Non-Linear System State Analysis via Takagi-Sugeno Fuzzy Modelling

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Abstract: - The fuzzy and neuro-fuzzy modeling approaches represent extremely powerful tool for non-linear dynamic systems approximation. By using this tool it is possible to overcome difficulties in conventional techniques for dealing with nonlinearity. This paper presents the design of the diagnostic system exploits these fuzzy modeling approximation abilities together with fault detection and isolation algorithm (FDI) to detect the presence of the fault at the system. The idea is based on using a Takagi-Sugeno fuzzy model to describe the non-linear dynamic system by its decomposition onto number of linear submodels. Having these submodels, the Kalman filters are designed for each of the local models to generate the fault indicating signals – residuals. Because of the assumption that the non-linear system under consideration is stochastic, the hypothesis testing technique (Generalized likelihood ratio test) is applied to the residuals along with the fuzzy regression to make a decision whether the system is subjected by the fault or not. The paper also provides the application study of the proposed approach using the three tank system example.

Key-Words: - Fault, fault diagnosis, residual signals, state space model, nonlinear system, fuzzy nonlinear regression model, linear subsystem, state observer, Kalman filter, hypothesis testing, generalized likelihood ratio

1 Introduction

With the growing complexity of modern engineering systems and the ever increasing demand for safety of these systems, there has been effort to develop new control and supervision techniques. Modern systems are large scale, highly complex, and operate with a large number of variables under closed-loop control. To ensure safety and product quality level the design of the fault compensating system seems to be sufficient. In this case the fault is defined as an unpermitted deviation of at least one characteristic property or variable of the system. Early and accurate detection and diagnosis of the faults can minimize downtime and increase safety of the monitored system. To tackle a fault diagnosis problem, it is very useful to have all the knowledge concerning a system behavior. Such the knowledge can be represented by the adequate model of the monitored system. Most model-based methods for fault diagnostics rely on a linear state-space model and since the most of industrial systems exhibit a non-linear behavior, only way to obtain this linear model is to linearise the process model around the operating point. However, linearization does not provide a good model for the strong non-linear system. Another way to handle the fault diagnosis in non-lineal system is to design a non-linear observer. But the systems that can be represented by these observers are limited to a few standard types of non-linearity. It follows from the above, that it is very profitable to use the fuzzy modeling approximation abilities to overcome difficulties in conventional techniques for dealing with nonlinearity.

This paper presents the fault diagnostics scheme using Takagi-Sugeno fuzzy model together with the Kalman filters as the observers generate the fault indicating signals – residuals. To make a decision whether the system is subjected by the fault the generalized likelihood ratio test is applied to the residuals.

The paper is organized as follows. Section 2 describes the Takagi-Sugeno fuzzy model structure that is essential for the fault diagnosis task. Section 3 presents method for residual signals generation using the bank of Kalman filters and subsequently decision algorithm based on the generalized likelihood ratio test. Application of the proposed fault diagnostic scheme for a three tank system is presented in Section 4.

2 Structure of TS fuzzy model

A Takagi-Sugeno fuzzy model is a way to describe a non-linear dynamic system using locally linearized linear models. Each linear model represents the local system behavior around the operating point. The global system is described by a fuzzy fusion of all linear model outputs. Necessary number of fuzzy IF-THEN rules describes the global system behavior. The consequent of and particular rule represents local linear relations of the non-linear system.

Suppose that the non-linear system can be described by the following general n – dimensional non-linear function:

$$y = F(x_1, x_2, \ldots, x_n), \tag{1}$$

where y is output of the system and $x_1, x_2, \ldots, x_n$ are input variables. Since function (1) can be approximated by the piecewise liner function, it is possible to describe this non-linear function by the set of the fuzzy IF-THEN rules. The local input subspace, where the particular consequent holds true, is enclosed by the r_{th} antecedent of the specific r_{th} rule ($r = 1,2,\ldots,R$). TS model which approximated the non-linear function (1) has the following structure:

