Fractal Image Processing and Analysis
by Programming in MATLAB

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Abstract: The present paper studies computing method of fractal dimension by using the image storing principles on computer. As an example of its application on char surface structure, by applying Matlab, the binary image and matrix about the surface pore were learned by processing and analyzing the SEM photographs of char samples during combustion. In accordance with the characteristics of binary digital image, have been minutely discussed based on the fractal theory that the principle and method of calculating SPFD by use of box-counting dimension.

Key-words: Matlab；SPFD；SEM；binary image；box-counting dimension

1   Introduction

Fractal geometry is an excellent mathematical approach in the study of irregular geometric objects. Fractal dimension can indicate some characteristics of images obtained from nature. The application of fractal theory allows description of various states of fragmenting and branching in biological, ecological and other systems[1-7]. The char is a carbon material with complicated surface structure and is featured with fractal characteristics, and the char SPFD (surface pore fractal dimension) could reflect the process of char burning in substance. Utilizing the powerful matrix operation function of Matlab，the char SPFD of SEM image could be obtained using Matlab language. Matlab has the structured control language, such as *for* circulation, *while* circulation, *break* sentence and *if* sentence, and has the programming characteristic facing object. In Matlab, the grammar can not be restricted strictly, the degree of freedom is large in writing program, and the transplantation of the program is good. The figure function of Matlab is powerful, provided with various kinds of toolboxes applied to image processing. Because processing the image is in the divided windows using Matlab to compute SPFD, the whole result of counting the boxes and every pixel of fractal image can be observed. At the same time, because of its powerful function in displaying figure, Matlab can display the binary image with fractal characteristic. Oppositely, the program written with Vc++6.0 can't see the whole result. This paper proposes a quantitative method to analyze the char SPFD from SEM images using image processing and statistical analysis in Matlab, which is a pixel-covering method to cover the object being analyzed with grids having the same dimensionality as the containing space. The Matlab program of this method is illustrated using the Cantor-Set and Koch-Set.

2   Theory of calculating SPFD

According to the basal theories of calculating fractal dimension, the calculating method could be divided into box-counting, PENTLAND[8] and KELLER[9]. As a result, box-counting is, at present, more widely used. According to the different arithmetic approaches, the box-counting are divided into such methods as PELEG, GANG, NIRUPAM, XIE[10,11]], each of which has its respective applicable objects, advantages, and disadvantages. This article gives the calculating method of SPFD on the basis of the counting probability of the boxes covering the object being analyzed.

Assuming that F is a random non-empty boundary subset in R^n, and the $N_\delta(F)$ is the minimal set

number that overlaying F, with δ as its maximum diameter. The box-counting of set F can be defined essentially as follows [12]:

$$Dim_B F = \lim_{\delta \to 0} \frac{\log N_\delta(F)}{-\log \delta} \qquad (1)$$

In application, the set that covering F could be a small ball, a net cube, or a δ-plane grid. What we will study is the distribution and transformation of the pore on char surface, that is to say, the changes

in area as concerned on the SEM image in size, quantity and area. Therefore, for a two-dimensional SEM image, this implies coverage with squares of a specific size. The image can be framed with orthogonal line grids of increasing lattice constant. The number (N) of meshes containing any part of char surface pore aggregate is counted for each square size (lattice constant δ).

(a)

(b)

Fig.1 binary SEM image and its data matrix in Matlab

For example, for the binary SEM image as shown in Fig.1, the fractal dimension could be calculated as follows: assume that using a grid whose every small square's side length is δ ($\delta \le 1$, the lattice scale is the size of fractal image as $\delta = 1$)to cover the fractal image. The small square including concerned pixel is white square, while the background square is black. As seen in Fig.1, $\delta = 1/4$, $N_\delta(F) = 13$, thereinto $N_\delta(F)$ is the number of nonzero matrix covering the fractal image. Upon the analysis above, the matrix (Fig.1b) is divided into blocks with equal line and column number (δ^{-1}), while the total number of blocks containing is counted as N_δ. Normally, δ^{-1} is selected as 1, 2, 4, . . . , 2^i, and for each δ^{-1}, there is a corresponding N_δ. Herein, δ stands for the square size, and N_δ stands for the number of squares containing char surface pore aggregate. On the double-logarithmic plot of $\log(\delta^{-1})$ against $\log N_\delta$, the slope of the fitting straight line through least-square method is just the box-counting fractal dimension of the SEM image [13].

