An Algorithm for Realtime High Resolution Octave Analysis Based on Multirate Signal Processing Theory

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Abstract: - Octave analysis is widely used in acoustics. Traditional bandpass filtering based octave analysis algorithms could not meet today's real time high resolution analysis requirements because of the huge number bandpass filters they would need. To resolve this problem, a digital algorithm without using bandpass filtering is introduced in this paper. This algorithm is realized in frequency-domain and could be easily implemented on common DSP (Digital Signal Processing) systems. According to the experiments conducted on a digital dynamic signal analyzer based on DSP chips, this algorithm could meet the requirements of multi-channel real time octave analysis.

Key-Words: - Realtime High Resolution Octave Analysis, Spectrum Estimation, Decimation, Halfband Filter, DSP

1 Introduction

The purpose of this paper is to introduce an algorithm for realtime high resolution octave analysis. Octave analysis is widely used in acoustic signal analysis. The results of octave analysis are often presented on a chart called a spectrogram, which shows power distributions of the input acoustic signal according to its frequency components. Octave band is defined by Constant Proportional Bandwidth (CPB) method. 1/n octave is a frequency band whose highest frequency is $2^{1/n}$ times of the lowest frequency.

Compared with traditional bandpass filtering algorithms, the algorithm introduced here saves a great deal of computations. This advantage makes it a feasible way to meet the realtime requirements. Publications about multirate signal processing are already accessible to most people, but there are few ones about applying this theory to octave analysis. It is thus hoped that this paper could provide some necessary materials to people who are interested in this field.

2 The Theory and Deficiency of Traditional Bandpass Filtering Based Octave Analysis Algorithms

Octave analysis algorithms based on bandpass filtering (Fig.1) have been widely used by many octave analyzers for a long time.

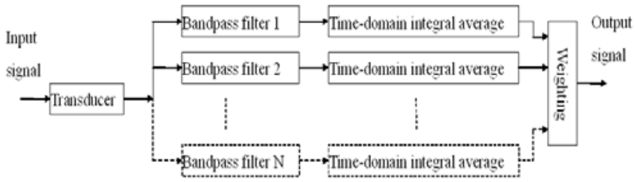

Fig.1 Theory of octave analysis algorithms based on bandpass filtering

After the transducer converts the input acoustic signal into electrical signal, this electrical signal passes a series of bandpass filters, which have the standard octave center frequencies and bandwidths. These octave bandpass filters separate the signals with desired frequency components from the input signal. Then conduct time-domain integral average operation to every signal from these filters. This process gains the power density of each signal with the standard octave frequency components. After the weighting operations, the results could be displayed in octave spectrogram.

The severest deficiency of these algorithms is that the number of bandpass filters increases exponentially as the analysis resolution enhances. For example, if the analysis frequency range is [20Hz, 20kHz], 1/24 octave analysis needs 248 bandpass filters according to *IEC61260-1995*[1]. Although the huge number bandpass filters could be replaced by a few DSP chips with proper software, namely digital filters, this digital filtering process needs very huge computations. To the example mentioned above, just the digital filtering process needs to execute 248 times filtering computations to signal from every input channel. Even the fastest DSP chips today could not meet the requirements of modern multi-channel realtime high resolution octave analysis.

[1] IEC 61620-1995: Electroacoustics-Octave band and fractional octave band filters

3 Theory and Deficiency of a Digital Octave Analysis Algorithm Realized in Frequency-domain

Parseval Theorem points that the total energy contained in a signal *x(t)* summed across all of time *t* is equal to the total energy of the signal's Fourier transform *X(f)* summed across all of its frequency components *f*. So, could octave analysis be realized in frequency-domain? (Fig.2)

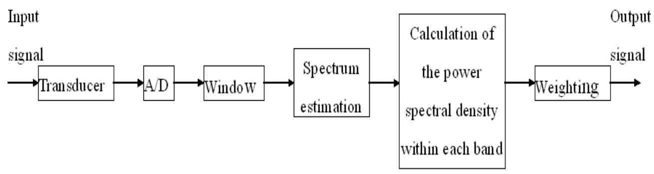

Fig.2 Theory of an octave analysis algorithm realized in frequency-domain

After the transducer and A/D(Analog/Digital) module the input acoustic signal turns into a digital signal, which passes the following *window* module. The window truncates this signal to proper extent. The main purpose of this truncating process is to reduce the energy leakage in frequency-domain. Then calculate the power spectral density distributions of the signal in the whole frequency-domain using some proper spectrum estimation algorithm. Doing integral average to the power within each analysis band gets the value of power spectral density of each analysis band. After the weighting operations, the results could be displayed in octave spectrogram.

