Retrieving the Most Probable Solution in a Temporal Interval Algebra Network

HAIYI ZHANG & XINYU XING & ANDRE TRUDEL
Jodrey School of Computer Science,
Acadia University
Wolfville, Nova Scotia, B4P 2R6
CANADA

Abstract: - In this paper, we propose a probability-based approach to retrieve the most probable solution in a temporal Interval Algebra (IA) network. In our approach, all probable solutions are abstracted as a Markov Chain. Based on this Markov Chain, we utilize Chapman-Kolmogorov functional equation to compute the most probable solution. Furthermore, in order to achieve easy and friendly operations for IA network's researchers, we attempt to adopt constraint logic programming to implement software which is able to support temporal an IA network. The implementation of the software is based on finite domain non-binary CSPs. In addition, we briefly provide an application of retrieving the most probable solution to show the functions of software. Through experiments, we conclude that the implementation can satisfy the desired goal.

Key-Words: - Logic, temporal reasoning, CSP, Constraint logic programming, Chapman-Kolmogorov.

1. Introduction

An IA (interval algebra) network is a graph where each node represents an interval. Directed edges in the network are labeled with subsets of I. By convention, edges labeled with I are not shown. An IA network is consistent (or satisfiable) if each interval in the network can be mapped to a real interval such that all the constraints on the edges hold (i.e., one disjunct on each edge is true) [6],[7].

The implementations mentioned above are non-trivial. The user must be an expert in the implementation language and software. The average user is not capable of making even simple extensions or modifications. We present an implementation which is probably not as efficient as the ones previously mentioned. Our goal is user friendliness and ease of use. The user draws an IA network, and then clicks a button for a solution. The target audience is researchers that need to quickly verify or generate a few solutions, and students entering the temporal area.

2. Markov Chain in Glance[8]

A Markov chain is a series of states of a system that has the Markov property. At each time the system may have changed from the state it was in the moment before, or it may have stayed in the same state. The changes of state are called transitions. If a sequence of states has the Markov property, it means that every future state is conditionally independent of every prior state given the current state.

Markov Chain can be modeled as a sequence of random variables X_1, X_2, X_3, … with the Markov property, namely that, given the present state, the future and past states are independent. i.e.,

$$p\left(X_{n+1} \mid X_n = x_n, \ldots, X_1 = x_1\right) = p\left(X_{n+1} = x \mid X_n = x_n\right).$$

3. IA Networks & Binary CSPs

We adopt Tsang's [3] binary CSP definition. A binary CSP of n variables x_1, …, x_n has a domain D_i of possible values associated with each variable x_i. Each D_i is finite, and it may not necessarily be the case that all the domains are equal. A binary constraint, R_{ij}, between variables x_i and x_j is a subset of the Cartesian product of their domains. Each R_{ij} is finite. We also require that $(a,b) \in R_{ij}$ if and only if $(b,a) \in R_{ji}$. An IA network is a binary CSP with infinite domains. The intervals are the variables. The domain of each variable is the set of pairs of reals of the form (x,y) where x < y. The constraint between two variables i and j is the label on the edge (i,j) in the IA network. During the past two decades, research on IA networks and finite domain CSPs has progressed relatively independently. The reason is that algorithms specifically designed for finite domains are usually not applicable to infinite domains. It was not widely known that IA networks are indeed finite domain CSPs. For example, van Beek and Manchak [5] write that "two of their heuristics cannot be applied in our context as the heuristics assume a constraint satisfaction problem with finite domains, whereas IA networks are examples of constraint satisfaction problems with infinite domains".

Recently, Thornton et al. [2] show how to convert an IA network into an equivalent non-binary CSP with finite integer domains. They observe that the relative positions of the interval endpoints in an IA network can be used to determine consistency. For example, X = (10,15) and Y = (100.5,110) is a solution to X{b}Y. This solution imposes the ordering X- < X+ < Y- < Y+ on the endpoints where X= (X-,X+) and similarly for Y. A simpler solution is to number the endpoints from left to right which results in X = (1,2) and Y= (3,4).

An IA network with two intervals is consistent if and only if each interval can be mapped to a pair of integers (a, b) where a < b, and a, b∈{1,2,3,4} such that the constraint on the edge holds. Note that it might be the case that endpoints from different intervals get mapped to the same integer (e.g., as in the case of X {=}Y). Thornton et al. [2] generalize the integer mapping to:

Theorem: Each interval in an IA network with n intervals can be mapped to a real interval such that all the constraints on the edges hold if and only if each interval in the IA network can be mapped to an interval with integer end-points in the range 1…2n such that all the constraints on the edges hold.

