The Feasibility Study of Applying Fuzzy Structural Modeling on Knowledge Structure Analysis

YUAN-HORNG LIN
Department of Mathematics Education
National Taichung University
140 Min-Shen Rd., Taichung City 403, Taiwan
TAIWAN

HE-KAI CHEN
Graduate School of Educational Measurement and Statistics
National Taichung University
140 Min-Shen Rd., Taichung City 403, Taiwan
TAIWAN

Abstract: The purpose of this study is to apply an integrated algorithm for knowledge structure analysis. The integrated algorithm includes Item Response Theory (IRT) and Fuzzy Structural Modeling (FSM). The authors use the psychometric model IRT for calculation the ability of task-takers. Based on original response data and item-concept matrix, the individualized fuzzy subordinate matrix is acquired. FSM is applied in the fuzzy subordinate matrix so that individualized knowledge structures is depicted by the hierarchical graphs. According to the empirical data analysis of mathematics test, it shows that the integrated algorithm is an effective methodology for knowledge analysis and cognition diagnosis.

Key-Words: fuzzy theory, fuzzy structural modeling, interpretive structural modeling, knowledge structure analysis.

1 Introduction

Knowledge representation is one purpose of psychometric research field. It had been discussed abundantly in recent years [13] [15]. Whether it's an expert system or a computer vision system, all are currently experimenting with knowledge-base approaches [1]. One way of knowledge representation is to display knowledge structures by visual system, such as hierarchical graphs [5] [9]. Therefore, the knowledge structure analysis is a complexity system methodology [8] [14] [18]. Among the complexity system analysis, Interpretive Structural Modeling(ISM) developed on the basis of graph theory is considered as an effective way to construct structural models of complex systems [2] [5]. However, the constraint of binary relation among elements decreased the application [3] [19]. The binary relation among elements can not indicate the real situation in system [7].

Fuzzy Structural Modeling (FSM) was proposed by Tazaki and Amagasa [3]. FSM can organize hierarchy for several complex problems. FSM is based on fuzzy sets theory and it could replace the binary relation among elements in complex system with fuzzy relations. As for overview on the related research FSM, FSM could be applied to solve complex problems in systems of varied fields [3] [19] . However, little is known about its application on education measurement and psychometrics [6] [17]. On the other hand, knowledge structures vary with personal knowledge storage and usage [11]. Therefore, individualized knowledge structure analysis is an important psychometric methodology [16].

In this study, the authors will apply the Item Response Theory (IRT) for data analysis so that individualized ability and item-concept attribute matrix could constitute individualized fuzzy subordinate matrix. Therefore, the authors could use FSM to analyze the hierarchical knowledge structure of each task-taker.

The authors provide the empirical data analysis of Equality Axiom Concept Test. The authors implement FSM software and analyze the results and IRT analysis [4]. Consequently, the structural graph is made and the hierarchical features are discussed for cognition diagnosis.

2 Theoretical Background

2.1 Generating Fuzzy Subordinate Matrix

For a test which measures several concepts, we can get the information about item response probability and correct probability on concepts given ability of task-takers. Suppose M ($m=1,2,\cdots,M$) express the number of items in a test which measures L ($l=1,2,\cdots,L$) concepts. Besides, there are K task-takers ($k=1,2,\cdots,K$) who take this test. The item-concept matrix is denoted by $A=(a_{ml})_{M\times L}$. $a_{ml}=1$ means item m exactly measures concept l ; otherwise $a_{ml}=0$ means item m could not measure concept l . Let $SA=(\sum_{m=1}^{M} a_{ml})_{1\times L}=(a_{\bullet l})_{1\times L}$ be a matrix and represent the number concepts in which each item measures.

With response data matrix $\mathbf{X}=(x_{nm})_{N\times M}$ and under two-parameter logistic IRT model analysis, the ability of task-taker k is θ_k and his correct response on item m is

$$P_m(\theta_k)=\frac{1}{1+e^{-\alpha_m(\theta_k-\beta_m)}} \qquad (1)$$

Where α_m means item discrimination parameter and β_m is the item difficulty parameter [10]. Then the degree of master on each concept for task-taker k could be denoted as following matrix [4] [6]

$$\begin{aligned} MA(\theta_k) &= [P_m(\theta_k)]_{1\times M}[a_{ml}/a_{\bullet l}]_{M\times L} \\ &= [ma_l(\theta_k)]_{1\times L} \end{aligned} \qquad (2)$$