$$\log N_\delta = D_b \cdot \log\left(\delta^{-1}\right) + C \qquad (2)$$

Where D_b is the fractal dimension. For discrete digital image, the minimum lattice scale contains 1 pixel, here δ^{-1} is the pixel number of the original image side. If δ^{-1} is overly large, displaying that index exceeds matrix dimensions in Matlab，the resulting number of samples may not be sufficient for statistical analysis, which is important for Matlab to achieve automatic computation.

SPFD is the description about the pore's distributing condition on char surface, and factors influencing it includes the number of pores, the smooth degree of pore edge, the distribution of pore and aperture in SEM image, etc.. Generally, The value of SPFD is between $1\sim2$ [14,15].

3 Matlab algorithm of calculating SPFD

3.1 The processing and analysis of SEM

image

Let us take the SEM image of PINGDINGSHAN pyrogenation coal at low temperature as an example, and see how it is processed and analyzed:

3.1.1 Image pretreatment

All the images that Matlab processes are digital images, which are actually quantified two-dimension sampling array. The gray variation of SEM image could represent the roughness and distribution of char surface pore, so the SEM image need to be processed in order to convert it into black-white image whose gray value is between 0～255 by software Photoshop.

3.1.2 Image segmentation

In image analysis on a machine sense of vision, the key issue is how to distinguish the concerned image from the complicated background, the process of which is image segmentation. Approximately, there are three ways of image segmentation: threshold method, district method and edge examining method. As seen in Fig.2, threshold method is the proper way to segment SEM image. As shown in Fig.2, the gray histogram consists double apices and one valley, therefore,

the gray value of the valley could be as the threshold of image segmentation. There are various calculating methods obtain the threshold. In Matlab conic $y=ax^2+bx+c$ can be used to calculate threshold to fit the histogram between double apices, with $x=-b/2a$ as the segmentation threshold.

3.1.3 Obtaining binary image and matrix
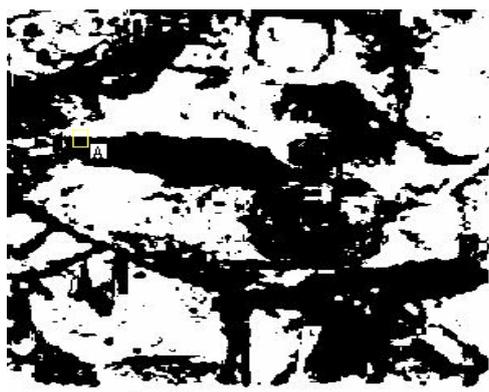
After image segmentation, conversing SEM image to binary image, each pixel in the new one is either black or white (Fig.3a). Then, through a program, the binary image was converted to a data file, which was in the form of a matrix. All the elements of the
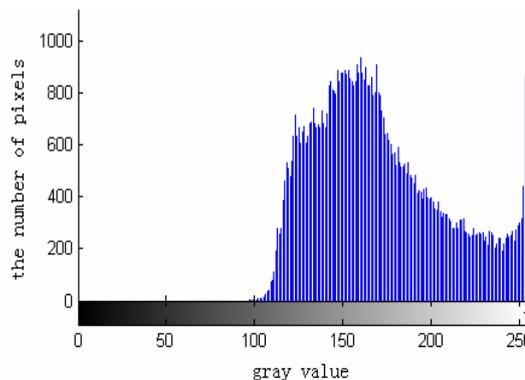

Fig.2 gray histogram

matrix correspond to the pixels of the binary image, and the element value is either 1 (white) or 0 (black) (Fig.3b), which is determined by the color of the pixel in the binary image. For the matrix, the mathematical computation software Matlab, which has a variety of toolboxes including image processing, was used in computations.

(a)

(b)

Fig.3 binary image and binary data matrix(A)

In Fig.3, the area that has been marked 1 is just a cluster in Matlab analysis, and the size and number of the area show the dimension and quantity of the cluster. The physical meaning of them are the char pore size and number of SEM image respectively.