Based on theorem, Thornton et al. [2] convert an IA network to a non-binary CSP with finite domains. Each endpoint becomes a variable with domain {1,…,2n}. A label on an edge from X to Y in the IA network imposes a constraint on some or all of the variables X-, X+, Y-, and Y+. For example, X {d} Y generates the constraint (Y- < X-) & (X+ < Y+) which is non-binary since the constraint involves 4 variables. They then apply local search techniques on the non-binary CSP.

We use the finite domain transformation described above. But instead of local search, we use Eclipse and constraint logic programming techniques to solve the IA network.

4. Architecture and Implementation

An overview of our implementation's components is given in figure 1 (copied from Paper [4]). The user interacts with the implementation via the graphical user interface (GUI). The GUI, built using jGraph (http://www.jgraph.com/), has 2 windows. The user enters a graph in the top window. There are buttons for drawing nodes and edges. The nodes represent intervals and are each numbered 1, 2, etc. Allen's interval relationships are entered on the edges separated by commas. There are no restrictions on the size or shape of the graph. When the user clicks on the button to request a solution, the solution is drawn in the GUI's bottom window. The graph is re-drawn as entered by the user. Each edge will be assigned a unique label. The entire graph is consistent. If not consistent, a warning message is displayed instead.

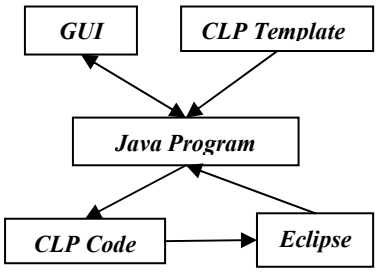

Fig. 1. Implementation

Example: 3-colour problem

Assume we have the binary CSP shown in figure 2 where the domain for each variable is {r,g,b} and the constraint on each edge is {(r,g), (r,b), (g,r),

(g,b), (b,r), (b,g)} (i.e., no adjacent nodes are assigned the same color).

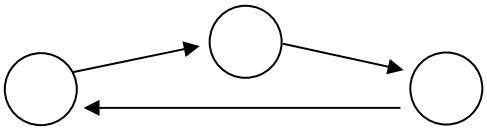

Fig. 2. CSP

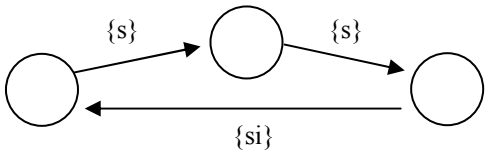

Fig.3. IA network solution

To construct P2, we associate 1 with r, 2 with g, and 3 with b. In P2, each node has domain {1,2,3} and the constraint on each edge is {(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)}. We have |U|=3, n=3, and n(2n-1)=15. To convert P2 to a reduced-integer-CSP, case 1 applies. In the reduced-integer-CSP, each domain is {1,…,15}, and the constraints are the same as P2.

The ordered domain for each variable in the reduced-interval-CSP is {(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5),(4,6),(5,6)} and each constraint is {((1,2),(1,3)),((1,2),(1,4)), ((1,3),(1,2)), ((1,3),(1,4)), ((1,4),(1,2)), ((1,4),(1,3))}.

The IA network has the same variable domains as the reduced-interval-CSP, and each constraint is {s,si}. A solution is shown in figure 3. Note that IA network consistency solvers typically return a solution that involves a single label (i.e., one of b,bi,m,mi,o,oi,d,di,s,si,f,fi,=) associated with each edge in the network. They do not return integer endpoint or real interval instantiations for the intervals (nodes).

The standard conversion of the constraint label {s} in figure 3 into a reduced-interval-CSP label would include tuples such as ((3,4),(3,6)). This tuple is not allowed in the original reduced-interval-CSP and therefore should not be included in the conversion. The permitted constraint tuples in the reduced-interval-CSP corresponding to the labels in figure 9 are shown in figure 4. Note that the number of tuples in each constraint has been reduced by half.

