Application of Fuzzy Lead Time to a Material Requirement Planning System

R. TAVAKOLI-MOGHADDAM
Department of Industrial Engineering, Faculty of Engineering
University of Tehran
P.O. Box 11365/4563, Tehran
IRAN

M. BAGHERPOUR
Industrial Engineering Department
Islamic Azad University, Shiraz, 71968-45615
IRAN

A.A NOORA
Department of Mathematics, Science and Research Branch,
Islamic Azad University, Tehran
IRAN

F. SASSANI
Department of Mechanical Engineering
The University of British Columbia
6250 Applied Science Lane, Vancouver, B.C., V6T 1Z4
CANADA

Abstract: - Material requirement planning (MRP) is used as an important technique in production planning topics. Several researches have been carried out in MRP systems, however in the case of fuzzy lead time (FLT), there are few researches available in the literature. In this paper, fuzzy lead time is discussed to cover an uncertainty condition in MRP systems. Hereby, the history of a vendor delivery and the volume of an order are considered as inputs and lead time is considered as an output of the proposed fuzzy system. Finally, lead time is estimated based on generating fuzzy rule bases and some linguistic rules, which are logical relationships between inputs and output. To establish the validation of the proposed approach, results of lead time obtained by the Monte Carlo simulation are compared with fuzzy lead times, both in 20 independent observations, each in 1000 simulation runs, using two-way analysis of variance (ANOVA). This statistical analysis also confirms the superiority of the proposed approach significantly.

Key-words: - Material requirement planning, Fuzzy rule based, Lead time, ANOVA

1 Introduction

Material requirement planning (MRP) is used in planning and control systems for batch manufacturing systems. The robustness of the MRP logic comes from its ability to deal with complexity, variability, and uncertainty. Key characteristics include the coordination of assembly and purchased component requirements through time-phased order releases and the reduction of setups through the aggregation of common part requirements. To obtain a basic understanding of MRP systems, you may refer to Vollmann et al. [1] and Sipper and Bulfin [2]. Lot sizing issues have been studied extensively; however, it is not clear how relevant the results are to MRP environments. MRP lot sizing research should be based on stochastic, multi-item, capacity-constrained assumptions with setup times. Time-phased order releases are also relevant since they affect downstream inter-arrival time variability and therefore influence queuing behavior. Finally, assembly requirements further complicate analysis since part coordination becomes an issue. Karmarkar [3] provided some excellent insights on lot sizing under capacity constraints.

Molinder [4] provided an example, in which safety stock and safety lead times were examined. Actual lead-time distributions within the production environment are treated as being independent of any forecasting or master scheduling inputs. Finished product can be assumed to be completed exactly on the due date dictated by the master production schedule (MPS). In a study of forecasting effectiveness, Fieldes and ingsman [5] applied an MPS generated on the basis of forecasts, safety stock, and lot-sizing policies.

Enns [6] focused on the effects of lot size and planned lead-time settings in a shop producing assembled products. Their results show that the required inventory levels can be minimized by selecting the proper lot sizes and by using planned lead times to control delivery performance. Biggs [7] showed that the effect of lot sizing and sequencing rules and the interaction between these are significant at the significance level of 0.01. He argued that it is possible that particular sequencing rule may work in "opposite direction" to a particular lot sizing rule and vice versa. He also studied the interaction of priority rules with the capacity levels and reported that these are significant. He found that the critical ratio rule (CRR) preformed consistently well over different levels of capacity. Later, Goodwin and Weeks [8] reported that the CRR for scheduling gives consistently better results over a variety of criteria and with a variety of lot sizing rules. Collier [9] and Billington, *et al.* [10] pointed out that certain lot sizing rules may hurt the capacity utilization. Grasso and Taylor [11] studied main effects and interaction effects among four factors as follows; lead time, buffering alterative, lot size rule, and cost value on the total cost. They reported that main effects of each factor and the interaction between lead time distribution and lot size rule and the interaction between buffering alternative and cost value on the total cost are significant. Now, we briefly review the work where the initial decision in this area is modified and presented later.

