Concept Structure Analysis Method based on Integration of FLMP and ISM with Application in Equality Axiom Concepts

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Abstract: The purpose of this study is to provide an integrated method which aims to analyze individualized concept structure. This method integrates algorithm of fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM). The combined algorithm of this integrated model could analyze the individualized concepts structure based on the comparisons with expert. The authors provide the empirical data analysis of equality axiom testing for pupils. The results show that task-takers with different response patterns and total score own varied concept structures. Finally, based on the findings and results, some suggestions and recommendations for future research are provided.

Key-Words: fuzzy logic model of perception, interpretive structural modeling, concept structure, α -cut .

1 Introduction

One purpose of cognition science is to understand the information processing and knowledge storage [6]. Concepts are basic elements of knowledge and they constitute attributes of knowledge [8]. Most researcher consider that knowledge is stored in a form of network. This kind of network reveals the relationship and hierarchies of concepts. Hence, analysis of concept structures is quite important for pedagogy, e-learning or computerized cognition diagnosis [2]. One benefit of educational measurement is to realize the learning condition of students. Methodologies of concept analysis are an important issue of psychometrics and computer science.

There are varied approaches as to methodologies of concept structure analysis. Knowledge space is mathematical-psychological models of knowledge structures [1]. Its knowledge state represent structures of learning results. Learning path reveal relationship of concept and learning process [3]. ALEKS is the computerized assessment system based on knowledge space.

Pathfinder is another approach of concept analysis based on foundations of graph theory [5]. Proximity matrix of concepts is acquired. By similarity rating between concepts and its algorithm, the network relationship of concepts or nodes is clearly understood.

Rule space is an integrated method which combines cluster analysis, S-P chart and item response theory. Task analysis is required to confirm the cognition attributes of items. With the analysis of rule space, response pattern of task-takers is explained by way of rules of problem-solving. M. Tatsuoka and K. K. Tatsuoka applied rule space in the cognition diagnosis of fraction concepts [20].

Another approach of analysis is based on traditional and modern psychometric method. The former is classical test theory (CTT) and the latter is item response theory (IRT). Classical test theory aims to analyze the difficulty, and discrimination of items and reliability of test. However, it mentions little about the concept structure analysis. As to item response theory, some researcher decompose the parameters of items into cognitive operations or cognition components [13]. Thus, they provide advanced models in order to present the relationship of concepts for items. For example, linear logistic test model is an extended model of one-parameter logistic model and difficulty of items is the linear combination of concepts within items [18]. It is the considerations of mixture strategies model that unique task-taker will use only one strategy to solve question [19]. Furthermore, mixture strategies model owns the characteristics of item response model and latent class model. However, most models based on item response theory provide little about the hierarchies of concepts between items.

In this study, the integrated method of fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM) will be extended into a combined algorithm. Viewpoints of α -cut operation from fuzzy theory will also be used [4]. The integrated algorithm will reveal the individualized concept structure and relationship. The authors will analyze the empirical data of equality axiom test.

2 Literature Review

The algorithm of fuzzy logic model of perception and interpretive structural modeling are the foundations of this study. The algorithm will be discussed as follows.

2.1 Fuzzy Logic Model of Perception

Fuzzy logic model of perception is to describe the probability with which combination of two stimuli could fit prototype T [12]. Suppose there is a combination of two factor C and O . There are I levels and J levels for factor C and O respectively. It is expressed as $C=\{C_1,C_2,\cdots,C_I\}$ and $O=\{O_1,O_2,\cdots,O_J\}$. The fuzzy truth value of C_i and O_j is c_i and o_j respectively. Fuzzy truth value c_i and o_j express the degree that the combination of C_i and O_j will support prototype T [12] [16]. It is derived from the viewpoints of choice rule and relative goodness rule (RGR) [14]. The probability that the combination of (C_i,O_j) could be viewed as prototype T can be expressed as follows [15]