Unlike the bandpass filtering based algorithm whose precision is mainly decided by the performances of the bandpass filtering process, there is no bandpass filtering in this algorithm. Instead, the number of spectrum lines within each band is an important factor affecting precision. According to experiments, there

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should be at least 6 spectrum lines within each analysis band in order to guarantee the energy leakage between neighbor bands meets the requirements of *IEC61260-1995*.

Since octave band is defined by CPB method, the bandwidth increases exponentially as the center frequency increases. However, FFT has the single frequency-domain resolution, so the spectrum lines are much fewer in a band with lower frequency than the one with higher frequency. Therefore, in order to guarantee there are at least 6 spectrum lines within each band, there should be at 6 spectrum lines in the band with the lowest center frequency.

Suppose that the analysis frequency range is [20Hz, 20kHz], the sampling frequency is 48kHz, which does not cause aliasing according to *sampling theorem*[2]. According to *IEC61260-1995*, the center frequencies of 1/24 octave band is given as

$$f_0 = 10^{(n+0.5)/80} \tag{1}$$

The upper limit and lower limit frequencies are given as

$$\begin{cases} f_{lower} = 2^{-1/48} f_0 \\ f_{uper} = 2^{-1/48} f_0 \end{cases} \tag{2}$$

From expressions (1)and (2), 1/24 octave band with the lowest center frequency is [19.95164Hz， 20.53626Hz]. To guarantee there are at least 6 spectrum lines in this band, the frequency resolution must be at least (20.53626-19.955164)/6 =0.0968493Hz. Therefore, there are at least 48000/0.0968493=495165 analysis data in one frame, that is, the refresh time of one data frame is at least 1/0.0968493=10.3253 seconds. Obviously, this refresh time is too long to meet the requirements of *realtime* analysis.This example shows that the conflict of the single resolution of FFT and the logarithmic frequency expression of octave analysis spectrogram is the reason that makes this algorithm unpractical.

4 An Algorithm for Octave Analysis Based on Multirate Signal Processing Theory

Multirate simply means multiple sampling rates. A multirate DSP system uses multiple sampling rates within the system. The process of increasing the sampling rate is called *interpolation*, and the process of reducing the sampling rate is called *decimation*. In this section, we use the *decimation* process to resolve the problem of the frequency-domain octave analysis algorithm in section 3.

4.1 Theory of the Algorithm

The basic theory of this octave analysis algorithm is demonstrated in Fig.3, where *LP* represents the lowpass filter, and $\downarrow 2$ represents the decimation process with *decimation factor*[3]=2.

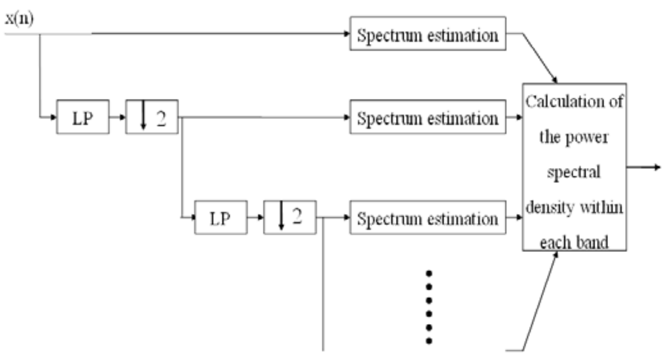

Fig.3 Theory of an octave analysis algorithm based on multirate theory

After the transducer and A/D module convert the

2 Sampling theorem: a signal can be reconstructed completely and exactly from the sampling signal if the sampling frequency is greater than or equal to the highest frequency of the original signal.