South and Stewart [12] reported their experience at Hix Corporation wherein the fifty percent increase in lot sizes and planned manufacturing lead time resulted in low system tardiness without increasing in WIP inventory costs. An intuitive explanation could be that the increase in lot sizes reduces the number of setups while releasing the shop capacity and hence leads to the reduction in a queue length before the processing centers, which leads to low tardiness. In the case of Hix Corporation, the effects balanced in such a manner that it did not increase the work in process inventory. Billington, *et al* formulated a capacity /lot size/ lead-time problem as a mixed-integer linear programming problem and they suggested a product structure compression to reduce the problem size. Models with lot sizing and sequencing together become large and are intractable to the conventional integer programming techniques. Single stage optimization procedure such as Wagner-Whithin [13] is used due to its simplicity and it is likely to yield sub-optimal solutions when stage wise interactions are ignored.

2 Fuzzy Rule Base

In any diagnostic or prognostic study in meteorology for the application of fuzzy reasoning, there are three interdependent steps. A successful execution of these steps leads to the solution of the problem in a fuzzy environment; i.e. the solution procedure digests any type of uncertainty in the basic evolution of the event concerned.

2.1 Fuzzification Review

All meteorological events are considered as having ambiguous characteristics and therefore their domain of change are divided into many fuzzy subsets that are complete, normal, and consistent with each other. Hence, the domain of change is fuzzified. This stem is applied to each meteorology factor considered in the solution of the problem.

2.2 Fuzzy Inference System (FIS)

In fact, this step relates systematically pair wise all the factors taking place in the solution, which depends on the purpose of the problem. This part includes many fuzzy conditional statements to describe a certain situation. For instance, if two events X and Y are interactive, then they are dependent on each other. Conditional statements express the dependence as verbally without any equation, which is used in the classical approaches:

IF X is A(1) THEN Y is B(1)
ALSO
IF X is A(2) THEN Y is B(2)
ALSO
IF X is A(3) THEN Y is B(2)
ALSO
IF X is A(n) THEN Y is B(n)

where A(1), A(2),: : :, A(n) and B(1), B(2),: : :, B(n) are the linguistic description of X and Y respectively. They are fuzzy subsets of X and Y that cover whole domain of change of X and Y. The fuzzy conditional statements in Eq. (1) can be formalized in a form of the fuzzy relation R(X,Y) as R(X,Y)=ALSO (R1; R2; R3; : : :; RN) Where, ALSO represents a sentence connective which combines Ri's into the fuzzy relation R(X,Y), and Ri denotes the fuzzy relation between X and Y determined by the i-th fuzzy conditional statement. After having established the fuzzy relationship R (X,Y), then the compositional rule inference is applied to infer the fuzzy subset B for a given fuzzy subset A for X as B=AOR(X,Y) where "o" is a compositional operator.

2.3 Defuzzification

The final result from the previous step is in a form of the fuzzy statement in order to calculate the deterministic value of a linguistic variable Y. The defuzzification method must be applied as follows [10]:

$$y_j = \frac{\sum_{i=1}^{L} y_i}{L} \quad (1)$$

$$p_j(x) = \frac{\prod_{i=1}^{M} \mu_{A_{ij}}(x_i)}{\sum_{j=1}^{L} \prod_{i=1}^{M} \mu_{A_{ij}}(x_i)} \quad (2)$$

$$y = f(x) = \sum_{j=1}^{L} p_j(x) y_j \quad (3)$$

TABLE 1. Obtained lead time based on the fuzzy system

History of Vendor delivery	Volume of order: Small	Medium	Big
Bad	4.4	5.0	5.6
Medium	1.2	5.0	5.0
Good	4.0	4.1	4.0

where, p(x) is a fuzzy basis function and y is a particular value of the linguistic variable. y_j is the support value, in which the membership function reaches its maximum grade of membership. L and M are a number of rules and inputs respectively. In this paper, the center-average method is selected and applied to defuzzify the proposed problem.