3.2   Matlab program of the fractal dimension

With the digital binary matrix, as shown in Fig.3, we could calculate the fractal dimension of char surface pore represented in SEM image making use of the theory of calculating SPFD that has been introduced before. Fig. 4 is the Algorithm pattern of calculating SPFD with Matlab.

In view of the binary matrix characteristics shown in Fig.3, on every kind of gridding size, we could calculate the square number $N_\delta(F)$ covering concerned area in the binary image by scanning all sub-matrixes, the nonzero matrix of which just make up of $N_\delta(F)$. This kinds of gridding partition and matrix numeration automatically

suit Matlab actualization. The arithmetic adopted by this article is

gradually halve the binary matrix. In order that this halving could be carried out, we should choose the short side of the binary matrix as the warranty to fix on the halving times, so assuming $b=\min(h, w)$. The structure of the descending sequence δ_k is also important, which decides whether $\delta_k \to 0$ can be actualized to the greatest degree.

$$\delta_k = \{\frac{1}{2^p} \Big| 0 < p \le \frac{\log b}{\log 2}, \ p \in N\} \qquad (3)$$

We validate the Cantor-Set and Koch-Set by the program, which have the strict self-comparability，the results as shown in figure 5 and figure 6. It may be bring errors when we construct these normal sets, thereby there are small deviation between actual dimension and theroretical dimension, as shown in table 1. The errors are among the allowable scope, .
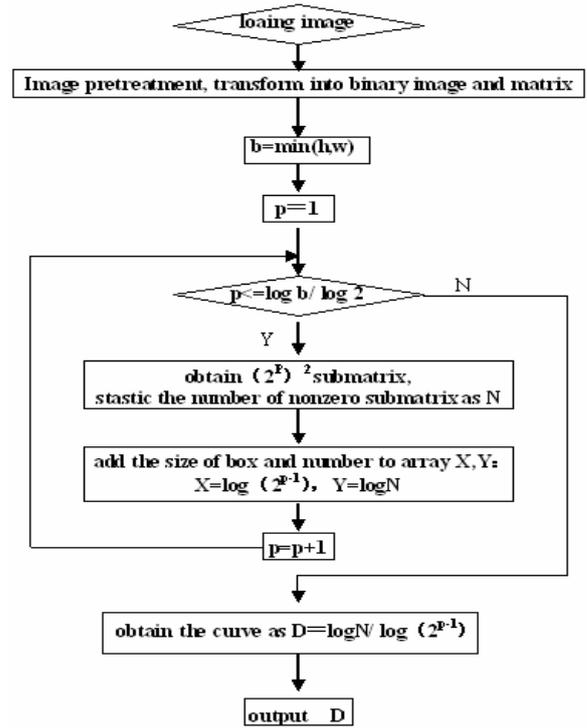

Fig.4 Algorithm pattern of calculating SPFD

so it is feasible to achieve automatic computation of SPFD by this program. The SPFD presented in Fig.3 calculated by this Matlab program is 1.8552
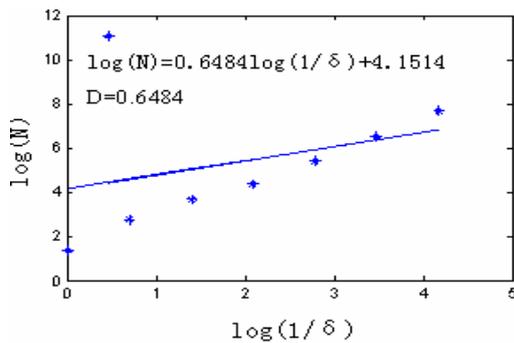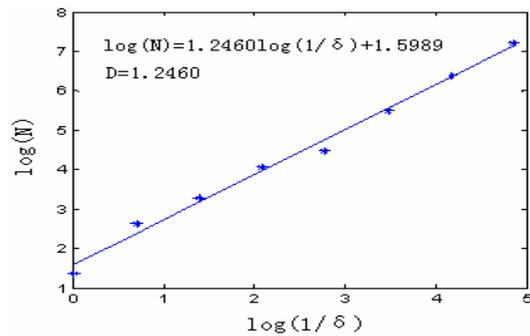

Fig.5   Result of Cantor-Set SPFD

Fig.6   Result of Koch-Set SPFD

Table 1   comparison between theoretical dimension and actual dimension

Fractal image	theoretical dimension	actual dimension	error ％
Cantor-Set	0.631	0.648	2.7
Koch-Set	1.262	1.246	1.3

4 Conclusions
(1) The distribution and transformation of char pore

on the char surface, as exhibited on SEM image, are the changes of concerned area in size, quantity and

area, so direct box-counting is an efficacious method to calculate SPFD.