3 Decimation factor: the ratio of the input rate to the output rate, so here input rate / output rate=2.

input acoustic signal into digital signal *x(n)*, through a series of successive decimations with the same decimation factor 2 get several groups of digital signals with different sampling rates (Fig.3). If using the same number of data in each signal group in the following spectrum estimation processes, the results of these spectrum estimations have different frequency resolutions. Furthermore, since the sampling rates of these groups of signals get lower and lower as the decimation times increases (from top to bottom in Fig.3), the resolutions of the spectrums of these signals get higher and higher.

Octave analysis needs to gain the power spectral density in each band, so the next task is calculating the power of the signal in each band from the spectrum estimation results gained above.

$$P_i = \sum PSD(k) \times \Delta f_k \qquad (3)$$

Here P_i is the power of the signal in band *i*. *PSD(k)* is the value of spectrum line *k* of the power spectral density, and Δf_k is the frequency resolution of spectrum line *k*. Since an analysis band may include spectrum lines with different frequency resolutions, the weighting factor Δf_k is necessary here. The range of the sum in expression (3) covers all the spectrum lines in a band. In the other hand, since the value of a spectrum line stands for the power spectral density in a range whose center is the frequency of this spectrum line and width is Δf_k, the values some spectrum lines standing for probably do not belong to one band alone. In this case, the values of these spectrum lines should be divided by proportions.

4.2 Design of the Lowpass Decimation Filter

The decimation factor in this algorithm is 2, so the cutoff frequency of the lowpass decimation filter's passband should be less than or equal to $\pi/2$ to avoid the aliasing[1].

In order to cut down computations, the decimation filters used in the algorithm are 47th halfband FIR (Finite Impulse Response) filters, whose nonzero impulse response coefficients are in Table 1 and frequency response is illustrated in Fig.4. As shown in Fig.4, the ripple in the filters' passband is less than 0.0001, and the stopband attenuation is 80dB. Non-aliasing normalized frequency range of the signal after the lowpass filtering is [0, 0.4], which becomes [0, 0.8] after the decimation.

Table 1 Nonzero impulse response coefficients of the 47th halfband filters

h(0), h(46)	-1.975722792229084e-004
h(2), h(44)	5.760704520844917e-004
h(4), h(42)	-1.350975944268072e-003
h(6) , h(40)	2.727516389781759e-003
h(8) , h(38)	-4.986043929625469e-003
h(10),h(36)	8.496970950995700e-003
h(12), h(34)	-1.378583469426858e-002
h(14), h(32)	2.171044581860376e-002
h(16), h(30)	-3.397756532486159e-002
h(18), h(28)	5.494288268556362e-002
h(20), h(26)	-1.006560861576759e-001
h(22), h(24)	3.164569315564292e-001

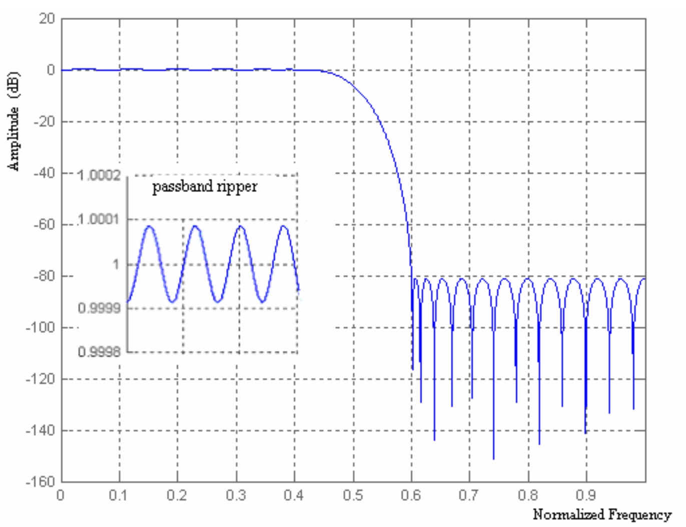

Fig.4 Frequency response of the 47th halfband filters

4.3 Frequency Range Division

The purpose of frequency range division is to combine the results of these spectrum estimations with different resolutions to get the power distributions in the whole analysis frequency range.