$$\begin{aligned}
&R_1 : \text{IF}(x_1 \ is\ A_{11}) \text{ and} \ldots \text{and } (x_n \ is\ A_{1n}) \text{ THEN} \\
&\quad y_1 = k_{01} + k_{11}x_1 + \cdots + k_{1n}x_n \\
&R_2 : \text{IF}(x_1 \ is\ A_{21}) \text{ and} \ldots \text{and } (x_n \ is\ A_{2n}) \text{ THEN} \\
&\quad y_2 = k_{02} + k_{21}x_1 + \cdots + k_{2n}x_n \\
&\quad \vdots \\
&R_R : \text{IF}(x_1 \ is\ A_{R1}) \text{ and} \ldots \text{and } (x_n \ is\ A_{Rn}) \text{ THEN} \\
&\quad y_R = k_{0R} + k_{R1}x_1 + \cdots + k_{Rn}x_n
\end{aligned} \tag{2}$$

where $x_1, x_2, \ldots, x_n$ are premise variables $A_{11}, A_{12}, \ldots, A_{Rn}$ are fuzzy sets and R is number of IF-THEN rules. The consequents in (2) take the form of linear functions of input variables.

Given the inputs, the global output of the system is inferred as follows:

$$y_G = \frac{\sum_{r=1}^{R} w_r y_r}{\sum_{r=1}^{R} w_r} \tag{3}$$

where w_r is the tensor product of grade memberships of the premise variables:

$$w_r = \min_i \left[\mu_{A_{ri}}(x_i^0)\right] \tag{4}$$

$\mu_{Ari}\ (x_i^0)$ is the grade of membership of the premise variable actual value x_i^0. The membership grade functions w_r ($r = 1,2,\ldots,R$) satisfy the following constraints:

$$\begin{aligned}
&\sum_{r=1}^{R} w_r = 1 \\
&0 \le w_r \le 1 \quad \forall r = 1,2,\ldots,R
\end{aligned} \tag{5}$$

Since the monitored system is supposed to be stochastic, to obtain the diagnostic signal – residual that are computed as a difference between the estimated and real system output, the Kalman filter can be used for construction of the output system estimate. In order to compute this global estimate it is convenient to apply the Kalman filter not to the global system, but to the local linear model appeared in the consequent of each of the rule. To do so, it is necessary to convert the linear regression functions to the corresponding state space representation.

It is possible to show, that if the regression equation which describe the local system behavior is given by:

$$\begin{aligned}
&y^r(k) = a_0^r + \sum_{j=1}^{zy} a_j^r y^r(k-j) + \sum_{j=0}^{zu_1} b_j^r u_1(k-j) + \ldots \\
&\ldots + \sum_{j=0}^{zu_S} c_j^r u_S(k-j)
\end{aligned} \tag{6}$$

where $y^r(k)$ is the output of the local linear model, a_0^r, a_j^r, b_j^r, c_j^r are the scalar coefficients, $u_1(k),\ldots,u_S(k)$ are the input variables and zy, $zu_1 \ldots zu_S$ represents the order of delays for the corresponding signals, the state space matrices for the particular local model can be obtained in the following observable canonical form:

$$\begin{aligned}
&\mathbf{\Phi}_1^r = \begin{bmatrix} -a_1^r & 1 & 0 & \cdots & 0 \\ -a_2^r & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{zy-1}^r & 0 & 0 & \cdots & 1 \\ -a_{zy}^r & 0 & 0 & \cdots & 0 \end{bmatrix} \\
&\mathbf{C}_1^r = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \\
&\mathbf{\Gamma}_1^r = \begin{bmatrix} b_1^r - a_1^r b_0^r & \cdots & c_1^r - a_1^r c_0^r & -a_1^r a_0^r \\ b_2^r - a_2^r b_0^r & \cdots & c_2^r - a_2^r c_0^r & -a_2^r a_0^r \\ \vdots & & \vdots & \vdots \\ b_{zu_1-1}^r - a_{zy-1}^r b_0^r & \cdots & c_{zu_S-1}^r - a_{zy-1}^r c_0^r & -a_{zy-1}^r a_0^r \\ b_{zu_1}^r - a_{zy}^r b_0^r & \cdots & c_{zu_S}^r - a_{zy}^r c_0^r & -a_{zy}^r a_0^r \end{bmatrix} \\
&\mathbf{D}_1^r = \begin{bmatrix} b_0^r & \cdots & c_0^r & a_0^r \end{bmatrix}
\end{aligned} \tag{7}$$

and the consequent linear models take the form of the MISO state space equations given by:

