A Fuzzy Mixed-Integer Goal Programming Model for a Parallel Machine Scheduling Problem

R. TAVAKKOLI-MOGHADDAM, A.H. GHAREHGOZLI, M. RABBANI, N. ZAERPOUR
Department of Industrial Engineering, Faculty of Engineering
University of Tehran
P.O. Box: 11365/4563, Tehran
IRAN

Abstract: - This paper presents a new mixed-integer goal programming (MIGP) model for a parallel machine scheduling problem with sequence-dependent setup times and release dates under the hypothesis of fuzzy processing time's knowledge. Two objectives are considered in the model to minimize the total weighted flow time and the total weighted tardiness simultaneously. At the end, the effectiveness of the proposed model is demonstrated through some test problems.

Key-words: - Parallel machine scheduling; Mixed-integer goal programming; Fuzzy sets theory; Release date; Sequence-dependent setup time.

1 Introduction

Production scheduling consists of planning jobs that need to be performed in an orderly sequence of operations. It is a tool that optimizes the use of available resources. However, scheduling problems are very complex computationally and therefore it is so difficult to optimally solve in reasonable time due to its combinatorial nature. As a result, finding an optimal solution in reasonable time is not always possible [1]. Many researchers have assumed the parameters such as job processing times, release dates, and due dates to be deterministic [2]. However, in the real-world situation, these parameters are often encountered with uncertainties. Accordingly, there are basically two approaches to deal with the machine scheduling problems that encounter with uncertainties [3] including stochastic probabilistic theory and possibility theory [4] or fuzzy set theory [5]. The fuzzy approach represents an alternative way to model imprecision and uncertainty, that is more efficient than the latter, especially when no historical information is available.

The use of the fuzzy sets theory in treating different scheduling problems has been so successful, particularly where judgment and intuition play an important role such as customer demand [6], processing times [7] or job precedence relations [8]. Tavakkoli-Moghaddam *et al.* [9] suggested a fuzzy mixed-integer goal programming model for single machine scheduling problem.

In production scheduling, most researches are concerned with the minimization of single criterion. However, real-life production scheduling tends to consider multi criteria simultaneously. Different problems in production scheduling have been treated using fuzzy multi-criteria decision making (MCDM) methods. Among lots of examples, some of them are the most considerable ones such as: scheduling of single machine [8] or job shop [10]. Although it is extremely important, little attention has been given to multi criteria scheduling problems, especially in the case of multiple machines, which is because of the extreme complexity of these combinatorial problems [11]. A typical multi-objective optimization problem can be presented as follows:

$$Z_l = \min Z_l(x) \qquad \forall l \qquad (1)$$

s.t.

$$(Ax)_i \le b_i \qquad \forall i \qquad (2)$$

$$x \ge 0 \qquad (3)$$

where, functions Z_l are the objectives and X is the vector of variables. A feasible solution X^* is efficient if and only if for all other feasible solutions X, $Z_l(X^*) \le Z_l(X)$ with at least one strict inequality.

In the literature concerning multi-objective scheduling problems, five main approaches are distinguishable: (a) Hierarchical approach, (b) utility approach, (c) goal programming (or satisfying approach), (d) simultaneous (or Pareto), and (e) interactive approaches. Each approach has its own strengths and weaknesses as described in the general literature on multi-objective optimization problems. Clearly depending on the aim of study or the context of the application treated, a proper approach is used.

In the rest of the paper, we focus on a parallel machine scheduling problem. In parallel processing, jobs are processed by one of several machines, allowing considerable reduction in makespan. The study of parallel machine problems is relevant from both the theoretical and the practical points of view [12]. From the practical point of view, it is important because we can find many examples of the use of parallel machines in the real world. From the theoretical point of view, it is a generalization of the single machine problem and a particular case of problems arising in flexible manufacturing systems.

Chen and Powell [13] considered the scheduling problem *n* jobs on *m* identical, uniform, or unrelated machines with two particular objectives: 1) to minimize the total weighted completion time and 2) to minimize the weighted number of tardy jobs. Chen and Powell [14] studied the minimization of the total weighted completion time of the jobs, when multiple job families should be scheduled on identical parallel machines with sequence-dependent or sequence-independent setup times. A limited amount of the literature has been devoted to fuzzy parallel machine scheduling problems [15].