The halfband filters used in this algorithm are passband equal-ripper filters, so the passband is much easier to satisfy the design requirements than the stopband[2]. Therefore, the aliasing band is mainly decided by the start frequency of stopband. The normalized start frequency of stopband of the filters in this algorithm is 0.6 (Fig.4), so the non-aliasing band is [0,0.4], which becomes [0, 0.8] after the 2:1 decimation. Define k_D as the *non-aliasing band coefficient*, namely k_D =0.8 here.

Suppose the sampling frequency of the input signal *x(t)* is F_s, and the non-aliasing band coefficient of anti-aliasing analog filter is k_A. Therefore the possible highest non-aliasing frequency of the spectrum estimation result of *x(t)* is $F_s k_A / 2$ according to *Sampling Theorem*. Since the first group of signal (Fig.3) is calculated spectrum estimation directly without decimation, the highest non-aliasing frequency of the estimation result got from this group is $F_s k_A / 2$. After the first 2:1 decimation, the highest non-aliasing frequency got from the second group of signal is $F_s k_D / 4$. Similarly, the highest non-aliasing frequency got from the *nth* group of signal is $F_s k_D / 2^n$.

Suppose the number of signal groups is *N*, the whole analysis frequency range could be divided into:

$$[0, F_s k_A / 2] = (F_s k_D / 4, F_s k_A / 2] \cup (F_s k_D / 8, F_s k_D / 4] \cup (F_s k_D / 16, F_s k_D / 8] \cdots \cup [0, F_s k_D / 2^N] \tag{4}$$

Expression (4) indicates that the power of signal with frequency range $(F_s k_D / 4, F_s k_A / 2]$ should be got from the spectrum estimation result of the first group of signal; the power of signal with frequency range $(F_s k_D / 8, F_s k_D / 4]$ should be got from the spectrum estimation result of the second group of signal, and so on.

5 Experiments

All the following experiments are conducted on a *TMS320C67* DSP chip based dynamic signal analysis system.

The input signal is a sine signal with 1V amplitude and 750Hz frequency. The analysis frequency range is [20Hz,20kHz]; the sampling frequency is 48kHz. Fig.5 and Fig.6 are the spectrograms of the 1/12 and 1/24 octave analysis respectively. The single block in the right side of the figure represents the overall sound lever. Letter *L* below this block implies that the analysis uses linear weighting.