Let $c_i=ma_i(\theta_k)$ and $o_j=1-ma_j(\theta_k)$, with the definition of fuzzy logic model of perception [2] [12], then $P_{ij}(\theta_k)$ represents the probability for concept i being precondition of concept j and

$$\begin{aligned} P_{ij}(\theta_k) &= \frac{c_i o_j}{c_i o_j+(1-c_i)(1-o_j)} \\ &= \frac{ma_i(\vartheta_k)[1-maj(\theta_k)]}{ma_i(\theta_k)[1-ma_j(\theta_k)]+[1-ma_i(\theta_k)]ma_j(\theta_k)} \end{aligned} \qquad (3)$$

We get the fuzzy subordination matrix $D(\theta_k)=[P_{ij}(\theta_k)]_{L\times L}$ on all L concepts for task-taker k . This matrix is the original data matrix for FSM.

2.2 Algorithm of Fuzzy Structure Modeling

2.2.1 Basic Definition

Before the procedure of structural modeling algorithm, some properties with respect to fuzzy subsets and fuzzy relations are shown as mathematical preliminaries [3].

The fuzzy complement of A, denoted by $\bar{A}$, is characterized by the following relation

$$\mu_{\bar{A}}=(1-\mu_A)/(1+\lambda\mu_A) \qquad (4)$$

Where a parameter λ is a real number in $-1<\lambda<\infty$.

A fuzzy binary relation and its complement in a direct product space of S and S are respectively characterized by f_R and $f_{\bar{R}}$ as follows

$$\left.\begin{aligned} f_R &: S\times S\to[0,1] \\ f_{\bar{R}} &: S\times S\to[0,1] \end{aligned}\right\} \qquad (5)$$

Where the relation of f_R and $f_{\bar{R}}$ is given by

$$f_{\bar{R}}(s_i,s_j)=\frac{(1-f_R(s_i,s_j))}{(1+\lambda f_R(s_i,s_j))} \qquad (6)$$

The fuzzy semi-transitive law plays an important role on the process of FSM. The definition is as follows

Let $m_{ik}=\vee_{j=1}^{n}(f_R(s_i,s_j)\wedge f_R(s_j,s_k))\ge p$ for $\forall(s_i,s_j),(s_j,s_k)\in S\times S$ $(i\ne j\ne k)$. When $f_R(s_i,s_k)\ge m_{ik}$ for any (s_i,s_k) exists, the relation is called fuzzy semi-transitive.

Let A be a fuzzy subordination matrix among the elements of S where $S=\{s_1,s_2,\cdots,s_n\}$ and

$$A=\lfloor a_{ij}\rfloor,\quad i,j=1,2,\cdots,n \qquad (7)$$

Where A is a square $n\times n$ matrix and the element a_{ij} of A is given by the fuzzy binary relation f_R as follows

$$a_{ij}=f_R(s_i,s_j)\quad 0\le a_{ij}\le 1,\quad i,j=1,2,\cdots,n \qquad (8)$$

a_{ij} displays the grade of which s_i is subordinate to s_j. We set one parameter p as a threshold by which we can determine whether s_i is subordinate to s_j or not. When the grade of subordination is greater than p, s_i is subordinate to s_j.

When the fuzzy semi-transitive law is satisfied in a fuzzy subordination matrix, A is called a fuzzy semi-reachability matrix.

2.2.2 Process of Structural Modeling

The procedure of structural modeling algorithm is given by the following steps.

***Step* 1**

Give a fuzzy subordination matrix $A = \lfloor a_{ij} \rfloor$ and let the fuzzy semi-reachability matrix A' satisfying the fuzzy semi-transitive law from A.

***Step* 2**

Let $S = \{s_1, s_2, \cdots, s_n\}$ is divided into level sets, which consists of a top level set $L_t(s)$, an intermediate level set $L_i(s)$, a bottom level set $L_b(s)$ and an isolation level set $L_{is}(s)$. The definitions are defined as follows:

$$L_t(s) = \left\{ s_i \left| \bigvee_{j=1}^{n} a_{ij} < p \le \bigvee_{j=1}^{n} a_{ji} \right. \right\} \tag{9}$$

$$L_i(s) = \left\{ s_j \left| \bigvee_{k=1}^{n} a_{kj} \ge p, \bigvee_{k=1}^{n} a_{jk} \ge p \right. \right\} \tag{10}$$

$$L_b(s) = \left\{ s_i \left| \bigvee_{j=1}^{n} a_{ji} < p \le \bigvee_{j=1}^{n} a_{ij} \right. \right\} \tag{11}$$

$$L_{is}(s) = \left\{ s_j \left| \bigvee_{k=1}^{n} a_{kj} < p, \bigvee_{k=1}^{n} a_{jk} < p \right. \right\} \tag{12}$$