$$p(c_i\ ,o_j)=\frac{c_i\ o_j}{c_i\ o_j+(1-c_i)(1-o_j)} \qquad (1)$$

2.2 Interpretive Structural Modeling

The theory of interpretive structural modeling (ISM) is based upon discrete mathematics and graph theory [9] [17]. J. N. Warfield [9] provided ISM and it aims to arrange elements in a hierarchical relation. For any set that contains K elements, we can make a hierarchical graph of all elements if the binary relationship between elements is known [21]. Namely, the relationship of A_i and A_j must be acquired in advance. The relationship could be expressed in the form of matrix $A=(a_{ij})_{K\times K}$. If $a_{ij}=1$ exists, A_i is the precondition of A_j . On the other hand, if $a_{ij}=0$ exists, A_i is not the precondition of A_j . The analytical procedure of ISM is as follows [10].

The ISM adopt Boolean operation. The transitive closure is $\hat{A}=A\oplus A^2\oplus A^3\oplus\cdots A^P$ and reachability matrix is $R=\hat{A}\oplus I=(A\oplus I)^P$. With transitive closure $\hat{A}$ and reachability matrix R, the hierarchical graph of elements in matrix $A=(a_{ij})_{K\times K}$ could be plotted [9]. For example, let the $A=(a_{ij})_{K\times K}$ be

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The relationship and hierarchies of elements is depicted in Fig. 1.

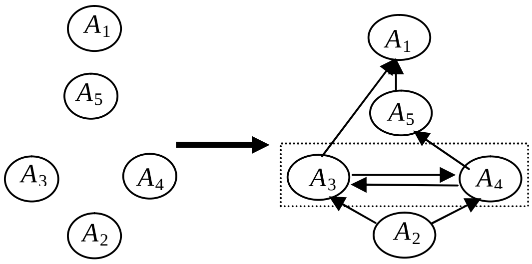

Fig. 1. The linkage of elements in hierarchies

3 Method of Concept Structure Analysis

Suppose M ($m = 1,2,\cdots,M$) express the number of items in a test which measures A ($a = 1,2,\cdots,A$) concepts. Besides, there are N task-takers ($n = 1,2,\cdots,N$) who take this test. The following symbols should be defined before the explanation of algorithm.

(1) The response data matrix from a test is denoted by $\mathbf{X} = (x_{nm})_{N\times M}$. $x_{nm} = 1$ if task-taker n gives correct answer on item m; otherwise $x_{nm} = 0$ if task-taker n gives wrong answer on item m.

(2) The item attribute matrix is denoted by $\mathbf{Y} = (y_{ma})_{M\times A}$. $y_{ma} = 1$ means item m exactly measures concept a; otherwise $y_{ma} = 0$ means item m could not measure concept a.

(3) These A concepts construct 2^A ideal concept vectors, which are denoted by $\mathbf{z}_i = (z_{ia})_{1\times A} = (z_{i1}, z_{i2}, \cdots, z_{iA})$, $i = 1,2,\cdots,I$ and $I = 2^A$. Each ideal concept vector expresses one certain kind of concept structure for task-takers. $z_{ia} = 1$ means the ideal concept vector $\mathbf{z}_i$ contains concept a; otherwise $z_{ia} = 0$ means the ideal concept vector $\mathbf{z}_i$ does not contain concept a. Based on total number of A concepts, these 2^A ideal concept vectors construct the ideal concept matrix $\mathbf{Z} = (z_{ia})_{I\times A}$.

(4) Each ideal concept vector $\mathbf{z}_i$ could correspond with only one ideal response vector $\mathbf{r}_i = (r_{im})_{1\times M} = (r_{i1}, r_{i2}, \cdots, r_{iM})$, where $i = 1,2,\cdots,I$ and $I = 2^A$. Ideal response vector $\mathbf{r}_i$ means the response pattern on all items based on the correspondence of ideal concept vector $\mathbf{z}_i$ with item attribute matrix $\mathbf{Y} = (y_{ma})_{M\times A}$. $r_{im} = 1$ means ideal response vector $\mathbf{z}_i$ could provide correct answer on item m; otherwise $r_{im} = 0$ means ideal response vector $\mathbf{z}_i$ could provide correct answer on item m. These total number of I ideal response vectors constitute the ideal response matrix $\mathbf{R} = (r_{im})_{I\times M}$.