Motivated by the literature discussed above, this paper presents a parallel-machine scheduling problem with sequence-dependent set-up times under the hypothesis of fuzzy processing time's knowledge and two fuzzy objectives. The rest of the paper is organized as follows. Section 2 discusses about the multi-objective mixed-integer programming of parallel machines. Section 3 describes about the fuzzy mixed-integer goal programming model. Section 4 is devoted to numerical experiments conducted on the basis of generated random data that shows the effectiveness of the proposed approach.

2 Proposed Multi-Objective Mixed-Integer Programming Model

In this paper, the problem can be formally formulated as follow: N jobs with sequence-dependent set-up times, different release dates, non-common due dates and varying processing times are to be processed on M parallel machines. Let d_i and p_i be the due date and processing time of job i respectively. Let s_{ij} denotes the set-up time for job j immediately following job i in the sequence. s_{0i} is the set-up time for job i in sequence position 1. Let M be a large positive number. Moreover, let the real variables t_i, c_i and r_i denote the tardiness, completion, and release time of job i respectively. In addition, we consider the following variables: x_{ij}^k is a binary variable which is equal to 1 if job j is immediately after job i in sequence on machine k; y_j^k is also a binary variable which is equal to 1 if job j is assigned to machine k. The multi-objective mixed-integer programming (MO-MIP) model of the parallel machine scheduling problem can be written as follows:

$$Z_1 = \min \sum_{i=1}^{N} w_i^1 t_i \tag{4}$$

$$Z_2 = \min \sum_{i=1}^{N} w_i^2 c_i \tag{5}$$

s.t.

$$c_i + M\left(1 - x_{0i}^k\right) \geq s_{0i} + r_i + p_i^k \qquad \forall i,k \tag{6}$$

$$c_j + M\left(1 - x_{ij}^k\right) \geq \max\{r_i, c_i + s_{ij}\} + p_j^k \qquad \forall i,j,k \quad ; i \neq j \tag{7}$$

$$\sum_{i=1}^{N} x_{0i}^k = 1 \qquad \forall k \tag{8}$$

$$\sum_{i=0\, i\neq j}^{N} x_{ij}^k = y_j^k \qquad \forall j,k \tag{9}$$

$$\sum_{j=1\, j\neq i}^{N} x_{ij}^k \leq y_i^k \qquad \forall i,k \tag{10}$$

$$\sum_{k=1}^{M} y_j^k = 1 \qquad \forall j \tag{11}$$

$$t_i \geq \max\{o, c_i - d_i\} \qquad \forall i \tag{12}$$

$$x_{ij}^k, y_j^k \in \{0,1\}\ \forall i,j; i \neq j\ ,\ c_i, t_i \geq 0\ \forall i \tag{13}$$

The objective functions (4) and (5) minimize the total weighted tardiness and total weighted flow time respectively. A dummy job 0 is introduced on each machine in order to take care of the setup time for the real job assigned to the first position in the sequence on each machine. Constraint (6) represents that the completion time of a real job assigned to the first position in the sequence on any machine should be greater than or equal to the sum of its initial set-up time and its processing time. Constraint (7) ensures that the completion time of a real job in a sequence on a machine will be at least equal to the sum of the completion time of the preceding job, the sequence-dependent set-up time, and the processing time of the present job. Constraint (8) guarantees that the dummy job 0 is positioned at the beginning of the sequence before all the real jobs on each machine. Constraint (9) depicts that if a real job is assigned to a machine, then it will be immediately preceded by one job (i.e., the real job in sequence position one will be preceded by the dummy job 0). Similarly, Constraint (10) portrays that if a real job is assigned to a machine then it can be succeeded by at most one job. The job in the last position of the sequence on a machine will not have a succeeding job. Constraints (9) and (10) together affirm that N real jobs are assigned over M machines. They also ensure that if job i immediately precedes job j on machine k, then both jobs i and j belong to machine k. Constraint (11) confirms that a real job is assigned to exactly one machine. Constraint (12) specifies the tardiness of each job.

