# Newsvendor Pricing with Fuzzy Demand

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*Abstract:* - In this paper a single product pricing model is studied with fuzzy demand quantities. It is assumed that uncertainties may appear in demand. Fuzzy demand is used to describe a subjective estimate, linguistically expressed by the phrase "demand is about d". Price is taken as the main criterion that is used to take expert opinion on demand. Decision variables of the model are the price of the product and the quantity that should be ordered for a fixed time period that maximizes the possible profit. The computational aspects of the fuzzy model and its interpretations are illustrated by an example.

*Key-Words:* - Fuzzy Demand, Newsvendor Pricing, Order Quantity

## 1 Introduction

Establishment of the sales price of a product is one of the fundamental management decisions. Pricing decisions are of crucial importance and unless taken seriously, they can cause a major threat to the sustainability of the company.

Conventional pricing models depend on demand forecasts or demand projections that include uncertainty. The uncertainty in those models is based on the concept of randomness and on probability theory. Bertsimas and Boer used stochastic demand functions to solve multi-period multi-product dynamic pricing problems [5]. In real life, there are situations that the probability distribution of demand is not obtainable due to lack of historical data. The introduction of a new product is a typical example. In these kinds of situations decision maker faces a fuzzy environment and demands are described by linguistic terms such as "approximately equal to" a certain amount that subjectively estimated by an expert. The fuzzy set theory provides a possible solution approach for that kind of vague model. [7]

In literature several researchers developed models for fuzzy demand. Yao and Wu [1] studied consumer surplus and producer surplus for fuzzy demand and fuzzy supply. Chang [4] studied optimal fuzzy revenue for fuzzy demand quantity. Yao and Shih [2] investigated fuzzy revenue for fuzzy demand quantity based on interval-valued fuzzy sets.

In this paper a fuzzy model for product pricing problem is studied. In the model it is assumed that uncertainty appears in demand. Depending on the pricing alternatives vague demand is described by a fuzzy set which formally expresses a subjective estimate given by the phrase "demand is approximately 10'. The extension principle is used to find the membership function of the profit function and its centroid in a fuzzy sense.

When a firm (Newsvendor) faces stochastic price sensitive demand, he/she has to make pricing and inventory decisions before the demand is realized [3]. Petrovic and Vusjosĕvic [6] studied the fuzzy models for the newsvendor problem in which only inventory levels are taken as the only decision variable. In this paper a newsvendor problem with pricing decisions is studied with fuzzy demand.

The paper is organized in the following way. In second section a newsvendor problem with pricing decisions is discussed in fuzzy environment. The modeling of the fuzzy demand is described briefly. In Section 3 the model dealing with fuzzy demand to find the optimum price and inventory level that maximizes the profit is constructed with proposed solution methods. The solution method is developed based on the solution method proposed in [3]. Section 4 contains the conclusion of the paper.

## 2 Newsvendor Pricing Problem in Fuzzy Environment

The conventional, well-known stochastic newsboy problem is: given a demand probability, determine the order quantity Q* for a fixed time period that will maximize the expected total profit. The newsvendor pricing problem is the extension of conventional model where demand depends on pricing decisions. Thus price "p" and the order quantity "Q" are the decision variables of that problem. The total cost is linear sum of various cost items such as purchase cost, overage cost and shortage costs [6].

The demand probability distributions are usually derived from the past data. The question arises when there is not meaningful past data available, when the related data is recorded in different environments or simply does not exist. In that case demand estimation is based on the subjective management judgment and it can be vaguely expressed. For example, "demand is about d" or "demand is much larger than d" or in a more complex form "demand will be in the interval [dl, du] with a high degree of possibility, but there is a moderate degree of possibility that demand will be zero".

Fuzzy set theory provides the appropriate framework to describe and treat uncertainty related to aforementioned vagueness of natural language expressions and judgments. Approximate variables may be represented by fuzzy sets with the membership functions. A triangular fuzzy set that represents uncertain demand is shown in Fig. 1.

