RTFDF Description for ARMA Systems

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Abstract:-This paper describes the Real-time Fuzzy Digital Filters (RTFDF) considering for it the fuzzy logic description (to see: [1], [3], [9], [11], [14], [15], [16], [20], [21]), adaptive digital filters concepts (to see: [2], [5], [11] and [19]) with Real-time properties (to see: [12], [13] and references in it).
The main characteristic operation of this kind filter is a region knew as linguistic natural knowledge base. Illustratively, the output signals system are classified with respect to desired answer in linguistic natural form (to see: [12] and [19]), requiring to establish the membership function set with respect to these for the characterization in computational sense as nature linguistic [3]. The RTFDF needs a group of rules inside of the knowledge base (in mathematical sense using logic connectors (IF…. THEN...) in agreement to [19]). Novel RTFDF description considered quality answer into natural languages, and the interaction time restrictions with adaptive properties.
The basic results described in this document in formal sense, using the set definitions developed in the papers referenced below without forgetting the Nyquist, Shannon, Zadeh and Passino criteria into convolution filter scheme ([7],[16],[18] and [21]).

Key-Words:- Fuzzy logic, digital filter, adaptive filter, knowledge base, Real-time.

1 Introduction

A Fuzzy filter system [1] works in loop form adapting the parameters set dynamically ([2], [6], [7], and [16]). Commonly, the adaptation is a criterion based in the error signal $e(k) \in \mathrm{R}$ ([6] and [7]) optimizing the velocity function estimation. This error $e(k)$ is defined as the difference between the desired process responses (could be described as $y(k) \in \mathrm{R}$) and the actual signal $\hat{y}(k) \in \mathrm{R}$ ([6], [7], and [16]) generated by an identification ♣ scheme.

The adaptive criterion (using the loop form concept [16]) previously selected, is a prime material required to establish the membership function, looking for the most approximate signal identified $\hat{y}(k)$ around of signal desired $y(k)$, adjusting the filter parameters dynamically, such that the error value $e(k)$ converges near to the ball with radio $\gamma>0$ previously defined [12].

Conceptually, each membership function is inside of error distribution function. It means, that the best approximation among membership function with respect to distribution description in Borel sense, is the triangular form.

In general, the fuzzy filter has the following elements considering the concepts studied in [1], [2], [6], [7], [8], [11], [12], [14], and [15]:

1. *Input Fuzzy Inference*: In this stage, the natural process answer is an input filter but transforming it in metric sense.
2. *Rule base*: Dynamic range metrics respect to input filter use the logical binary connector known as *IF*.
3. *Inference Mechanism*: The expert action respect to the rule base knowing as consequence, use logical binary connector *THEN*.
4. *Output Fuzzy Inference:* Finally, it is the last filter stages, where the digital result, converting in natural answer process based in the knowledge base predefined.

♣ The difference between estimator and identifier is that the first describes the parameter evolution and the second of them describes the state velocity [12].

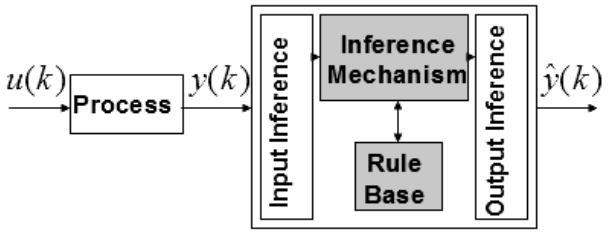

Fig. 1 Fuzzy filter process: Description.

2 Membership Functions

A fuzzy filter uses an error signal functional in distributive sense ([6] and [12]) for to generate a membership function ([1], [3], [10], [11], [14], and [15]) by each n-cil interval, required these into adaptable algorithm ([2], [6], [9], and [11]) in agreement to objective function.

Although each member function is dynamically adjusted with gradualism, consider the objective function, without forget the adaptation, because in probability sense, for example, the objective function adjusts its parameters directly considering the n-cil intervals wide in each time respect to the density information input fuzzy inference.

The most used criterion into literature as objective function is the quadratic mean error (QME), described it by the second probability moment (to see: [5], [6], and [12]) in a recursive form (1) as:

$$J(k)=\left[\frac{1}{k}\left((k-1)J(k-1)^2+e(k)^2\right)\right]^{\frac{1}{2}}, \quad \text{with } J(k)\in \mathrm{R}_+ \qquad (1)$$

In fuzzy filter concepts, each rule defines a membership function limited by a specific sequence with respect to correct operation system range in a distribution function desired answer sense.

The objective into the fuzzy filter is to give a specific natural response $\hat{y}(k)$ respect to the desired signal *y(k)*, considering the black box idea, but each specific response limited by error criterion functional determining a fuzzy filter action response in agreement to knowledge base, generating a natural answer as low, medium, or high in linguistic manner.