3 Fuzzy Mixed-Integer Goal Programming

A typical fuzzy multi-objective linear programming problem can be stated as follows:

$$Z_k = \widetilde{\min}\, Z_k(x) \qquad \forall k \qquad (14)$$

s.t.

$$(Ax)_i \leq \tilde{b}_i \qquad \forall i \qquad (15)$$

$$x \geq 0 \qquad (16)$$

Consider that the aspiration level of the objective is defined as Z_0. In addition we assume b_i and its corresponding tolerance $\Delta_{b_i}\ \forall i$. The fuzzy objective and the fuzzy constraints are then considered without difference. Thus, the fuzzy mixed-integer goal programming model (F-MIGP) can be stated as follows:

$$find \quad x \qquad (17)$$

s.t.

$$Z_k(x) \leq Z_k \qquad \forall k \qquad (18)$$

$$(Ax)_i \geq b_i \qquad \forall i \qquad (19)$$

$$x \geq 0 \qquad (20)$$

The fuzzy objective functions and the fuzzy constraints are defined by their corresponding membership functions. For simplicity, let us assume that the membership functions μ_l of the fuzzy objectives are non-increasing continuous linear functions, and the membership functions $\mu_i\ \forall i$, of the fuzzy constraints are non-increasing linear membership functions. Thus, the membership functions of the constraints are as follows:

$$\mu_i(x) = \begin{cases} 1 & if\ Ax \succ b_i \\ 1 + \dfrac{(Ax)_i - b_i}{\Delta_{b_i}} & if\ b_i - \Delta_{b_i} \leq Ax \prec b_i \\ 0 & if\ Ax \leq b_i - \Delta_{b_i} \end{cases} \qquad \forall i \qquad (21)$$

To be more specific, let us describe the membership functions of the F-MIGP objectives as:

$$\mu_{Z_k}(x) = \begin{cases} 1 & if\ Z_k(x) \prec Z_k^l \\ \dfrac{Z_k^u - Z_k(x)}{Z_k^u - Z_k^l} & if\ Z_k^l \leq Z_k(x) \prec Z_k^u \\ 0 & if\ Z_k(x) \geq Z_k^u \end{cases}$$

$$\forall k \qquad (22)$$

where, Z_k^u and Z_k^l are the upper bound and lower bound of the k^{th} objective function, respectively. The membership functions can be illustrated as Fig. 1.

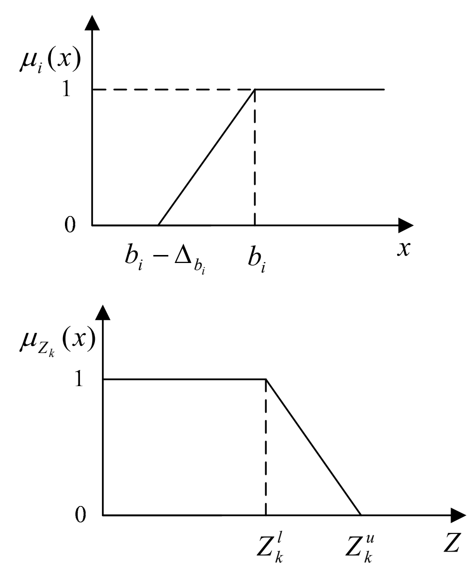

Fig. 1. The membership functions

Zimmerman [16] then used Bellman and Zadeh's [17] max-min operator to solve the equations. So the Crisp Mixed-Integer Goal Programming model (C-MIGP) will be equivalent to:

$$\max \quad \alpha \tag{23}$$

s.t.

$$\mu_k(x) \geq \alpha \qquad \forall k \tag{24}$$

$$\mu_i(x) \geq \alpha \qquad \forall i \tag{25}$$

$$\mu_k(x), \mu_i(x), \alpha \in [0 \quad 1] \quad \forall k, i \tag{26}$$

The above model can be also expressed as:

$$\max \quad \alpha \tag{27}$$

s.t.