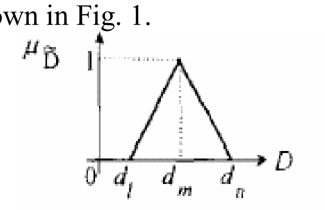


Fig. 1. Fuzzy set that represents uncertain demand

Now, the fuzzy newsvendor pricing problem statement is: for different possible pricing decisions, given a discrete fuzzy demand $\overline{D}$ with membership function $\mu_D(d_j)$, constant purchase cost $c_p$, unit overage cost $c_o$, shortage cost $c_s$, determine optimum order quantity, Q* and optimum price, p* which will maximize the possible total profit. The total cost is equal to the sum of purchase cost, possible overage cost and possible shortage costs.

## 3 Model

Let there are i = 1, 2,.., n price alternatives, which cause different fuzzy demand projections. Each fuzzy demand value $\overline{D}_i$ for a given price alternative pi is given by a domain $D_i = \{d_{i,1}, d_{i,2}, \ldots, d_{i,m}\}$ and $\mu_{\tilde{D}i}(d_j)$, j = 1, . . . ,m. Constant unit purchase, unit overage and unit shortage costs are $c_p$, $c_o$, and $c_s$, respectively for all possible pricing decisions.

Each uncertain demand estimation causes an uncertain overage cost $\tilde{U}_{i,o}$ , and an uncertain shortage cost $\tilde{U}_{i,s}$ for a given price alternative pi. For each Q and each D є Di the possibilities of the overage cost being $\tilde{U}_{i,o} = c_{i,o} \max(Q - D, 0)$ and the shortage cost being $\tilde{U}_{i,s} = c_{i,s} \max(D - Q, 0)$, are equal to the possibility of the demand being D:

$$\mu_{\overline{U}_{i,o}}\left(U_{i,o}(Q,D)\right) = \mu_{\bar{D}}(D)$$

$$\mu_{\overline{U}_{i,s}}\left(U_{i,s}(Q,D)\right) = \mu_{\bar{D}}(D) \qquad [1]$$

Denote $\tilde{U}_{i,os} = \tilde{U}_{i,o} + \tilde{U}_{i,s}$, by the domain [$\tilde{U}_{i,o}$ (Q, $D_i$) + $\tilde{U}_{i,s}$ (Q, $D_i$) ] $\in \{U_{i,os,1}, U_{i,os,2}, \ldots, U_{i,os,n}\}$. Then, according to the properties of possibility measure,

$$\mu_{\tilde{U}_{i,os}}\left(U_{i,os,j}\right) = max\, \mu_{\bar{D}_i}(\mathrm{D}) \qquad j = 1, 2, ..., n$$

$$U_{i,os} = U_{i,o}(Q,D) + U_{i,s}(Q,D) \qquad [2]$$

The possible total cost $U_{tot}$ is calculated by:

$$U_{i,tot}(Q) = U_p \times Q + \text{ defuzz } \left(U_{i,os}\right)$$

Where the operator "defuzz" denotes arithmetic defuzzification. It is given by the following equation.

$$\text{defuzz}\left(U_{i,os}\right) = \frac{\sum U_{i,os,j} \times \mu_{\overline{U}_{i,os}}(U_{i,os,j})}{\sum \mu_{\overline{U}_{i,os}}(U_{i,os,j})} \qquad [3]$$

Again for each D $\in D_i$ the possibility of the revenue being $\overline{R}_i = p_i \max(Q, D)$ is equal to the possibility of the demand being D:

$$\mu_{\overline{R}_i}\left(R(D_i)\right) = \mu_{\bar{D}_i}(D) \qquad D \in D_i \qquad [4]$$

The possible profit ∏ is calculated by:

$$\Pi_i = \text{defuzz}\left(\bar{R}_i\right) - \left\{c_p \times Q + \text{defuzz}\left(\tilde{U}_{i,os}\right)\right\} \qquad [5]$$

### 3.1 Example

Let the costs per product unit be precise: $c_p = 2$, $c_o = 1$, $c_s = 3$. The demand estimates for various price alternatives are given in Table 1.