(5) The standardized closeness of item response vector $(x_{n1}, x_{n2}, \cdots, x_{nM})$ for task-taker n between ideal response vector $\mathbf{r}_i = (r_{i1}, r_{i2}, \cdots, r_{iM})$ is sc_{ni}. The higher the sc_{ni} is, the more similar $(x_{n1}, x_{n2}, \cdots, x_{nM})$ and $(r_{i1}, r_{i2}, \cdots, r_{iM})$ is. Let $\mathbf{SC} = (sc_{ni})_{N\times I}$ expresses the standardized closeness matrix of all N task-takers with regard to all ideal response vectors.

Only the two matrices $\mathbf{X} = (x_{nm})_{N\times M}$ and $\mathbf{Y} = (y_{ma})_{M\times A}$ are known already. The following subsections of algorithms, which are AMC, ASC and AFISM, describe the steps to analyze individualized concept structures.

3.1 Algorithm for Mater of Concepts (AMC)

(1)With the known item attribute matrix $\mathbf{Y} = (y_{ma})_{M\times A}$ and the ideal concept matrix $\mathbf{Z} = (z_{ia})_{I\times A}$, the ideal response matrix $\mathbf{R} = (r_{im})_{I\times M}$ is defined as follows.

$$r_{im} = \begin{cases} 1 \quad , & (z_{ia})(y_{ma}) = y_{ma} \ , \ \forall\ a = 1,2,\cdots,A \\ 0 \quad , & \text{else} \end{cases} \tag{2}$$

(2) The closeness between response pattern of task-taker n and ideal response vector $\mathbf{r}_i$ is c_{ni}. It is

$$c_{ni} = \sum_{m=1}^{M} (x_{nm}) \circ (r_{im}) \Big/ M \tag{3}$$

where $(x_{nm}) \circ (r_{im}) = \begin{cases} 1 \quad , & x_{nm} = r_{im} \\ 0 \quad , & x_{nm} \neq r_{im} \end{cases}$

(3) For task-taker n, his standardized closeness sc_{ni} in matrix $\mathbf{SC} = (sc_{ni})_{N\times I}$ is defined as

If K ($K \geq 1$)different c_{ni} values satisfy $c_{ni} = 1$, the crisp recognition is used to calculated sc_{ni} and

$$sc_{ni} = \begin{cases} 1/K & , \quad \forall\, c_{ni} = 1 \\ 0 & , \quad else \end{cases} \tag{4}$$

If $c_{ni} \neq 1 \quad \forall\, i = 1,2,\cdots,I$, the fuzzy recognition is used to calculated sc_{ni} and

$$sc_{ni} = c_{ni} \Big/ \sum_{i=1}^{I} c_{ni} \tag{5}$$

No matter crisp recognition or fuzzy recognition, the standardized closeness satisfies $0 \leq sc_{ni} \leq 1$ and $\sum_{i=1}^{I} sc_{ni} = 1$.

3.2 Algorithm for Subordination of Concepts (ASC)

(1) Llet $\mathbf{D} = (d_{na})_{N \times A} = (\mathbf{SC})(\mathbf{Z})$ denote the magnitude of master for task-taker n on concept a. It is

$$d_{na} = \sum_{i=1}^{I} (sc_{ni})(z_{ia}) \quad \text{and} \quad 0 \leq d_{na} \leq 1 \tag{6}$$