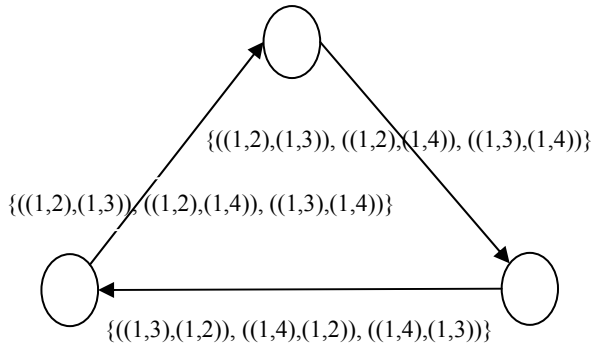

Fig. 4. Reduced-interval-CSP partial solution

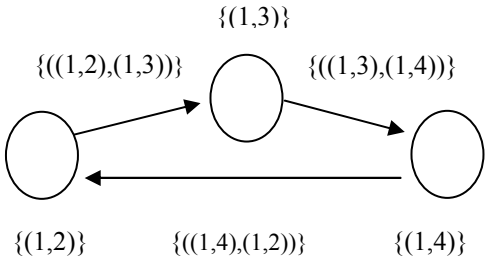

Fig. 5. Reduced-interval-CSP solution

Although each constraint tuple in the resulting reduced-interval-CSP solution is locally consistent, not every constraint tuple in figure 4 is part of a globally consistent solution. Binary CSP algorithms can be applied to figure 4 to generate a solution as shown in figure 5. The corresponding reduced-integer-CSP and P2 solutions are shown in figure 62. The final solution is shown in figure 7.

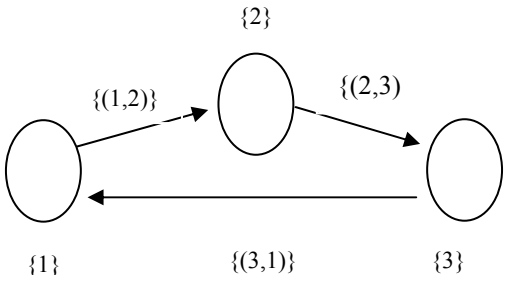

Fig. 6. Final solution

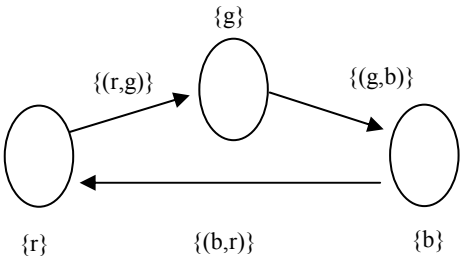

Fig.7. Reduced-integer-CSP and P2 solution

5. Execution Flow

After the user has entered a graph and requested a solution, control is given to the Java program in figure 1. The program first reads in a constraint logic program template. The template is updated with information from the particular graph to solve. The completed logic program is then passed on to Eclipse. Eclipse will solve the graph and then pass the solution to the Java program. The java program will then display the solution in the GUI.

The CLP template file contains constraint code for Allen's relations [1]. The relations are implemented by placing restrictions on the endpoints. For example, interval (XL,XR) is before (YL,YR) if and only if XR < YL. In Eclipse, we write: b(XL,XR,YL,YR,1) :- XR < YL. The "1" in the last parameter of b is a numeric representation of b and is used to keep track of the relationships on the edges. The relations are numbered from 1 to 13. The after relationship bi is implemented in terms of before: bi(XL,XR,YL,YR,2) :- b(YL,YR,XL,XR,1). The other relations are similarly implemented. The CLP template file also contains clauses to enforce that the left endpoint of each interval precedes its right endpoint.

The Java program copies the contents of the CLP template file to the CLP code file. Graph specific code is then added to the file. Assume we are given an IA network with n intervals (nodes) numbered from 1 to n. The left and right endpoints of the i'th interval are labeled Li and Ri respectively. The set of endpoints is represented in Eclipse as: EndPoints = [L1,R1, L2,R2,…,Ln,Rn]. For example, if n=4 we have: EndPoints = [L1,R1, L2,R2, L3,R3, L4,R4].

The range of each interval endpoint must be explicitly specified and is between 1 and 2n. In Eclipse, this is written in the following format: EndPoints :: 1..2n. For example, if n=4 we write: EndPoints :: 1..8.

The edges are also numbered. For example, if there are 5 edges: Edges = [E1, E2, E3, E4, E5].

