A New Fuzzy Logic Controller and Its Performance

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Abstract: - A new fuzzy logic controller is designed, which can enhance the efficiency of a control process. This controller employs the implication logic to determine the weights of the defuzzification of the control signals. To verify its performance, the new controller is applied to stabilizing a torsion pendulum. Compared with the traditional fuzzy control approach, simulation shows that the new design can improve the control performance with shorter settling time.

Key-Words: - fuzzy control, implication logic, torsion pendulum

1 Introduction

Fuzzy logic, fuzzy systems and fuzzy control have been developed for several decades since 1965, with a large number of textbooks [1-4], research monographs [5-6], and research papers [7-8] published.

To minimize the total number of rules so as to reduce the complexity of a fuzzy controller, a rule base containing only two input parameters and one output parameter is the most commonly used form, as showed in Table 1. An inevitable step in a fuzzy control process is the execution of the fuzzy rule base. A typical control rule has the following form: “IF a is A AND b is B THEN c is C ”, where A, B and C are fuzzy sets, consisting of defining domains (usually, intervals) I_A , I_B , I_C , and their associate membership functions μ_A , μ_B , μ_C , respectively.

In most previous works, the membership value of the output of each fuzzy rule is the minimum (or product) of the membership values of the two input parameters of the controller. Thus, in the above-mentioned control rule, by letting c = (a AND b), one will have $\mu_c = \mu_a \wedge \mu_b = \min(\mu_a, \mu_b)$.

Table 1 A simple fuzzy control rule base

a \ b	P	Z	N
P	NB	N	Z
Z	N	Z	P
N	Z	P	PB

On the other hand, to stabilize a given system, the fuzzy controller usually uses a rule base similar to the following one (in which c is the control force):

$$u = \frac{\sum_{i=1}^{9} \mu(c_i) \times c_i}{\sum_{i=1}^{9} \mu(c_i)} = \frac{\sum_{i=1}^{9} \min\{\mu_{a,i}, \mu_{b,i}\} \times c_i}{\sum_{i=1}^{9} \min\{\mu_{a,i}, \mu_{b,i}\}}$$

It has been noticed that in controlling a system by the common fuzzy method, the efficiency of the fuzzy controller may not be always satisfied. To find a way to enhance the control efficiency, a fuzzy logic controller is proposed in this paper, with a new method to compute the membership value of each rule and thus to perform the defuzzification. In section 2, the implementation of the new fuzzy logic controller is described. In section 3, the new method is applied to stabilizing a torsion pendulum, followed by some conclusions are finally given in section 4.

2 The New Fuzzy Logic Controller

The core of the new approach is the use of the logical implication “A implies B”, with the corresponding membership value given by the logical formula $\mu(A \Rightarrow B) = \min\{1, 1 + \mu_B - \mu_A\}$ [1,2], to execute the rule base.

First, since every rule has the form “IF a is A AND b is B THEN c is C”, or simply, “IF a AND b THEN c”, by defining *x* = (a AND b), the IF part has the corresponding membership value given as usual by $\mu_x = \mu_a \wedge \mu_b = \min\{\mu_a, \mu_b\}$. Second, in the THEN part, to give the control signal *c* a membership value μ_c, in a way that μ_c is changing with μ_a and μ_b, one may simply let $\mu_c = \mu_a \times \mu_b$. Third, as the new idea in this design, each rule "IF *x* THEN *c*" is executed by the logical implication formula $z = \mu(x \Rightarrow c) = \min\{1, 1 + \mu_c - \mu_x\}$ [1,2], which yields the corresponding membership value. Finally, the control signal of each rule is generated in the weighted form of $z_i \times c_i$.