Table 1. Fuzzy Demand Estimates for Price Alternatives

| Alternative Unit Price | Fuzzy Demand |
|---|---|
| $p_1 = 4$ | 250 units |
| $P_2 = 5$ | 210 units |
| $p_3 = 6$ | 150 units |

Table 2. Membership Functions of Fuzzy Demand Values

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1 = 4$ | $D_1$ | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 |
| $p_2 = 5$ | $D_2$ | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 |
| $p_3 = 6$ | $D_3$ | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
| $\mathbf{p_i}$ | $\boldsymbol{\mu_{Di}}$ | 0 | 0,25 | 0,5 | 0,75 | 1 | 0,85 | 0,65 | 0,3 | 0 |

Table 3. The costs $\tilde{U}_{1,os}$ incurred by order Q = 260 and $D_1 \epsilon$ {210, 220, …, 290) when $p_l = 4$

| | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ | $d_7$ | $d_8$ | $D_9$ |
|---|---|---|---|---|---|---|---|---|---|
| D | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 |
| $\mu_D$ | 0 | 0,25 | 0,5 | 0,75 | 1 | 0,85 | 0,65 | 0,3 | 0 |
| $F_o$ | 50 | 40 | 30 | 20 | 10 | 0 | 0 | 0 | 0 |
| $F_s$ | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 20 | 30 |
| $F_{so}$ | 50 | 40 | 30 | 20 | 10 | 0 | 30 | 60 | 90 |
| $\mu_{Fo}$ | 0 | 0,25 | 0,5 | 0,75 | 1 | 0,85 | 0,65 | 0,3 | 0 |

For example demand is estimated to be "about 250 products" for "unit price $p_1 = 4$". The membership function $\mu_{\tilde{D}1}$ is defined on the discrete domain {210,220, …, 290}. Related membership values are represented in Table 2. For all price alternatives, i.e. fuzzy demand values, same possibility distributions are taken for computational purposes.

Consider $\bar{D}_1$ , the order quantity Q* will be selected from the set {210, 220, …, 290) to maximize the possible net profit.

Determination of the total cost is performed through the following procedure. Let, for example, Q = 260. Imprecise demand generates imprecise costs $\tilde{U}_{1,o}$ and $\tilde{U}_{1,s}$, and their sum $\tilde{U}_{1,os}$. If D = 210 then $\tilde{U}_{1,os} = 50$ and the possibility of this event is 0; if D = 220 then $\tilde{U}_{1,os} = 40$ with the possibility of 0.25 and so on (see Table 3).

The possibility distribution of Ũ1,os is presented in Table 4. For example, Ũ1,os = 30 in two cases: for D = 230 with possibility 0,5 and for D = 270 with possibility 0,65. This implies that the possibility of costs sum taking value 30 is 0,65. (see Table 4)

Table 4. Fuzzy Sum $\tilde{U}_{1,os}$

| $\tilde{U}_{1os}$ | $\mu_{\tilde{U}1os}$ |
|---|---|
| 10 | 1 |
| 20 | 0,75 |
| 30 | 0,65 |
| 40 | 0,25 |
| 50 | 0 |
| 60 | 0,3 |

The scalar that appropriately represents $\tilde{U}_{1,os}$, is selected. The moment rule is applied

$$U_{1,tot}(260) = c_p \times 260 + \text{defuzz}\left(\tilde{U}_{1,os}\right) = 544,58$$

Finally, the possible total cost when Q = 260 is

$$\text{defuzz}\left(\tilde{U}_{1,os}\right) = \frac{(40\times0.25+30\times0.65+20\times0.75+10\times1+60\times0.3)}{(0.25+0.65+0.75+1+0.3)} = 24.58$$

The same procedure is performed for every possible Q. The results obtained by using the fuzzy model are presented in Table 5.