(2) The char SPFD can be calculated automatically on computer utilizing the powerful function of Matlab on matrix calculation. In program, there are two main factors to influence the result of the calculation: first, choosing the threshold correctly. Since the function of the Matlab program is basing on the binary image, some concerned area would be neglected or confused with background, if the choosing of threshold is improper, which will result in an error in calculation. Second, in order that $\delta_k \to 0$ be actualized to the furthest. For reaching this require, the image obtained by SEM is may as well be foursquare, otherwise p must be choosed properly.

References:

[1] P.A. Burrough. Fractal dimensions of landscapes and other environmental data[J]. Nature,Vol.294, 1981,pp.240-242.

[2] D.L. Critten. Fractal dimension relationships and values associated with certain plant canopies[J]. J. Agric. Engng. Res.,Vol.67, 1997.pp.61-72.

[3] F. He, P. Legendre, C. Bellehumeur, J.V. Lafrankie. Diversity pattern and spatial scale: A study of a tropical rain forest of Malaysia[J]. Environ. Ecol. Stat. Vol.1, 1994,pp.265-286.

[4] B.T. Milne. Measuring the fractal geometry of landscapes[J]. Appl. Math. Comput. ,Vol.27, 1988,pp.67-79.

[5] K.L. Nielsen, P.L. Jonathan, H.N. Weiss. Fractal geometry of bean root systems: Correlations between spatial and fractal dimension[J]. Am. J. Bot., Vol.84, 1997,pp.26-33.

[6] R.D. Otto. An evaluation of forest landscape spatial pattern and wildlife community structure[J]. For. Ecol. Manage, Vol.89, 1996,pp.139-147.

[7] B.J. West, A.L. Goldberger. Physiology in fractal dimensions[J]. Am. Sci., Vol.75, 1987,pp.354-365.

[8] Xie Heping, Wann Jin an. Direct fractal measurement of fracture surfaces[J]. Int Solids Struct, Vol.36, No.5, 1999, pp.3073-3084.

[9] Kelley J M, Chen S, Texture description and segmentation through fractal geometry Computer vision[J], Computer vision, Graphics, Image Process, Vol.45, No.1, 1989, pp.150-166.

[10] Shmuel Peleg, Joseph naor, Ralh Hartley, et al., Multiple resolution texture analysis and classifications[J], IEEE Trans Pattern Anal Mach Int, Vol.6, No.4, 1984, pp.1221-1234.

[11] Nirupam S, Chaudhuri B B, An efficient approach to estimate fractal dimension of textural images[J], Pattern Recognition, Vol.25, No.9, 1992, pp.1035-1041.

[12] Dejian Lai, Marius-F. Danca, Fractal and statistical analysis on digits of irrational numbers Chaos[J], Solitons and Fractals, 2006.

[13] Zhi-bin Liu, Bin Shi, Hilary I. Inyang, Yi Cai. Magnification effects on the interpretation of SEM images of expansive soils[J]. Engineering Geology,Vol.78, 2005,pp.89-94.

[14] Gulijk, C. Van, Marijnissen, J.C.M., Makkee, M., Moulijn, J.A., Schmidt-Ott, A.. Measuring diesel soot with a scanning mobility particle sizer and an electrical low-pressure impactor: performance assessment with a model for fractal-like agglomerates[J]. Journal of Aerosol Science,Vol.35, 2004,pp.633-655.

[15] Wu, M.K., Friedlander, S.K.. Enhanced power law agglomerate growth in the free molecular regime[J]. Journal of Aerosol Science,Vol.24, 1993,pp.273-283.