESC, Coimbra, Portugal. p. 17.

10 Shen, C.-C. and K.-L. Hsieh. *The Study of Fuzzy Performance Evaluation*. in *11th WSEAS International Conference on APPLIED MATHEMATICS*. 2007. Dallas, Texas, USA: WSEAS.

11 Zimmermann, H.-J., *Fuzzy Set Theory And Its Applications*. Fourth ed. 2001: Springer.

12 Dubois, D. and H. Prade, *Operations on fuzzy numbers.* International Journal of Systems Science, 1978. **9**: pp. 613-626.

13 Yager, R.R., *A procedure for ordering fuzzy subsets of the unit interval.* Information Sciences, 1981. **24**(2): pp. 143-161.

14 Papadopoulos, A. and A. Karagiannidis, *Application of the multi-criteria analysis method Electre III for the optimisation of decentralised energy systems.* Omega, 2006. **Article in Press**.

15 LAMSADE, *Electre III-IV*. 1994.