Table 5. Results of Cost Calculation, pi = 4

| Ordered Quantity | Possible Cost |
|---|---|
| 210 | 543,84 |
| 220 | 547,6 |
| 230 | 541,29 |
| 240 | 532,25 |
| 250 | 535,74 |
| 260 | 544,58 |
| 270 | 564,93 |
| 280 | 590,88 |
| 290 | 621,11 |

Determination of the total revenue is performed through a similar procedure. Again for p1=4, vague demand generates vague revenue Ř1. Table 6 gives the possible revenue values with their membership functions for Q = 260.

Table 6. Fuzzy Revenue and Its Membership Values, p1 = 4 and Q=260

| $\check{R}_1$ | $\mu_{\check{R}1}$ |
|---|---|
| 840 | 0 |
| 880 | 0,25 |
| 920 | 0,5 |
| 960 | 0,75 |
| 1000 | 1 |
| 1040 | 0,85 |

The scalar that appropriately represents Ř1, is calculated by moment defuzzification rule as:

$$\text{defuzz}\left(\bar{R}_1\right) = \frac{(880\times0.25+920\times0.5+960\times0.75+1000\times1+1040\times0.85)}{(0.25+0.5+0.75+1+0.85)} = 980.3$$

The same procedure is performed for every possible Q. The revenue values obtained by using the fuzzy model are presented in Table 7.

Table 7. Fuzzy Revenue Values for each Q, p1 = 4

| Ordered Quantity | Revenue |
|---|---|
| 210 | 840 |
| 220 | 880 |
| 230 | 912 |
| 240 | 937,14 |
| 250 | 960 |
| 260 | 980,3 |
| 270 | 996,5 |
| 280 | 1005,12 |
| 290 | 1005,12 |

Now the profit values calculated by $\prod$ = defuzz ($\check{R}$) - {$c_p \times Q$ + defuzz ($\tilde{U}_{os}$)} for all possible order quantity decisions, with $p_1$ = 4, is given in Table 8. The recommended order quantity is Q* = 260

Table 8. Possible Profit Values for $p_1$=4

| Ordered Quantity | Possible Profit |
|---|---|
| 210 | 296,16 |
| 220 | 332,40 |
| 230 | 370,71 |
| 240 | 404,89 |
| 250 | 424,26 |
| 260* | 435,72 |
| 270 | 431,57 |
| 280 | 414,24 |
| 290 | 384,01 |

Same procedure is repeated for all alternative prices and related fuzzy demand distributions. Consolidated data is represented in Table 9.

Table 9. Results of Fuzzy Model

| $p_i$ | Ordered Quantity | Possible Profit |
|---|---|---|
| 4 | 260 | 435,72 |
| 5* | 210* | 544,26 |
| 6 | 170 | 529,82 |

According to the fuzzy model proposed firm should choose the alternative price 5 for its product and should order 210 units of product which results the highest profit.

# 4 Conclusion

The fuzzy versions of the newsvendor pricing problem with vague demand are studied. It is shown that fuzzy sets are suitable for modeling the uncertain input data in the newsboy pricing problem that are subjectively estimated. The fuzzy model developed is flexible: it includes and operates with both precisely and imprecisely specific data.

*References:*


[1] J.S. Yao, K. Wu, Consumer Surplus and Producer Surplus for Fuzzy Demand and Fuzzy Supply, *Fuzzy Sets and Systems 103 (1999)*

[2] J.S. Yao, T.S. Shih, Fuzzy Revenue for Fuzzy Demand Quantity Based on Interval-Valued Fuzzy Sets, *Computers & Operation Research 29 (2002)*

[3] R.L. Zhan, Z.J.M. Shen, Newsvendor problem with pricing: Properties algorithm and simulation, *Proceedings of the 2005 Winter Simulation Conference*

[4] S.C. Chang, Optimal Fuzzy Revenue for Fuzzy Demand Quantity, *Fuzzy Sets and Systems 111 (2000)*

[5] D. Bertsimas, S. Boer, Dynamic pricing and inventory control for multiple products, *Journal of Revenue and Pricing Management ( Jan 2005)*

[6] D. Petrovic, R. Petrovic, M. Vusjosěvic, Fuzzy Models for the Newsboy Problem, Int. J. Production Economics 45 (1996)

[7] L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems 1(1978)*