Basically, the knowledge base contains the pairs in metric sense [4] formed by the product set $Y(k)\times\hat{Y}(k)\in \mathrm{R}^2$ defined symbolically as $T_N\in \mathrm{R}^2_{2[1,N]}$ where the desired set is $Y(k)=\{y(k)\}$ and the identification answer set is $\hat{Y}(k)=\{\hat{y}(k)\}$, respectively, labeling each of them from [1, N] to [1, N]:

$$T_N=\{(y(k),\hat{y}(k))\}_{k=1}^{2\times N} \qquad (2)$$

Each membership function establishes the maximum correspondence value between the output $\hat{y}(k)$ and the desired signal $y(k)$, where the best value (in ideal form) is the infimum cost for each sequence. In illustrative sense, looking inside of the control area (defined for T_N) having a subset convergence pairs ordered.

In addition, mathematically the pair's correspondence expressed with respect to the second probability moment:

$$J_{\min}=\min_N\{\inf J(y_0,\hat{y})\}_N. \qquad (3)$$

3 Fuzzy Digital Filters

The adaptation as a feedback law modifies the signal input filter such that the answer fuzzy filter system $\hat{y}(k)$ tends to desired signal $y(k)$, generating changes in its states.

The fuzzy filter classifies its different levels operation respect to the membership functions set, for to give an specific response value in natural language in agreement to J_{min} limited by ball radius γ in a metric sense [4].

Classification system response realized by fuzzy filter bounding parameters gains in agreement to distribution function (1), selecting the optimal criterion into the membership functions set considering that they are disjoint in a metric sense [13].

These regions permit to select into the knowledge base, the best values to adjust the input filter, considering the inference mechanisms. Inside of the Inference stage, the classification of the membership functions in agreement to [20] and [21] accomplishes the functional error respect to (1), in Borel sense.

Each membership function established inside the

distribution function error with range interval criterion (to see: (1)) based in the Lebesgue sense [17] permitting that each of them could be expressed in an iterate form describing in almost every point (to see: [6], [17], [10], and [12]).

4 Real-time Fuzzy Digital Filter

DEFINITION 1. (Real-time Fuzzy Digital Filter - RTFDF). *A Real-time Fuzzy Digital Filter is an adaptive filter performed in according to ([5], [7], and [15]):*

a) Extraction and emission of the fuzzy information through limited intervals respect to the process response within limited intervals according to the stability criteria as Lyapunov, Root-Hurwitz, and others (see: [5], [7]).

b) Extract and emit information through semi-open time intervals [4], synchronized with evolution time of the process [6] described in a relative way by semi-open time intervals, considering the criteria expressed by Kotel'nikov (1933), Nyquist (1928), Whittaker, (1915), Shannon (1948), and Ecker (2000) (to see references included in [15], [16], [17] and [18]).

c) A membership functions group forming a discourse linguistic universe, according to the properties considered in a) and b) points, respectively.

d) A set of fuzzy rules builds the knowledge base depending of the fuzzy desired signal $\hat{y}(k)$ respect to reference model $y(k)$ without objective function.

e) The adaptation algorithm actualizing the filter coefficients according to the membership function corresponding with the established error criteria symbolically expressed by γ_*

In fuzzy filter, the knowledge base has all information that the filter requires for to adjust its gains in optimal form[1] and give a good enough answer accomplishing the convergence range, inside of the time interval (indexed with $k \in \mathbf{Z}_+$) in agreement to Nyquist sense, without lose the stability properties:

[1] In natural language, the convergence is around of the point a it is knowing as optimal, but in metric description (inside of the filter), the convergence is robust because it is defined by intervals.

a) $y(k)$ knew as measurable value classified in ranges in a linguistic sense (describing all of them into space state variable bounded symbolically in a linguistic natural expressions as high, medium or low values),

b) $T(k)$ is the control area described in pairs formed by $\hat{y}(k)$ and $y(k)$ limited with time interval (it has a velocity change bounded in the sense exposed by [15]),

c) $e(k)$ is the fuzzy value defined by the difference among $\hat{y}(k)$ and $y(k)$, where this linguistic value is bounded by the set $\{\gamma_i : \gamma_i > 0, \ \forall i \in Z_+\}$, $\inf\{\gamma_i\} \to |\lambda_*|$, such that $|\lambda_*| > 0$, $\sup\{\gamma_i\} \to |\lambda^*|$, $|\lambda^*| < 1$, means that $\hat{y}(k)$ is approximately equal to $y(k)$ metrically speaking, but in linguistic sense both are the same natural value.

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5 Conclusions

This paper described the Real-time Fuzzy Digital Filters (RTFDF) concepts considering the fuzzy logic description (to see: [1], [3], [11], [14], [15], [16]), adaptive digital filters concepts (to see: [2], [4], [8] and [12]) with Real-time properties (to see: [12], [13] and references in it). The basic characteristic operation of this kind of filters expressed as operation regions knowing it as linguistic natural descriptions.

Illustratively, the output signals system classified with respect to desired answer in linguistic natural form (to see: [12] and [13]), requiring to establish the membership function set with respect to the linguistic description [3].

Then RTFDF needed a group of rules, to consult and to discriminate inside of the knowledge base build it in a probabilistic manner [19].

A novel RTFDF description evaluated in two senses, in quality answer (considering the natural languages), and the interaction time restrictions, considering the adaptive properties.

The basic results described in this document in formal sense, using the set definitions developed in the papers referenced below without forgetting the Nyquist and Shannon criteria into the convolution filter scheme. Fortunately, the basic concepts illustrated showing a RTFDF simulation.

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