We identify the level sets $L_t(s), L_b(s)$ and $L_{is}(s)$ on the basis of A'. Further, determine the subordination relation sets $B(s_i)$ between $L_t(s)$ and $L_b(s)$ $(s_i \in L_b(s))$ and the block sets $\{Q_j\}$. $B(s_i)$ is a subset of $L_t(s)$ and

$$B(s_i) = \left\{ s_j \left| a'_{ij} \ge p, s_i \in L_b(s), s_j \in L_t(s) \right. \right\} \tag{13}$$

Let Q_j be a block set of $L_t(s)$ which is a single hierarchy.

***Step* 3**

Eliminate all of the columns including elements belonging to $L_b(s)$ and the rows and columns including elements belonging to $L_{is}(s)$. The fuzzy subordination matrix consisting of remaining rows and columns is reconstructed as A' [3].

***Step* 4**

From A' obtained in Step 3, construct the single hierarchy matrix $A^{(j)}$ corresponding to each block set Q_j.

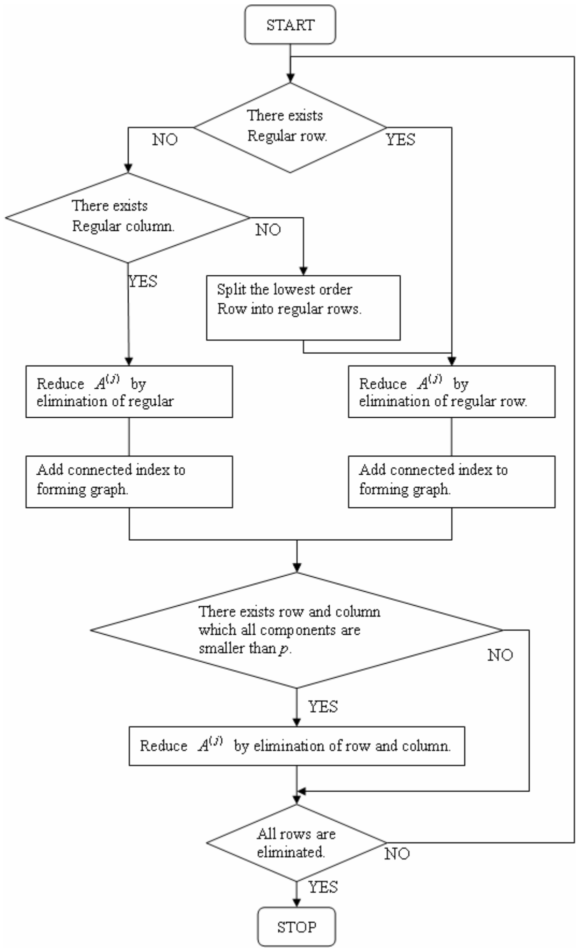

Fig. 1. Flowchart for Graphic Construction

***Step* 5**

Set up a fuzzy structure parameter λ and identify the graphic structure concerned with each single hierarchy matrix $A^{(j)}$ according to the flowchart in Fig. 1. [3]

3 Research Design

The Equality Axiom Concept Test contains 13 concepts and 465 sixth graders participating in the test. The contents of each concept are depicted in

Table 1.

Table 1. The Contents of Equality Axiom Concept

Concept	Contents
A_1	Recording the equation with symbolic expressions which contain one unknown number
A_2	Recording the equation with symbolic expression which contains two unknown number
A_3	Viewing the equation with symbolic expressions in the form of number-symbol as a representation of the result of multiplying the symbol by the number
A_4	Viewing the equation with symbolic expression in the form of symbol / number as a representation of the result of dividing the symbol by the number
A_5	Understanding the transitivity of Equality
A_6	Comprehending the equivalence of equation when adding a number to both side of the equal sign
A_7	Comprehending the equivalence of equation when subtracting a number from both side of the equal sign
A_8	Comprehending the equivalence of equation when multiplying both side of the equal sign by a number
A_9	Comprehending the equivalence of equation when dividing both side of the equal sign by a number
A_{10}	Using multiple-steps equation, reversal algorithm, or other methods to solve the equation with symbolic expressions
A_{11}	Solving the equation with symbolic expressions which the unknown symbol is before the equal sigh by obeying the form of Equal Axiom
A_{12}	Using Equality Axiom to solve problems in live situations
A_{13}	Solving the equation with symbolic expressions which the unknown symbol is after the equal sigh in the form of Equal Axiom.