$$\begin{aligned}
\mathbf{x}_1^r(k+1) &= \mathbf{\Phi}_1^r \mathbf{x}(k) + \mathbf{\Gamma}_1^r \mathbf{u}(k) \\
y_1^r(k) &= \mathbf{C}_1^r \mathbf{x}(k) + \mathbf{D}_1^r \mathbf{u}(k)
\end{aligned} \tag{8}$$

where $\mathbf{x}_1^r(k) \in \mathrm{R}^{zy}$ is the state vector, $y_1^r(k) \in \mathrm{R}$ is output of the local system, $\mathbf{u}(k) \in \mathrm{R}^{S+1}$ is the input vector and $\mathbf{\Phi}_1^r$, $\mathbf{\Gamma}_1^r$, $\mathbf{C}_1^r$ and $\mathbf{D}_1^r$ are system matrices with appropriate dimensions.

For the MIMO systems it is necessary to obtain the corresponding state space equations (8) for the each of existing output.

The final state space form of the r_{th} consequent linear model can be expressed as:

$$\begin{aligned} \mathbf{x}_{\mathrm{M}}^r(k+1) &= \mathbf{\Phi}_{\mathrm{M}}^r \mathbf{x}_{\mathrm{M}}^r(k) + \mathbf{\Gamma}_{\mathrm{M}}^r \mathbf{u}(k) \\ \mathbf{y}_{\mathrm{M}}^r(k) &= \mathbf{C}_{\mathrm{M}}^r \mathbf{x}_{\mathrm{M}}^r(k) + \mathbf{D}_{\mathrm{M}}^r \mathbf{u}(k) \end{aligned} \tag{9}$$

where $\mathbf{y}_{\mathrm{M}}^r(k) \in \mathrm{R}^m$ is now output vector of the local system. System matrices are given by:

$$\mathbf{\Phi}_{\mathrm{M}}^r = \begin{bmatrix} \mathbf{\Phi}_1^r & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{\Phi}_2^r & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \mathbf{\Phi}_m^r \end{bmatrix} \quad \mathbf{\Gamma}_{\mathrm{M}}^r = \begin{bmatrix} \mathbf{\Gamma}_1^r \\ \mathbf{\Gamma}_2^r \\ \vdots \\ \mathbf{\Gamma}_{m-1}^r \\ \mathbf{\Gamma}_m^r \end{bmatrix}$$
$$\mathbf{C}_{\mathrm{M}}^r = \begin{bmatrix} \mathbf{C}_1^r & 0 & \cdots & 0 \\ 0 & \mathbf{C}_2^r & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}_m^r \end{bmatrix} \quad \mathbf{D}_{\mathrm{M}}^r = \begin{bmatrix} \mathbf{D}_1^r \\ \mathbf{D}_2^r \\ \vdots \\ \mathbf{D}_{m-1}^r \\ \mathbf{D}_m^r \end{bmatrix} \tag{10}$$

Output vector of the global model can be computed using (3) and hence:

$$\mathbf{y}_{\mathrm{M}}(k) = \frac{\sum_{r=1}^{R} w_r \mathbf{y}_{\mathrm{M}}^r(k)}{\sum_{r=1}^{R} w_r} \tag{11}$$

where the membership grade functions w_r ($r = 1,2,\ldots,R$) are obtained using:

$$w_r = \min\left[\mu_{A_{S,j}^r}(\mathbf{U}(k))\right] \tag{12}$$

where $\mathbf{U}(k) = (u_1(k), u_1(k-1), \ldots, u_S(k), \ldots, u_S(k-j))$.

In this case, the all rules consequents of the TS model take the form of the state space equations. Therefore it is possible to design the Kalman filter for each of the submodels.