To illustrate the design and implementation steps in detail, take the first rule in Table 1 as an example:

R1: IF $\mu_{aP} \neq 0$ AND $\mu_{bP} \neq 0$

THEN $c_1 = z_1 \times NB$,

in which

$$\begin{cases} z_1 = \min\{1, 1 + \mu_{NB} - \mu_{x1}\} \\ \mu_{NB} = \mu_{aP} \times \mu_{bP} \\ \mu_{x1} = \min\{\mu_{aP}, \mu_{bP}\} \end{cases}$$

The defuzzification module is calculated by the commonly used "center-of-gravity" formula, but the weight of each rule is calculated as follows:

$$u = \frac{\sum_{i=1}^{9} \mu(c_i) \times c_i}{\sum_{i=1}^{9} \mu(c_i)} = \frac{\sum_{i=1}^{9} z_i \times c_i}{\sum_{i=1}^{9} z_i}$$

in which

$$\begin{cases} z_i = \min\{1, 1 + \mu_{c,i} - \mu_{xi}\} \\ \mu_{c,i} = \mu_{a,i} \times \mu_{b,i} \\ \mu_{xi} = \min\{\mu_{a,i}, \mu_{b,i}\} \end{cases}$$

3 Application to Torsion Pendulum Control

Torsion pendulum is the basic mechanical system behind clocks that are enclosed in glass domes, as shown in Fig. 1. The control objective is to stabilize the pendulum from its various initial conditions.

The equation describing the torsion pendulum is

$$J \times \frac{d^2\theta(t)}{dt^2} = \tau(t) - B \times \frac{d\theta(t)}{dt} - k \times \theta(t)$$

This equation represents the sum of the torques of the pendulum bob. The moment of inertia of the pendulum bob is denoted by *J*, the elasticity of the brass suspension strip by *k*, and the friction between the bob and the air by *B*. Moreover, $\tau(t)$ is used as the controlling torque applied at the bob, so the final control equation is given by

$$\frac{d^2\theta(t)}{dt^2} + 2\frac{d\theta(t)}{dt} + 5\theta(t) = \tau(t)$$

In simulations, $J = 1\ kg \cdot m^2$, $k = 5\ N \cdot m / rad$, and $B = 2\ N \cdot m \cdot s / rad$ [4].

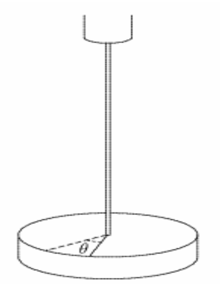

Fig. 1 Torsion pendulum schematic

Further, by defining the state variables

$x_1 = \theta(t)$ and $x_2 = \dot{\theta}(t)$

$\dot{x}_1 = \dot{\theta}(t)$ and $\dot{x}_2 = \ddot{\theta}(t)$

the equation can be written in a state-space form:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = \tau(t) - 2x_2(t) - 5x_1(t)$$

This model is used to create data for simulation and is not used in the controller design process.

While using the new fuzzy logic controller to control this pendulum, the membership values for $\theta, \dot{\theta}$, and τ are the same as they are used in the common fuzzy controller, as shown in Figs. 2, 3, 4.

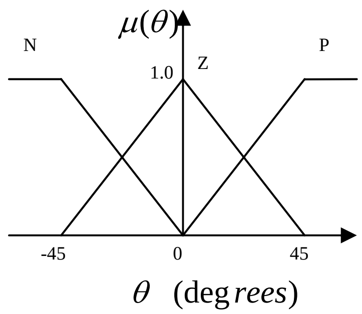

Fig. 2 Membership values for θ [4]

The rules for control are given in Table 2.

Table 2 Rule base for controlling torsion pendulum

θ \ $\dot{\theta}$	P	Z	N
P	NB	N	Z
Z	N	Z	P
N	Z	P	PB

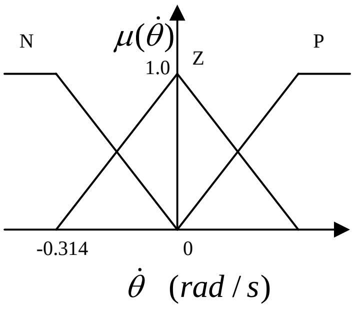

Fig. 3 Membership values for $\dot{\theta}$ [4]