$$Z_k(x) \leq Z_k^u - \alpha(Z_k^u - Z_k^l) \qquad \forall k \tag{28}$$

$$(Ax)_i \geq b_i - \Delta_{b_i}(1-\alpha) \qquad \forall i \tag{29}$$

$$x \geq 0, \alpha \in [0 \quad 1] \tag{30}$$

3.1 Proposed F-MIGP model for parallel machine scheduling problem

Due to the complexity and uncertainty involved in real world scheduling problems, it is sometimes unrealistic or even impossible to require exact data. A good scheduling model needs to tolerate vagueness or ambiguity. In this study, the model imprecision is represented by the fuzzy approach. A multi-objective programming model for a parallel-machine scheduling problem with sequence-dependent setup times under the hypothesis of fuzzy processing time's knowledge and two fuzzy objectives is presented as follows:

$$\widetilde{Z}_1 = \widetilde{\min} \sum_{i=1}^{N} w_i^1 t_i \tag{31}$$

$$\widetilde{Z}_2 = \widetilde{\min} \sum_{i=1}^{N} w_i^2 c_i \tag{32}$$

s.t.

$$c_i + M\left(1 - x_{0i}^k\right) \geq s_{0i} + r_i + \widetilde{p}_i^k \qquad \forall i, k \tag{33}$$

$$c_j + M\left(1 - x_{ij}^k\right) \geq \max\{r_i, c_i + s_{ij}\} + \widetilde{p}_j^k \qquad \forall i, j, k \quad ; i \neq j \tag{34}$$

$$\text{Constraints (8) to (13)} \tag{35}$$

The fuzzy objective functions (31) and (32) imprecisely minimize the total weighted tardiness and total weighted flow time respectively. Constraint (33) represents that the completion time of a real job assigned to the first position in the sequence on any machine should be greater than or equal to the sum of its initial setup time and its fuzzy processing time. Constraint (34) ensures that the completion time of a real job in sequence on a machine will be at least equal to the sum of the completion time of the preceding job, the sequence-dependent set-up time, and the fuzzy processing time of the present job. The other constraints are the same as MO-MIP model. So the F-MIGP model will be equivalent to:

$$\max \quad \alpha \tag{36}$$

s.t.

$$Z_1(x) \leq Z_1^u - \alpha(Z_1^u - Z_1^l) \tag{37}$$

$$Z_2(x) \leq Z_2^u - \alpha(Z_2^u - Z_2^l) \tag{38}$$

$$c_i + M\left(1 - x_{0i}^k\right) \geq s_{0i} + p_i^k + r_i - (1-\alpha)\Delta_{p_i^k} \qquad \forall i, k \tag{39}$$

$$c_j + M\left(1 - x_{ij}^k\right) \geq \max\{r_i, c_i + s_{ij}\} + p_j^k - (1-\alpha)\Delta_{p_j^k} \qquad \forall i, j, k \quad ; i \neq j \tag{40}$$

$$\text{Constraints (8) to (13)}, \ \alpha \in [0 \quad 1] \tag{41}$$

4 A Numerical Example

In this section, we demonstrate the effectiveness of the F-MIGP technique for the parallel-machine scheduling problem with sequence-dependent setup times with assuming fuzzy processing times through some test problems. All test problems have been solved using the Lingo 8.0 software. These tests have been done on a portable computer Intel(R) Pentium(R) M 1.86 GHz with 512 MB RAM and MS Windows XP professional Operating System. The initial inputs are produced as follows:

- The process times and release dates are created at random between $[1 \quad 10]$;
- The corresponding process time tolerances are assumed to be $\delta * (p_i^k)$ $\forall i, k$, $\delta \in [0 \quad 1]$;
- The weights of jobs ($w_i^1, w_i^2 \neq 0$) are created at random between $[1 \quad 6]$;
- The dependent setup times are created at random between $[2 \quad 4]$. As commonly assumed in the literature [18], setup times satisfy the triangle inequality: $s_{ij} + s_{jv} \geq s_{iv}$, for all $i, j, v \in N$;
- The corresponding due dates are also computed by $d_i = (\sum_{k=1}^{M} p_i^k / M) N (1 - \text{Rnd}[0 \quad 1])$ $\forall i$ as given

in [19]. N is the number of jobs, M is the number of machines, and p_i are the processing times.