3 The Proposed Approach

When manufacturer wants to begin production, it is important to estimate lead time accurately. Also, estimated planned lead time will affect on the MRP and MPS respectively. After a proper estimation of lead time, the manufacturer is able to improve delivery performance into the customer. In this paper, a fuzzy rule base is applied in order to estimate lead time accurately. In the proposed fuzzy system, the history of vendor delivery and volume of order are considered as inputs. Lead time is defined as an output of the fuzzy system. Selecting the inputs is significant because they have high impact on output. Then according to real conditions, some linguistic rules are used to model this system. After fuzzification and fuzzy inference steps, finally based on defuzzification results, expected lead time will be estimated based on real conditions (i.e., linguistic rules),in which it is impossible to model them except using a fuzzy approach.

4 A Typical Example

In this section, a typical test problem is given to demonstrate applicability of the approach.

4.1 Fuzzy rule base implementation

According to the steps mentioned in Section 2, the test problem is implemented in MATLAB 7.0 as follows:

4.1.1 Fuzzification step*:*

In this step, it is required to fuzzify inputs and output as depicted in Figures 1 to 3 respectively.

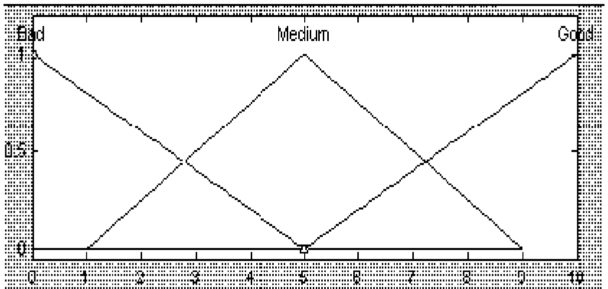

Fig. 1. Membership function for the vendor history

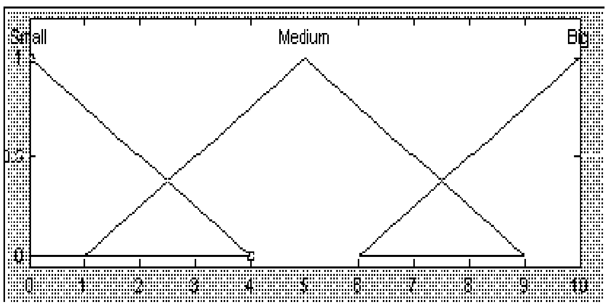

Fig. 2. Membership function for the order volume

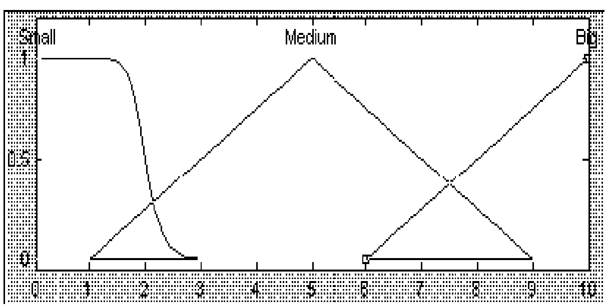

Fig.3. Membership function for the lead time

4.1.2 Fuzzy inference system

In this section, linguistic rules are used to attain the objective as follows:

- Rule 1: If history of vendor delivery is good and volume of order is small then lead-time is small.
- Rule 2: If the history of vendor delivery is good and volume of order is medium, then lead time is medium.
- Rule 3: If the history of vendor delivery is good or volume of order is small, then lead time is small.
- Rule 4: If the history of vendor delivery is bad and volume of order is medium, then lead time is medium.
- Rule 5: If the history of vendor delivery is bad and volume of order is big, then lead time is big.

4.1.2 Deffuzification step

After defuzzifing the proposed fuzzy system, estimated lead time will be achieved. The form of a fuzzy rule base is illustrated through a matrix. The number of fuzzy regions in the input defines the size of the matrix of the combined fuzzy rule base. For a two input – one output case, a combined rule base can be formed with a two-dimension matrix as shown in Figure 4. Data given in Table 1 illustrate the lead time value for various conditions.

4.2 Discussion

In the above well-indicated example, instead of considering deterministic lead time, it is estimated due to real conditions on the vendor delivery and order volume. The results show that each pair of two selected inputs has an impact on lead time, only exception is occurred when the history is medium and volume of order is small simultaneously. Also, it is possible to run a sensitivity analysis on affected parameters on lead time to make a good decision about lot sizing policies. It is clear that the obtained lead time will be used in the MRP system by applying available methods in the literature.