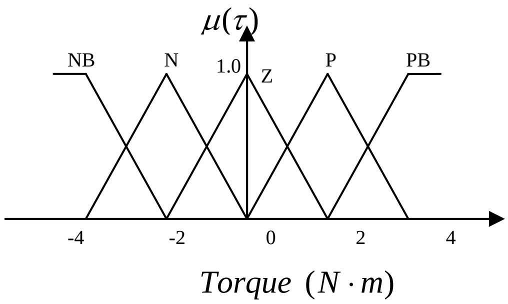

Fig. 4 Membership values for τ [4]

There are five sets of initial conditions used for comparison in the simulations:

(1) $\theta(0) = 40^\circ$, $\dot{\theta}(0) = -0.2 rad/s$

(2) $\theta(0) = 20^\circ$, $\dot{\theta}(0) = -0.2 rad/s$

(3) $\theta(0) = 5^\circ$, $\dot{\theta}(0) = -0.2 rad/s$

(4) $\theta(0) = 2^\circ$, $\dot{\theta}(0) = -0.2 rad/s$

(5) $\theta(0) = 0.7^\circ$, $\dot{\theta}(0) = -0.2 rad/s$

The simulation results, starting from the above five sets of initial conditions, based on the new control design and the conventional fuzzy control approach, are shown in Figs. 5, 6, 7, 8 and 9, in which the dotted curves are the simulation results controlled by the conventional fuzzy control method and the solid curves are the simulation results controlled by the new fuzzy logic controller. The *x*-axis represents the sampling time, and the *y*-axis the twisting angle of the torsion pendulum.

By comparing the dotted curves and the solid curves shown in Figs 5, 6, 7, 8 and 9, respectively, one can see that all the control results are satisfactory in the sense that the trajectories are smooth. Clearly, the new control design is as good as, but never is worse than, the conventional fuzzy control method in some cases (Figs. 5, 6), while has a much shorter settling time in some other cases (Figs. 7, 8, 9), showing quite significant improvements.

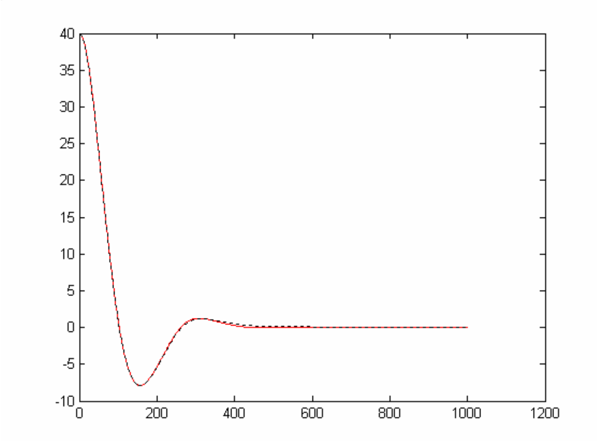

Fig. 5 Simulation results on θ from initial condition $\theta = 40^\circ$ *and* $\dot{\theta} = -0.2 rad/s$

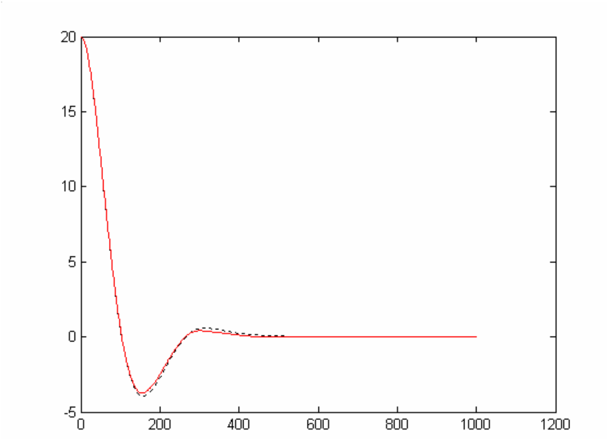

Fig. 6 Simulation results on θ from initial condition $\theta = 20^\circ$ *and* $\dot{\theta} = -0.2 rad/s$