4 Results

It is unfeasible to display hierarchical graphs of all task-takers. We will only take one task-taker for discussion. One task-taker with ability $\theta = 1.45$ is randomly sampled and we process the integrated algorithm mentioned above. The fuzzy subordinate matrix A of this task-taker is made as Table 2.

Table 2. Fuzzy Subordinate Matrix ($\theta_k = 1.45$)

Concepts	Concepts						
	A_1	A_2	A_3	A_4	A_5	A_6	A_7
A_1	0.00	0.50	0.35	0.70	0.11	0.49	0.63
A_2	0.50	0.00	0.35	0.70	0.11	0.50	0.64
A_3	0.65	0.65	0.00	0.81	0.19	0.65	0.76
A_4	0.30	0.30	0.19	0.00	0.05	0.30	0.43
A_5	0.89	0.89	0.81	0.95	0.00	0.88	0.93
A_6	0.51	0.50	0.35	0.70	0.12	0.00	0.64
A_7	0.37	0.36	0.24	0.57	0.07	0.36	0.00
A_8	0.57	0.57	0.42	0.75	0.15	0.57	0.70
A_9	0.44	0.43	0.29	0.64	0.09	0.43	0.57
A_{10}	0.85	0.85	0.75	0.93	0.42	0.85	0.91
A_{11}	0.99	0.99	0.99	1.00	0.96	0.99	1.00
A_{12}	0.56	0.55	0.40	0.74	0.14	0.55	0.68
A_{13}	0.26	0.25	0.16	0.44	0.04	0.25	0.37

Concepts	Concepts					
	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
A_1	0.43	0.56	0.15	0.01	0.44	0.74
A_2	0.43	0.57	0.15	0.01	0.45	0.75
A_3	0.58	0.71	0.25	0.01	0.60	0.84
A_4	0.25	0.36	0.07	0.00	0.26	0.56
A_5	0.85	0.91	0.58	0.04	0.86	0.96
A_6	0.43	0.57	0.15	0.01	0.45	0.75
A_7	0.30	0.43	0.09	0.00	0.32	0.63
A_8	0.00	0.63	0.19	0.01	0.52	0.80
A_9	0.37	0.00	0.12	0.00	0.38	0.69
A_{10}	0.81	0.88	0.00	0.03	0.82	0.94
A_{11}	0.99	1.00	0.97	0.00	0.99	1.00
A_{12}	0.48	0.62	0.18	0.01	0.00	0.78
A_{13}	0.20	0.31	0.06	0.00	0.22	0.00

In Table 2, we get the fuzzy subordination matrix $D(\theta_k) = [P_{ij}(\theta_k)]_{13\times13}$. The parameters p=0.65 and $\lambda = -0.5$ is chosen and fuzzy subordination matrix is processed by FSM analysis. FSM software is implemented by authors and this software processes data matrix under the Windows

XP personal computer system. The FSM graph is depicted in Fig. 3.

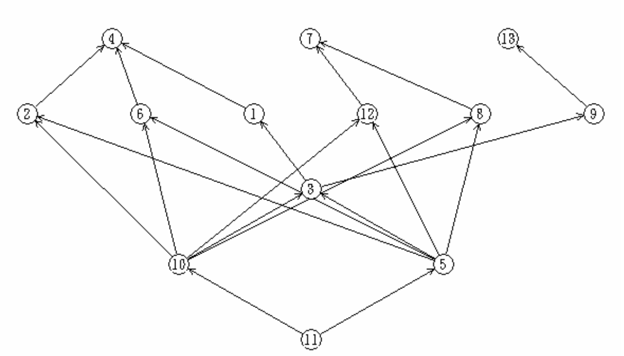

Fig. 3. Graphic Structure for Task-taker with Ability $\theta_k = 1.45$ ($p = 0.65$, $\lambda = -0.5$)

In Fig. 3, there are five hierarchies and each of which consists of several concepts. The concepts belong to each level, which is top level set $L_t(A)$, intermediate level set $L_i(A)$ and the bottom level set $L_b(A)$ respectively, are depicted in Table 3. Moreover, the isolation level set $L_{is}(A)$ was ϕ .

Table 3. The Contents of Each Level

Levels	Concepts
$L_t(A)$	A_4, A_7, A_{13}
$L_i(A)$	A_2, A_6, A_1, A_{12}, A_8, A_9
	A_3
	A_{10}, A_5
$L_b(A)$	A_{11}

A_{11} is considered as the basic concept of Equality Axiom Concept, and should be introduced at the beginning of teaching. On the other hand, A_4, A_7, A_{13} are the most difficult concepts for students to learn. Therefore, the individualized concept structure help us easily understand the knowledge structure of each task-taker.