3 Residual generation and evaluation

In order to design a residual generator based on the bank of the Kalman filters, assume that the state space representation of the local models with the possible fault can be expressed as follows:

$$\begin{aligned} \mathbf{x}_{\mathrm{M}}^r(k+1) &= \mathbf{\Phi}_{\mathrm{M}}^r \mathbf{x}_{\mathrm{M}}^r(k) + \mathbf{\Gamma}_{\mathrm{M}}^r \mathbf{u}(k) + \mathbf{R}_1^r \mathbf{f}(k) + \mathbf{w}^r(k) \\ \overline{\mathbf{y}}_{\mathrm{M}}^r(k) &= \mathbf{C}_{\mathrm{M}}^r \mathbf{x}_{\mathrm{M}}^r(k) + \mathbf{R}_2^r \mathbf{f}(k) + \mathbf{v}^r(k) \end{aligned} \tag{13}$$

where:

$$\overline{\mathbf{y}}_{\mathrm{M}}^r(k) = \mathbf{y}_{\mathrm{M}}^r(k) - \mathbf{D}_{\mathrm{M}}^r \mathbf{u}(k) \tag{14}$$

$\mathbf{f}(k)$ is unknown function of time corresponds to a specific fault while $\mathbf{R}_1^r$ and $\mathbf{R}_2^r$ are the known fault distribution matrices represents the effect of the faults on the system. $\mathbf{w}^r(k)$ and $\mathbf{v}^r(k)$ are supposed to be independent zero-mean white noise sequences with covariance matrices $\mathbf{Q}^r$ a $\mathbf{R}^r$, assumed to be known. Assume fault free case. To compute the state vector estimate of the stochastic system, the Kalman filter with the following structure is proposed:

$$\begin{aligned} &\hat{\mathbf{x}}_{\mathrm{M}}^r(k|k) = \hat{\mathbf{x}}_{\mathrm{M}}^r(k|k-1) + \mathbf{K}_{\mathrm{f}}^r(k)(\mathbf{y}(k) - \\ &-(\mathbf{C}_{\mathrm{M}}^r \hat{\mathbf{x}}_{\mathrm{M}}^r(k|k-1) + \mathbf{D}_{\mathrm{M}}^r \mathbf{u}(k))) \end{aligned} \tag{15}$$

where $\hat{\mathbf{x}}_{\mathrm{M}}^r(k|k-1)$ is state estimate extrapolation. $\mathbf{K}_{\mathrm{f}}^r(k)$ is the Kalman gain matrix and it is designed to achieve minimum variance estimation. To do so, Kalman gain matrix should be determined by:

$$\mathbf{K}_{\mathrm{f}}^r(k) = \mathbf{P}^r(k|k-1) {\mathbf{C}_{\mathrm{M}}^r}^{\mathrm{T}} (\mathbf{C}_{\mathrm{M}}^r \mathbf{P}^r(k|k-1) {\mathbf{C}_{\mathrm{M}}^r}^{\mathrm{T}} + \mathbf{R}^r)^{-1} \tag{16}$$

where $\mathbf{P}^r(k|k-1)$ is a priory estimation error covariance matrix and it is a function of its last a posteriori value. A posteriori estimation error covariance matrix is given by:

$$\mathbf{P}^r(k|k) = \mathbf{P}^r(k|k-1) - \mathbf{K}_{\mathrm{f}}^r \mathbf{C}_{\mathrm{M}}^r \mathbf{P}^r(k|k-1) \tag{17}$$

Local outputs estimation can be computed using the following expression:

$$\hat{\mathbf{y}}_{\mathrm{M}}^r(k) = \mathbf{C}_{\mathrm{M}}^r \hat{\mathbf{x}}_{\mathrm{M}}^r(k|k-1) + \mathbf{D}_{\mathrm{M}}^r \mathbf{u}(k) \tag{18}$$

and the global system estimation is evaluated according to (11):

$$\hat{\mathbf{y}}_{\mathrm{M}}(k) = \frac{\sum_{r=1}^{R} w_r \hat{\mathbf{y}}_{\mathrm{M}}^{r}(k)}{\sum_{r=1}^{R} w_r} \tag{19}$$

Residual vector takes the form:

$$\mathbf{r}(k) = \mathbf{y}(k) - \hat{\mathbf{y}}_{\mathrm{M}}(k) = \boldsymbol{\gamma}(k) \tag{20}$$

which is the global innovation process and can be computed form the local ones $\boldsymbol{\gamma}^{r}(k)$ using (11). Since the innovations $\boldsymbol{\gamma}(k), \boldsymbol{\gamma}(k-1), \ldots, \boldsymbol{\gamma}(0)$ are statistically independent of one another, generalized likelihood ratio test can be performed to decide if a fault has occurred. The hypotheses test can be expressed in terms of the innovation:

$$\mathrm{H}_0 : \boldsymbol{\gamma}(k) = \boldsymbol{\gamma}_0(k) \tag{21}$$

$$\mathrm{H}_1 : \boldsymbol{\gamma}(k) = \boldsymbol{\gamma}_0(k) + \boldsymbol{\rho}(k, k_{\mathrm{f}}) \tag{22}$$