First of all, we show the procedure of the proposed methodology for solving F-MIGP model on an intermediate-size test problem (three machines and seven jobs). The setup times are similar for each machine. Tables 1 and 2 summarize the results of the F-MIGP model in the test problem. The degree of goal achievement is $\alpha = 0.45$. Thus, it obviously confirms the effectiveness of the model. The last column of table 2 depicts the objective value of the MO-MIP model which is not a fuzzy model.

Table 1. The obtained goals by solving the MIP model

The obtained goals by solving the MIP model	Z_l^*	Z_u^*
Z_1^* =Total weighted tardiness	59.6	72
Z_2^* =Total weighted flow time	214	247

Table 2. The final output of the C-MIGP model

objectives	C-MIGP model	MO-MIP model
Degree of goals achievement	0.45	---
Optimal total weighted tardiness	66.41	73
Optimal total weighted flow time	231.2	251

In addition the optimal sequence of jobs is depicted in Fig. 2.

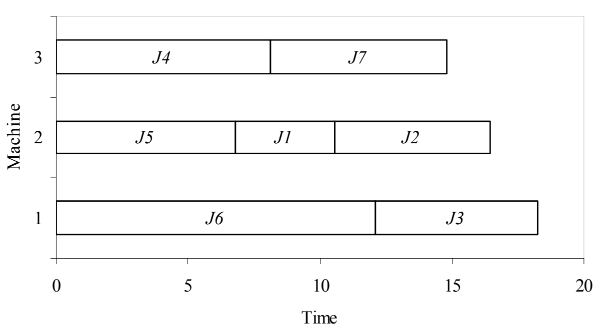

Fig. 2. Optimal sequence of jobs

We have also applied the F-MIGP model to some random generated test problems. The degrees of goal achievement (α) are illustrated in Table 3.

Experiments proved that the value of delta (δ) may affect on the results of test problems. Table 4 shows that the degree of goal achievement generally grows as the delta becomes larger (two machines).

Table 3. Degree of goal achievement (α) ($\delta = 0.4$)

Number of machines (m)	Alpha (α) Number of jobs (n) 4	5	6	7
1	0.5	0.47	0.363	0.4
2	0.322	0.336	0.419	0.487
3	0.471	0.286	0.325	0.45

Table 4. Degree of goal achievement (α) through some test problems with different δ

Number of jobs (n)	Alpha (α) δ 0.2	0.4	0.6	0.8
5	0.325	0.336	0.404	0.484
6	0.343	0.419	0.446	0.512
7	0.273	0.286	0.414	0.451

In this section, a simulation study has been performed to evaluate the average goal achievements and confidence intervals for different numbers of jobs and two machines. The study was repeated 120 times for each situation. The confidence intervals are calculated by means of Eq. 36 and reported in table 5. The confidence level is assumed to be 90% and delta value is also considered to be 0.4. $\overline{X}$ is the estimator of sample mean and S is the estimator of sample variance.

$$\left[\overline{X} - t_{\alpha/2;n-1}\frac{S}{\sqrt{n}} \quad \overline{X} + t_{\alpha/2;n-1}\frac{S}{\sqrt{n}}\right] \tag{36}$$

Table 5. Confidence intervals with 2 machines

	Number of jobs 4	5	6	7
$\overline{X}$	0.36	0.37	0.39	0.28
$t_{\alpha/2;n-1}\frac{S}{\sqrt{n}}$	0.008	0.008	0.006	0.001

5 Conclusion

It is well known that the optimal solution of single objective models can be quite different to those models consisting of multi objectives. In fact, the decision maker often wants to minimize the earliness/tardiness penalty or total flow time. Each of these objectives is valid from a general point of view. Since these objectives conflict with each other, a solution may perform well for one objective or it gives inferior results for others. For this reason, scheduling problems have a multi-objective nature.