5 Conclusions

The integrated algorithm of IRT and FSM could be feasible methodology for purpose of cognition diagnosis. The method will display individualized concept structures. FSM had been used to analyze structural hierarchies in the field science, engineering and management [17]. This research provided an educational application of FSM, and illustrated structural and hierarchical graphic which can be used to explain the individual knowledge structure of each test-taker with certain ability. To sum up, it shows the integrated algorithm will be prospective in the field of cognition diagnosis.

Reference:

[1] M. C. Mathur, Graph Theoretic Knowledge *Representation and Knowledge Organization, Proceedings of IEEE International Conference on Systems, Man and Cybernetics*, Vol. 2**,** 1992, pp.1712-1717.

[2] T. Mitamura, and A. Ohuchi, Idea Generation Supporting System based on FISM/fuzzy, *Proceedings of the 34th SICE Annual Conference*, 1995, pp. 1309-1312.

[3] E. Tazaki, and M. Amagasa, Structural Modeling in a Class of Systems Using Fuzzy Sets Theory, *Fuzzy Sets and Systems*, Vol. 2, 1979, pp. 87-103.

[4] Y. H. Lin, and S. M. Chen, The Integrated Analysis of S-P Chart and Ordering Theory on Equality Axiom Concepts Test for Sixth Graders. *WSEAS Transactions on Mathematics*, Issue 12, Vol. 5, 2006, pp. 1303-1308.

[5] J. N. Warfield, and A. R. Cardenas, *A Handbook of Interactive Management*, Iowa State University Press, 1994.

[6] J. N. Warfield, Crossing Theory and Hierarchy Mapping, *IEEE Transactions on System, Man, and Cybernetics*, Vol. SMC-7, 1977, pp. 505-523.

[7] A. Ohuchi, and M. Kurihara, An Efficient Profesure for Transitive Coupling in ISM, *IEEE Transaction on Systems, Man, and Cybernetics*, Vol. SMC-15, 1985, pp.426-431.

[8] J. N. Warfield, *Societal Systems Planning, Policy and Complexity*, Wiley, 1976.

[9] K. Sugiyama, S. Tagawa, and M. Toda, Method for Visual Understanding of Hierarchical System Structures, *IEEE Transaction on Systems, Man, and Cybernetics*, Vol. SMC-11, 1981, pp.109-125.

[10] W. J. van der Linden, and R. K. Hambleton, *Handbook of Modern Item Response Theory*, Springer, 1997.

[11] K. K. Tatsuoka, and M. M. Tatsuoka, Computerized Cognitive Diagnostic Adaptive Testing: Effect on Remedial Instruction as Empirical Validation, *Journal of Educational*

Measurement, Vol. 34, 1997, pp. 3-20.

[12] G. Klir, and B. Yuan, *Fuzzy Sets and Fuzzy Logic, Theory and Applications*, Prentice Hall, 1995.

[13] Y. Miao, and Z. Q. Liu, On Causal Inference in Fuzzy Cognitive Maps, IEEE Transactions in Fuzzy Systems, Vol. 8, 2000, pp. 107-119.

[14] P. D. Nichols, S. F. Chipman, and R. L. Brennan, *Cognitively Diagnostic Assessment*, Lawrence Erlbaum Associates, 1995.

[15] M. J. Horvath, C. E. Kass, and W. R. Ferrell, An Example of the Use of Fuzzy Set Concepts in Modeling Learning Disability, *American Educational Research Journal*, Vol. 17, 1980, pp. 309-324.

[16] Y. Du, C. Lewis, and P. J. Pashley, Computerized Master Testing Using Fuzzy Set Decision Theory, *Applied Measurement in Education*, Vol. 6, 1993, pp. 181-193.

[17] R. W. Hawthorne, and A. P. Sage, On Applications of Interpretive Structural Modeling to Higher Education Program Planning, *Socio-Economic Planning Sciences*, Vol. 9, 1975, pp. 31-43.

[18] J. N. Warfield, Toward Interpretation of Complex Structural Models, *IEEE Transactions on System, Man, and Cybernetics*, Vol. SMC-4, 1974, pp. 405-417.

[19] A. Mandal, and S. G. Deshmukh, Vender Selection Using Interpretive Structural Modelling (ISM), *International Journal of Operations & Production Management*, Vol. 14, 1994, pp. 52-59.