Here $\boldsymbol{\gamma}_0(k)$ is the innovation in the absence of the fault, and $\boldsymbol{\rho}(k, k_{\mathrm{f}})$ is additive fault signature, which can be recursively computed [.]. k_{f} is the unknown time at which the fault occurs. Provided that innovation has the Gaussian distribution the generalized likelihood ratio for hypothesis (21) and (22) takes the form [.]:

$$\lambda(k) = \max_{1 \le k_{\mathrm{f}} \le k} (\mathbf{d}^{\mathrm{T}}(k, k_{\mathrm{f}}) \mathbf{S}^{-1}(k, k_{\mathrm{f}}) \mathbf{d}(k, k_{\mathrm{f}})) \tag{23}$$

where $\mathbf{d}(k, k_{\mathrm{f}})$ is a least square estimate of the fault magnitude vector $\mathbf{f}$ and $\mathbf{S}(k, k_{\mathrm{f}})$ is the error covariance of estimate of $\mathbf{f}$. $\mathbf{d}(k, k_{\mathrm{f}})$ and $\mathbf{S}(k, k_{\mathrm{f}})$ are obtained from the following expressions:

$$\mathbf{d}(k, k_{\mathrm{f}}) = \frac{\sum_{r=1}^{R} w_r \mathbf{d}^{r}(k, k_{\mathrm{f}})}{\sum_{r=1}^{R} w_r} \qquad \mathbf{S}(k, k_{\mathrm{f}}) = \frac{\sum_{r=1}^{R} w_r \mathbf{S}^{r}(k, k_{\mathrm{f}})}{\sum_{r=1}^{R} w_r} \tag{24}$$

where $\mathbf{d}^{r}(k, k_{\mathrm{f}})$, $\mathbf{S}^{r}(k, k_{\mathrm{f}})$ can be computed recursively [.] and w_r are corresponding membership grade functions.

To make a decision whether the fault has occurred, it is necessary to compare (23) to a properly chosen threshold. But for reduction of memory and computation amount requirements it is convenient to make the comparison at each time k inside a finite data window. Threshold selection is a tradeoff between the false alarm rate and the detection time.

4 Experimental results

A laboratory three tank system model is used here to demonstrate functionality of the proposed fault diagnosis scheme. The 3 tank system shown in Fig.1 is a non-linear system consisting of 3 tanks of circular cross-section that are connected to each other through connecting pipes of circular cross-section.

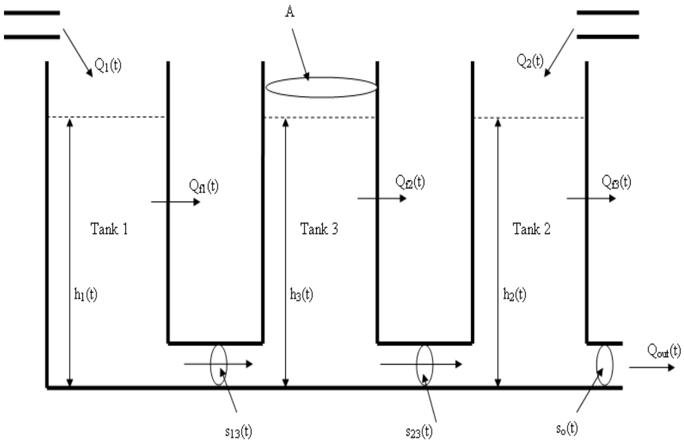

Fig. 1 Three tank system scheme

There are two inputs to the system – the incoming flows $Q_1(t)$ and $Q_2(t)$. Assume that water levels $h_1(t)$ and $h_3(t)$ are measurable output variables while water level in the tank 2 is immeasurable. System is parameterized as follows: $s_{13} = s_{23} = s_o = 0{,}31$ cm^2, A = 0,0707 m^2. The task is to detect the fault occurrence represented by the leaks in the tanks using the proposed fault diagnosis scheme.