In this paper, we have proposed a new mixed-integer goal programming model for a parallel-machine scheduling problem with sequence-dependent setup times, release dates, and two objectives. In decision making situations, the high degree of fuzziness and uncertainties is included in the data set. The fuzzy set theory provides a framework for handling the uncertainties of this type. We have suggested a novel fuzzy mixed-integer goal programming model for a parallel-machine scheduling problem with sequence-dependent setup, release dates, and fuzzy process times as well as a suitable methodology for solving it. Additional research from this study may follow several directions. One trend may make an empirical study of manufacturing organizations, comparing the effectiveness of their current production planning with the theoretical policy resulting from this model. Because of the fact that the large-sized parallel machine scheduling problem is NP-complete, another direction might be to employ metaheuristics algorithms to determine the solution of large-sized model.

References:

[1] D. Petrovic, & A. Duenas, A fuzzy logic based production scheduling/rescheduling in the presence of uncertain disruptions, *Fuzzy Sets and Systems,* Vol. 157, 2006, pp. 2273-2285.

[2] P. Brucker, *Scheduling Algorithms,* Second ed., Berlin: Springer, 1998.

[3] P.Y. Mok, C.K. Kwong, & W.K. Wong, Optimization of fault-tolerant fabric cutting schedules using genetic algorithms and fuzzy set theory, *European Journal of Operational Research*, Vol. 177, 2007, pp. 1876-1893.

[4] D. Dubois, & H. Prade, *Possibility Theory: An Approach to Computerized Processing of Uncertainty,* New York: Plenum Press, 1988.

[5] L.A. Zadeh, Fuzzy sets, *Information and Control*, Vol. 8, 1965, 338-353.

[6] R. Petrovic, & D. Petrovic, Multi-criteria Ranking of Inventory Replenishment Policies in the Presence of Uncertainty in Customer Demand, *International Journal Production Economics,* Vol. 71, 2001, pp. 439-446.

[7] M. Kuroda, & Z. Wang, Fuzzy job shop scheduling, *International Journal of Production Economics, Vol. 44*, 1996, pp. 45-51.

[8] H. Ishii, & M. Tada, Single machine scheduling problem with fuzzy precedence relation, *European Journal Operational Research,* Vol. 87, 1995, pp. 284-288.

[9] R. Tavakkoli-Moghaddam, B. Javadi, & N. Safaei, Solving a mixed-integer model of a single machine scheduling problem by a fuzzy goal programming approach, *WSEAS Transactions on Business and Economics*, Vol. 3, Issue 2, 2006, pp. 45-52.

[10] P.T. Chang, & Y.T. Lo, Modeling of job-shop scheduling with multiple quantitative and qualitative objectives and a GA/TS mixture approach, *International Journal of Computer Integrated Manufacturing,* Vol. 14, 2001, pp. 367–384.

[11] T. Loukil, J. Teghem, & D. Tuyttens, Solving multi-objective production scheduling Problems using metaheuristics, *European Journal of Operational Research,* Vol. 161, 2005, pp. 42-61.

[12] M.J.P Lopes, & J.M. Vale´rio de Carvalho, A branch-and-price algorithm for scheduling parallel machines with sequence dependent setup times, *European Journal of Operational Research,* Vol. 176, 2007, pp. 1508-1527.

[13] Z.L. Chen, & W. Powell, Solving parallel machine scheduling problems by column generation, *INFORMS Journal on Computing,* Vol. 11, No. 1, 1999, pp. 78-94.

[14] Z.L. Chen, & W. Powell. Exact algorithms for scheduling multiple families of jobs on parallel machines, *Naval Research Logistics,* Vol. 50, 2003, pp. 823-840.

[15] J. Peng, & B. Liu, Parallel machine scheduling models with fuzzy processing times, *Information Sciences,* Vol. 166, 2004, pp. 49-66.

[16] H.J. Zimmermann, Description and optimization of fuzzy systems, *International Journal of General Systems,* Vol. 2, 1976, pp. 209-215.

[17] E.R. Bellman, & L.A. Zadeh, Decision making in a fuzzy environment, *Management Sciences,* Vol. 7, 1970, pp. 141-164.

[18] C.N. Potts, & M.Y. Kovalyov, Scheduling with batching: A review, *European Journal of Operational Research*, Vol. *120*, 200, pp. 228-249.

[19] D.R. Sule, *Industrial Scheduling,* PWC Publishing Company, 1997.