Every edge constraint is a disjunction of relationships. We represent the disjunction directly. For example, let the constraint on edge E1 between intervals 1 and 2 be meets or overlaps (i.e., {m,o}). This constraint is represented in Eclipse as: (m(L1,R1, L2,Y2,E1); o(L1,R1, L2,Y2,E1)). Singleton labels are represented directly. For example if instead we have {m} we write: m(L1,R1, L2,Y2,E1).

The problem's constraints have now all been specified and we request Eclipse to generate a solution with the query: finda_solution(Edges). If a solution is found, Edges will be bound to a list of integers. Each integer represents an Allen relation for an edge.

6. Most Probable Solution

Figure 8 is a screenshot of the GUI. The top window contains the IA network entered by the user. The solution is shown in the bottom window. The CLP code file generated for this example is in paper [4]. Note that it is typical that only 1 page of Eclipse code is generated to solve the IA network.

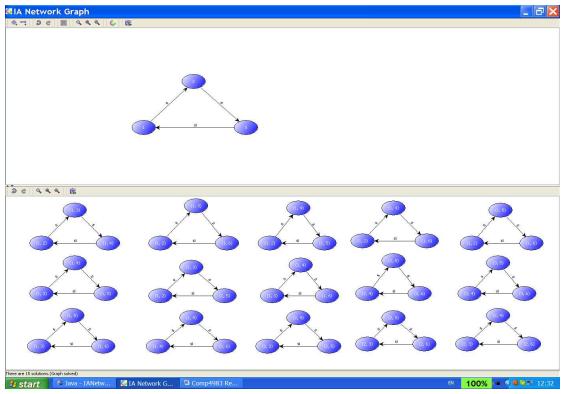

Fig. 8. The screenshot of the GUI

Consider the simple IA network in figure 4. Assume we are only interested in solutions that assign a single label to the edges. For example, we don't care about the temporal relationship between the two bottom nodes. Every label can appear in a solution for a total of 8 solutions. The straightforward approach for finding all these solutions is to first find one solution, and then

backtrack to find the others. But, we must be careful what we backtrack over:

Labels: Traditional IA network software assumes that each pair of nodes has an edge between them. If an edge is not explicitly shown, it is assumed to have a label of I. If we backtrack over the labels in the network, we must consider 17,576 possible solutions. Notice the combinatorial explosion with a network of only 4 nodes!

Endpoints: If instead, we use software based on Thornton et al's approach [4], we have a network of 4 nodes and must find an assignment of each interval's endpoints to an integer in the range from 1 to 8. Assume we have the label "b" between two nodes X and Y. There are 70 different ways that the endpoints of X and Y can be assigned to integers in the range from 1 to 8 so that X {b} Y holds. For example, one assignment is X = (1,2) and Y = (5,8). For meets, there are 56 different assignments for the endpoints.

Note that in the above, the worst case number of possibilities to backtrack over is given. Clever algorithms and heuristics can reduce the number of possibilities.

Another approach is to backtrack over the candidate solutions. We first generate a candidate solution which has one label on each edge. We check if this is a solution. We then generate the next candidate solution and so on. This is the approach we adopt and experiment with in this paper.

The code for finding a single solution was left untouched. We added code to the "CLP Code" file which first generated all the possible solutions. We then apply the original code for finding a single solution to each possible solution to verify if indeed it is a solution. The set of valid solutions are passed back to the "Java Program". The program stores the solutions in a two dimensional array and displays the solutions in the GUI one at a time.

Let us now add probabilities to Allen's relations on the edges. Each relation is assigned a probability, and the probabilities on an edge sum to 1. One interpretation for the numbers is likelihood. If there is no edge between two nodes, we assume the label is I, and each relation has equal probability 1/13.

The process of computing in IA network is based on Chapman-Kolmogorov functional equation. Through this approach, we formulate the solution finding as a Markov process, i.e., each solution is constructed as a Markov Chain. Each interval in IA network is abstracted as a state and the relation represent the transition probability as shown in Figure 9.