The first stage to design fault diagnosis system is to identify the three tank system. The goal of the identification is to obtain the required form of the TS model. The three tank system has two inputs and two outputs. A non-linear SIMULINK model is used to generate data for identification. The Anfis MATLAB environment has been used to identify TS model. The inputs of the TS model are $Q_1(t)$, $Q_2(t)$ and its past values up to second order together with delayed samples of output variables $h_1(t)$ and $h_3(t)$. Input signal variation is chosen from 0,0001 m^{3}s^{-1} up to 0,0005 m^{3}s^{-1} for $Q_1(t)$ and from 0,0004 m^{3}s^{-1} to 0,0008 m^{3}s^{-2} for $Q_2(t)$. For the antecedents double Gaussian membership functions are used with $Q_1(t)$ and $Q_2(t)$ being the antecedent variables. The consequents are linear regression models to be converted to its corresponding state-space representation. The mean squared error for the two outputs is found to be $e_{r1} = 1{,}529.10^{-4}$ and $e_{r2} = 5{,}4215.10^{-5}$. The final TS model is consists of nine rules whereas the consequents take the form of state-space equations.

A variety of abrupt faults (leaks) have been considered over the simulation experiments. Fig.2 shows that an abrupt leak occurs in tank 1 at 13 minutes and 20 seconds with the magnitude 0,0003 m3s-1and this leak is detect correctly. Generalized likelihood ratio exceeds the threshold at 4 seconds and the threshold is chosen with respect to a tradeoff between the false alarm rate and the detection time ant its value is ε = 14,89.

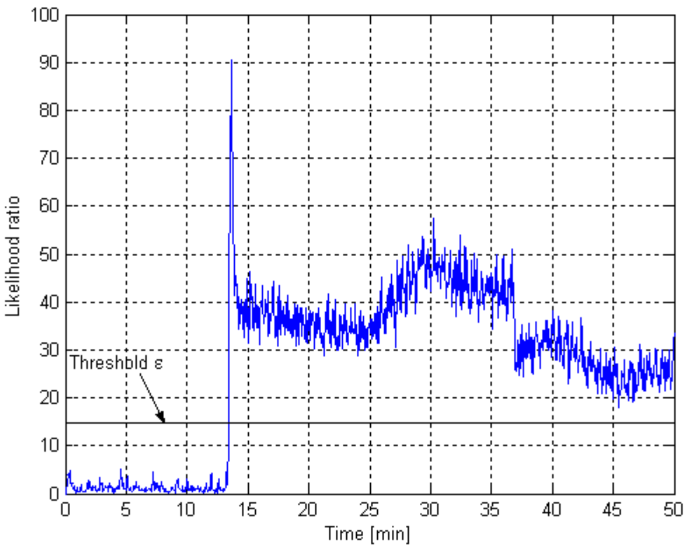

Fig. 2: Likelihood ratio when leak in tank 1 occurs

Naturally, if no faults occur, the likelihood ratio does not exceed the chosen threshold. It follows from Fig.2 that there is some behavior changes of the likelihood ratio after the detection, mainly at 36 minutes. This effect can be observed when the change of the operation point has been realized and is caused by unequal identified model accuracy in the different operation conditions. The disadvantage, mentioned above, can be minimalized by making the diagnosis system robust against the modeling uncertainty.

5 Conclusion

In this paper a fault diagnosis scheme using the modified TS model has been presented. Here Takagi-Sugeno fuzzy model has been used to describe the non-linear dynamic system by its decomposition onto number of linear submodels. Because of assumption that the monitored system is stochastic, the Kalman filter has been used to obtain the diagnostic signal – residual that are computed as a difference between the estimated and real system output. For application of the Kalman filters it is necessary to convert the consequents linear regression functions to the corresponding state space representations. To evaluate the residual vector, the generalized likelihood ratio test has been used. This test utilizes the local innovation processes generated by the Kalman filters and produces the results, based on which the decision of fault presence in the global system can be made. A three tank system has been studied to demonstrate of the proposed diagnostics system. The results indicate usefulness of the method for the early detection of the faults occurrence in the non-linear dynamic systems. Finally a demand of the extension of the proposed fault diagnosis method to be robust against the modeling errors has been emphasized.

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