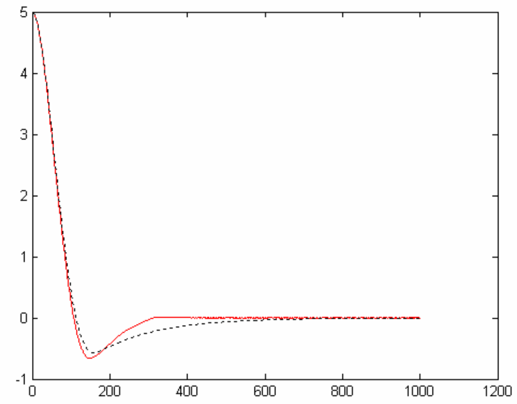

Fig. 7 Simulation results on θ from initial condition $\theta = 5^\circ$ *and* $\dot{\theta} = -0.2 rad/s$

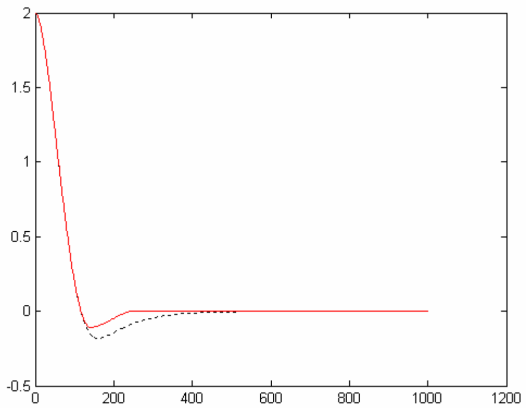

Fig. 8 Simulation results on θ from initial condition $\theta = 2^\circ$ *and* $\dot{\theta} = -0.2 rad/s$

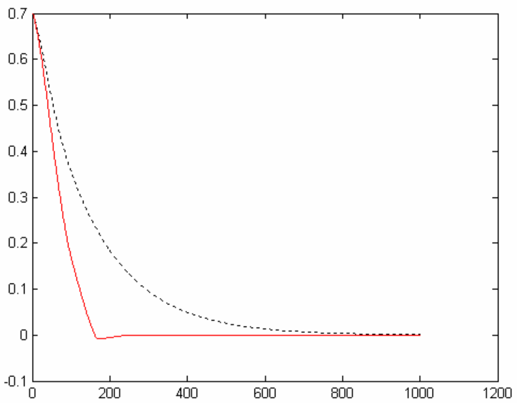

Fig. 9 Simulation results on θ from initial condition $\theta = 0.7^\circ$ *and* $\dot{\theta} = -0.2 rad/s$

4 Conclusion

A new fuzzy logic controller has been designed and simulated, in which the implication logic is used to determine the weights of the defuzzification of the control signals, which can enhance the efficiency and performance of fuzzy control. The new controller has been applied to stabilizing a torsion pendulum model. Compared with the conventional fuzzy control approach, simulation shows that the new design can improve the control performance in the sense of achieving shorter settling time.

References:

[1] G. Chen, and T. T. Pham, *Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems*, Boca Raton, FL: CRC Press, 2001.

[2] G. Chen, and T. T. Pham, *Introduction to Fuzzy System*, Boca Raton, FL: CRC Press, 2006.

[3] K. M. Passino, and S. Yurkovich, *Fuzzy Control*, Menlo Park, CA: Addison-Wesley, 1998.

[4] T. J. Ross, *Fuzzy Logic with Engineering Applications*, Hoboken, NJ: John Wiley, 2004.

[5] K. Tanaka, and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, New York, NY: IEEE Press, 1999.

[6] H. Ying, *Fuzzy Control and Modeling: Analytical Foundations and Applications*, New York, NY: IEEE Press, 2000.

[7] C. W. Tao and J. S. Taur, Robust fuzzy control for a plant with fuzzy linear model. *Fuzzy Systems, IEEE Transactions on*, Vol.13, No.1, 2005, pp. 30-41.

[8] B. Yang, D. Yu, G. Feng, and C Chen, Stabilization of a class of nonlinear continuous time systems by a fuzzy control approach. *Control Theory and Applications, IEE Proceedings-*, Vol.153, No.4, 2006, pp. 427-436.