5 Experimental Results

While common methods available in the literature considered lead time as a deterministic and pre-specified value, the proposed fuzzy approach emphasis on applying fuzzy rule based and estimating lead time resulting from the real conditions that may occur in the manufacturing environment. Also, the obtained lead time is considered as an input – as well as MPS- in the MRP system.

5.1 Monte Carlo Simulation

In this section, the validation of the proposed approach is evaluated. For this reason, simulated lead times are compared with lead times obtained from the fuzzy rule based approach. The model was tested in the cases of 1, 2, 3, and 4 orders in an MRP system to demonstrate the superiority of the proposed approach. Each observation has obtained through 1000 simulation runs, based on obtained data given in Table 1. Also, simulated lead times are obtained after 1000 simulation runs due to the uniform probability distribution function. Finally, the summation of lead times is considered to compare both methods. Also, to have a significant insight within comparative analysis, a two-way analysis of variance has been conducted. The results show that the fuzzy lead times are smaller than simulated lead times in all cases. As a result, when the number of orders in the MRP system increases, it seems that the existing difference between simulated and fuzzy lead time becomes bigger. However, as it was expected in both cases by considering large numbers of order, the summation of lead times becomes greater. Table 2 indicates a comparative analysis between fuzzy and simulated data. As indicated, fuzzy lead times in all observations are less than corresponding simulated lead times.

TABLE 2. Comparative analysis between fuzzy and simulated lead times

Observation	Number of order	Fuzzy lead time	Simulated lead time
1	1	4.2	4.9
2		4.3	5.1
3		4.2	5
4		4.3	4.9
5		4.2	4.8
6	2	8.5	9.9
7		8.6	9.8
8		8.4	10.2
9		8.6	9.9
10		8.5	9.8
11	3	12.7	15.2
12		12.8	14.9
13		12.8	14.9
14		12.7	14.8
15		12.8	15.1
16	4	17.12	19.9
17		17	20.2
18		17	19.8
19		16.9	20.1
20		17.1	19.8
Average		**10.636**	12.45

TABLE 3. Two-way analysis of variance

Source	Mean square	Degree of freedom	*F*- statistic	*P*-value
Column	32.9	1	8.84	0.0058
Row	964.11	4	64.74	0
Interaction	6.22	4	0.42	0.79
Error	111.69	30		

5.2 Two-way analysis of variance (ANOVA)

A two-way analysis of variance tests the equality of population's means when classification of treatments is by two variables or factors, which in this research the number of orders and the available methods are considered as two important factors affecting on lead time in the MRP system. The results of analysis of variance, as shown in Table 3, indicates the fuzzy approach is significantly different from the results obtained by simulation. The *p*-value for the approach effect is 0.0058. This is a strong indication that the lead time varies from one approach to another. Also, the *p*-value for the number of orders is near to zero, which is also highly significant. Thus, the proposed approach can be used appropriately.

6 Conclusion

In this paper, a fuzzy lead time application to a material requirement planning (MRP) environment has been addressed. The proposed approach has used a fuzzy rule base to cover linguistic rules, which it would be happen in a manufacturing environment. In the proposed fuzzy system, the history of vendor delivery and volume of order were considered as inputs to estimate lead time as out put in the proposed system. The results showed that after the defuzzification process, lead time could be estimated easily on the basis of various input conditions and obtained results would be used in the MRP system by applying available methods in the literature.

The approach was more reliable when the manufacturer wanted to estimate lead time and consequently MRP and master production schedule (MPS) more accurate. To evaluate the validation of the fuzzy approach, the results obtained by the proposed approach were compared with simulated lead times, both in 1000 simulation runs and 20 independent observations. A two-way analysis of variance confirmed our hypothesis significantly. Therefore, the proposed fuzzy approach can be used in MRP systems safely. Further research will be focused on applying stochastic processes to obtain lead time due to a probabilistic nature of MRP situations.

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