(2) According to the formula of FLMP, for task-taker n , the probability of concept a being precondition of concept a' (subordination relation probability) is

$$P_{aa'} = \begin{cases} 1 & , \quad d_{na} = d_{na'} = 1 \\ 0 & , \quad d_{na} = d_{na'} = 0 \\ \dfrac{(d_{na})(1-d_{na'})}{(d_{na})(1-d_{na'}) + (1-d_{na})(d_{na'})} & , \quad else \end{cases} \tag{7}$$

3.3 Algorithm for Fuzzy ISM (AFISM)

(1)For any task-taker n, his fuzzy relation matrix got from the above ASC algorithm is $F_n(p_{aa'})_{A \times A}$. α-cut on fuzzy relation matrix is applied so that the corresponding binary relation matrix is acquired [7]. Therefore, appropriate α value ($0 \leq \alpha \leq 1$) is selected and the binary relation matrix F_n^{α} is

$$F_n^{\alpha} = (p_{aa'}^{\alpha})_{A \times A} \quad \text{and} \quad p_{aa'}^{\alpha} = \begin{cases} 1 & , \quad p_{aa'} \geq \alpha \\ 0 & , \quad p_{aa'} < \alpha \end{cases} \tag{8}$$

(2)The adjacent matrix for task-taker n on all concepts is

$$p_{aa'}^{\alpha} = \begin{cases} 1 & , \quad p_{aa'} \geq \alpha \\ 0 & , \quad p_{aa'} < \alpha \end{cases} , \quad 0 \leq \alpha \leq 1 \tag{9}$$

(3)For binary relation matrix $p_{aa'}^{\alpha}$ of task-taker n, the ISM is applied so that the hierarchical graph represent the individualized concept structures.

4 Data Resource

The test of equality axiom is designed by authors. There are 465 sixth graders from Taiwan in this study. The test includes 10 items and each item contains one concept attribute. The concept attribute within each item are depicted in Table 1. Concepts of equality axiom are the basis of algebra learning. All these items are dichotomous. In this study, $\alpha = .65$ is selected in Algorithm for Fuzzy ISM (AFISM). The authors implemented the software .

Table 1. The Concept Attributes within Each Item

Item	Concept Attribute
1	List an unknown number in equation
2	List two unknown numbers in equation
3	Express the multiplicative relation between symbol and number
4	Express the relation of multiplication between symbol and number
5	Express the relation of division between symbol and number
6	Equality axiom of addition
7	Equality axiom of subtraction
8	Equality axiom of multiplication
9	Equality axiom of division
10	Solve the unknown number correctly

5 Results

The correct ration of each item are depicted in Table 2. As shown in Table 2, the correct ration of items varies a lot. It implies that concept structures may exist.

Table 2. Correct Ratio of Items

Item	Correct Ratio	Item	Correct Ratio
1	.809	6	.916
2	.398	7	.333
3	.785	8	.806
4	.757	9	.578
5	.370	10	.114

Although the combined algorithm of FLMP and ISM could provide the concept structure of each task-taker, it is unfeasible to display the concept structure of all task-takers here. Thus, the following two subsections will display concept structures by giving example of concept structures from several task-takers.

In the first subsection 5.1., we will provide two task-takers of different total score so that we could realize the characteristics of concept structures. In the second subsection 5.2., we will provide one pair of task-takers who have the same total score but with different response pattern. It is predicted that task-takers have the same total score with different response pattern will reveal varied concept structure.

5.1 Concept Structure of Different Total Score

Two task-takers with different total score are randomly selected. In Fig. 2 and Fig. 3, the symbol of each concept and magnitude of master on concept d_{na} is shown. These two task-takers have total score of 2, 8 respectively. They own different concept structure.