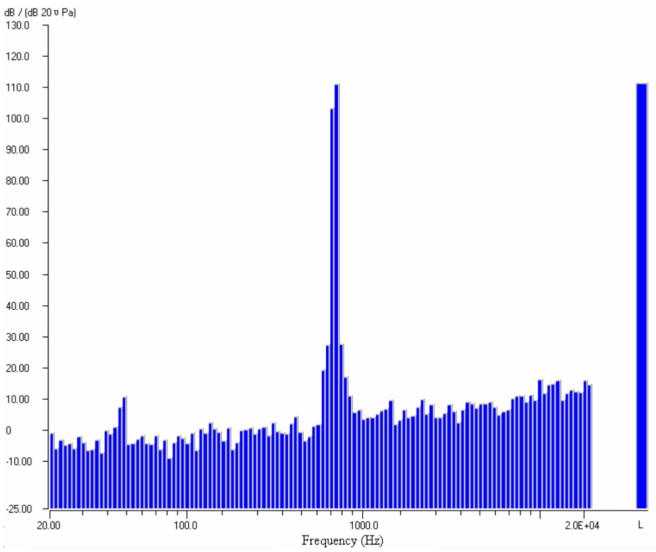

Fig.5 The spectrogram of 1/12 octave analysis

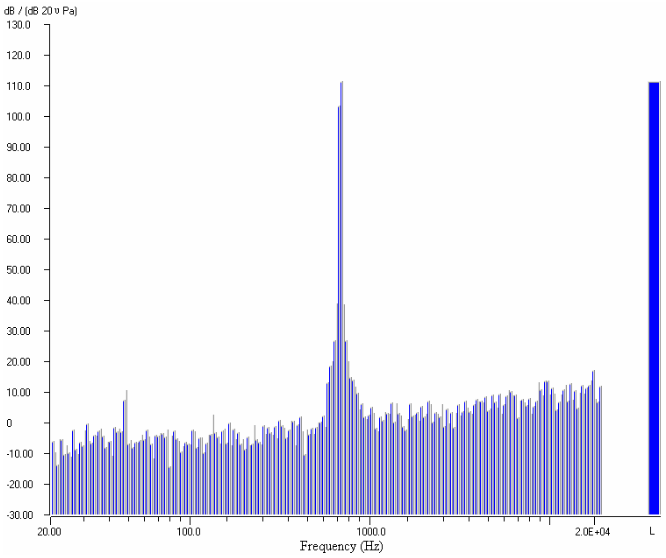

Fig.6 The spectrogram of 1/24 octave analysis

Fig.7 is the spectrogram of an 8-channel 1/24 octave analysis. The input signal of channel 1 is an 1V, 750Hz sine signal while other 7 channels are not connected.

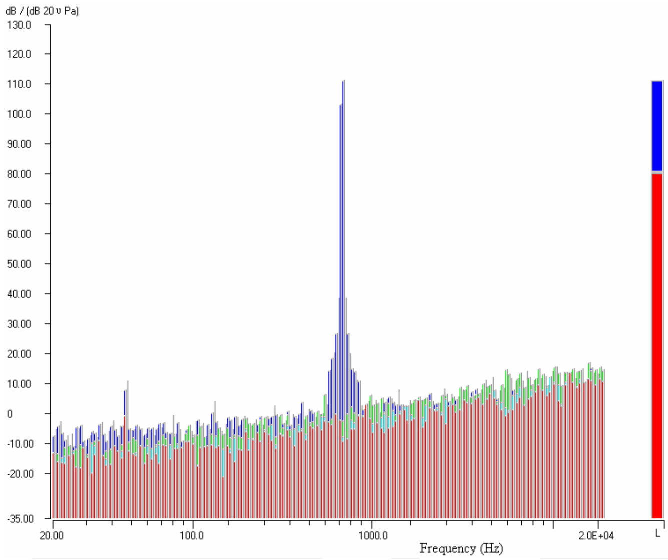

Fig.7 The spectrogram of 8-channel 1/24 octave analysis

As shown in Fig.5--Fig.7, the power spectral density of the noise signal has more than 80dB attenuation compared with the input signal. Such performance could meet the requirements of most acoustic octave analysis. Besides, all the experiments conducted using this algorithm could meet the requirements of *real time*. While on the same hardware, the traditional bandpass filtering based algorithm could only realize 2-channel 1/3 octave analysis in real time. To the 1/24 octave analysis, this algorithm based on multirate signal processing theory could save about 10 times computations than the algorithm based on bandpass filtering on the same hardware platform according to our experiments.

6 Conclusions

By integrating the multirate signal processing theory and classic spectrum estimation theory, a digital octave analysis algorithm is introduced here. The biggest innovation of this algorithm is that it abandons the bandpass filters. This makes the computations of the algorithm increases much fewer than the bandpass filtering based algorithms when analysis resolution enhances. This is because most computation time of this algorithm is occupied by FFT and the decimation filtering, which increases little as the analysis resolution enhances. Experiments on a dynamic signal analysis system have proven this algorithm to be a practical way of the real time high resolution octave analysis.

As mentioned above, the number of data in each spectrum estimation process is the same, while these data have different sampling rates, so the refresh time of bands with different frequencies is not uniform. That is, bands with higher frequencies have shorter refresh time than those with lower frequencies. Generally speaking, this character of refresh time is consistent with the physical character of broadband acoustic signal—the higher frequency components usually vary more quickly than lower ones. Yet, different refresh time in the whole analysis frequency range may also cause discontinuousness of the display of spectrogram. This discontinuousness is ignorable to the relatively stable signals according to our experiments. Yet, to some transient signals like the shock signal, the discontinuousness becomes visible.

Future work about the algorithm presented here may include finding ways to overcome display discontinuousness of the spectrogram of transient signals. Another possible work may be applying this idea to the sound intensity based acoustic analyzers.

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