During the computation, a set of solutions are generated based on fundamental searching algorithm in graph theory. And then various generated solutions are formulated as different Markov Chain. Based on the Chapman-Kolmogorov functional equation (1), we compare different computing result and pick up the greatest value as the most probable result.

$$q^{(n)}(j)=\sum_{i=0}^{\infty} q(i)p_{i,j}^{(n)} \tag{1}$$

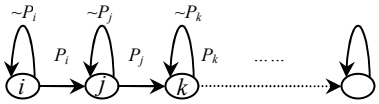

Fig.9. Markov solution Chain

As shown in figure 9, for example, when *n=0* the probability of the system being in the state *i* is *q(i)*, i.e., $q(i)=p\{X_0=i\}$, then the absolute probability of the system being in the state *k* is given by

$$\begin{bmatrix} p_i & \overline{p_i} \\ \overline{p_i} & p_i \end{bmatrix} \cdot \begin{bmatrix} p_j & \overline{p_j} \\ \overline{p_j} & p_j \end{bmatrix} = \begin{bmatrix} p & \overline{p} \\ \overline{p} & p \end{bmatrix} \tag{2}$$

$$p = p_i \cdot p_j + \overline{p_i} \cdot \overline{p_j} \tag{3}$$

We use the all solutions feature presented in the previous section to solve a probabilistic IA network. We first strip off the probabilities, and then generate and store all the solutions. For each solution, we compute a value by re-assigning a probability to each label in the solution and taking the product based on the Chapman- Kolmogorov functional equation. The solution with the highest value is the solution to the probabilistic network. This extra processing is added to the "Java Program" in Figure 1. The Eclipse code was not modified. It is trivial to add code to find the least likely, or median solution. Since finding a single solution to an IA network is an NP complete problem, finding all the solutions is not feasible for large difficult problem instances. Our implementation is targeted at small instances.

7. Conclusion and Future Work

IA networks are finite domain CSPs. They can be translated either into non-binary or binary CSPs

with finite integer domains. Given an arbitrary IA Network, it can be translated into an equivalent Integer-CSP. An Integer-CSP is a finite domain binary CSP. Off the shelf software can be used to solve the Integer-CSP. Solutions can then be translated back to IA Network solutions. Integer-CSPs are a subset of binary CSP problems. It is not the case that all binary CSPs can be translated to an equivalent IA Network problem. But, binary CSPs are equivalent to Reduced-Interval-CSPs. Solving the IA network associated with a Reduced-Interval-CSP can sometimes reduce the size of the Reduced-Interval-CSP problem and therefore simplify it.

Future work will involve generating large IA networks and comparing the efficiency of our implementation described in this paper. We need also do optimizing the software. One enhancement we are investigating, is exploiting the presence of "cut edges" or "bridges" in the network. These edges can be found in linear time using a DFS based algorithm. Either of the labels on the bridge can appear in a solution, and they do not influence other labels. If we remove the bridge, we are left with two smaller networks that can be solved quickly and their solutions combined. Our implementation is of benefit to non-technical users. The user does not need to learn specialized software and algorithms. The implementation allows the user to draw any IA network and solve it. Emphasis is on ease of use, not efficiency. We challenge the reader to find a simpler and more direct method for solving qualitative IA networks.

To our knowledge, this is the first time most probable solution to a probabilistic IA network has been solved. Unfortunately, only small size problems can be tackled with the approach described in this paper**.**

Acknowledgements

We thank Wei Zhu and Changxin Liu for coding version 1 of the implementation for their senior undergraduate project.

References:

[1] J.F. Allen. Towards a general model of action and time, Artificial Intelligence, 23(2), 1984, p. 123-154.

[2] J. Thornton, M. Beaumont, A. Sattar, and M. Maher. A local search approach to modeling and solving interval algebra problems, The journal of logic and computation, 4(1), 2004, p. 93-112.

[3] E. Tsang. Foundations of constraint satisfaction, Academic Press, 1993.

[4] A. Trudel, H. Zhang " Finding a solution. All

the soultions, or the most probable solution to a temporal interval algebra network" 5th World Enformatika Conference WEC 2005, Volume 7, Page 299-303, Prague, Czech Republic, August 2005.

[5] P. van Beek and D.W. Manchak. The design and experimental analysis of algorithms for temporal reasoning, Journal of Artificial Intelligence Research, 4, 1996, p. 1-18.

[6] Nebel. Solving hard qualitative temporal reasoning problems: evaluating the efficiency of using the ORD-Horn class, Constraints, 1, 1997, p. 175-190.

[7] V. Ryabov and A. Trudel. Probabilistic Temporal Interval Networks, 11th International Symposium on Temporal Representation and Reasoning (TIME 2004), Tatihou Island, France, 2004, p. 64-67.

[8] A.T. Bharucha-Reid. Elements of the Theory of Markov Processes and Their Applications. New York: McGraw-Hill, 1960.