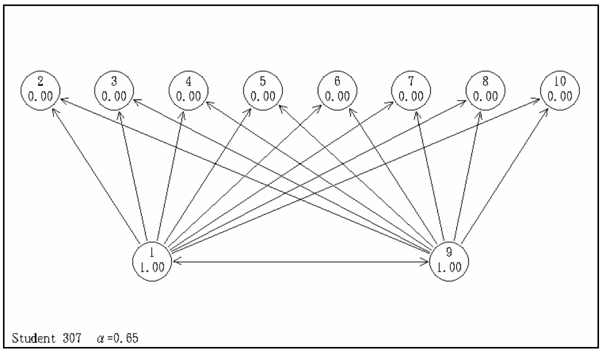

Fig. 2. The Concept Structure of Student ID 307 (Total Score= 2)

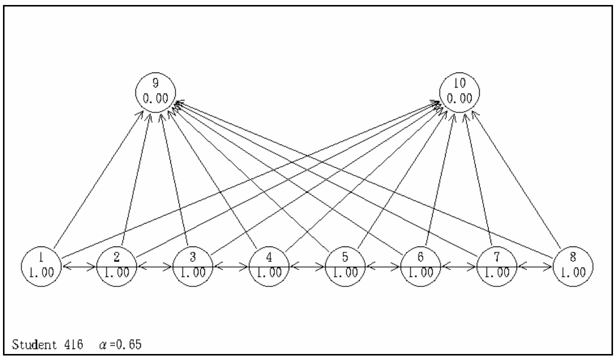

Fig. 3. The Concept Structure of Student ID 416 (Total Score= 8)

5.2 Concept Structure of Same Total Score with Different Response Pattern

As shown in Table 3, one pair of students with the same total score are randomly selected. Both students have total score of 4 with different response pattern.

Table 3. Two Pairs of Students with Response Pattern and Total Score

student ID	response pattern	total score
120	0011010100	4
421	0000110110	4

As shown in Fig. 4 and Fig. 5, these two students have varied concept structures. This phenomenon supports viewpoint of cognition psychology and psychometrics that response pattern could distinguish characteristics of concept structure, but not total score.

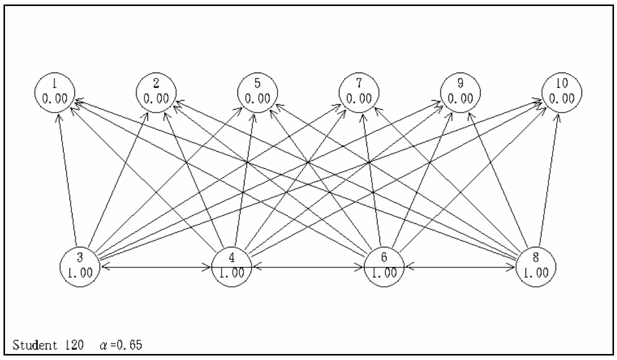

Fig. 4. The Concept Structure of Student ID 120

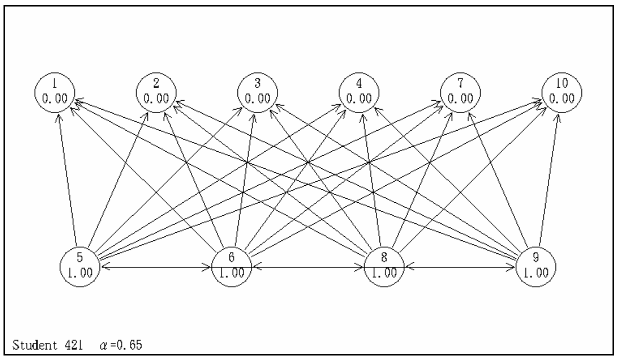

Fig. 5. The Concept Structure of Student ID 421

6 Conclusions

An integrated method of FLMP and ISM for analyzing individualized concept structure is provided in this study. With this integrated algorithm, the graphs of concept structures will display the characteristics knowledge structure.

The authors also implement the software and apply the integrated algorithm in the empirical data of equality axiom test from pupils. It shows that students with different total score own varied concept structure. Moreover, students have the same total score with different response pattern display distinct concept structure. This consequence corresponds with foundation of cognition diagnosis in psychometrics [11]. To sum up, this integrated algorithm could improve the assessment methodology of cognition diagnosis.

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