Application of the *S*-Transform to Identify the Localisation of Fatigue Features in a Variable Amplitude Loading

S. ABDULLAH[1], M. Z. NUAWI AND A. ZAHARIM
Engineering Faculty, Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor
MALAYSIA

Abstract: - This paper describes the use of the *S*-Transform to identify fatigue features in variable amplitude loadings. For this case, this type of loading exhibits nonstationary signal pattern, for which a normal frequency domain analysis cannot provide an accurate results for the analysis. In order to overcome this problem, the time-localisation approach provides a promising answer. A variable amplitude fatigue loading, which was measured from a lower suspension arm of a vehicle driven over a test track, was used for the analysis of this study. In order to identify fatigue damaging events, the data was processed using the orthogonal wavelet based algorithm, or known as Wavelet Bump Extraction (WBE). Since the *S*-transform if the simplification of the wavelet transform, it is a good idea to explore this transform to help the identification of these fatigue features. The results from the computational analysis results showed that the high amplitude events were detected in the variable amplitude loading based on the difference pattern of the time-frequency localisation. From the findings of this paper, it is suggested that further developments in the *S*-transform will find applications in a broad research area, particularly in the fatigue life assessment.

Key-Words: - Fatigue features, Fatigue data, *S*-transform, Time-Frequency, WBE, Variable amplitude.

1 Introduction

Many experimental fatigue loadings exhibit time-varying, or nonstationary characteristics, which provide a challenge in signal analysis. Traditional approach for the frequency domain analysis of the time series was performed using the Fourier transform. This kind of analysis is not suitable for nonstationary signals, as it cannot provide any information of the spectrum changes with respect to time [1].

Realising the limitation of the Fourier transform, therefore, the wavelet transform is more suitable method. In addition to the wavelet transform, which is a complex data analysis approach, an alternative method for the time-frequency domain analysis has also been introduced in order to process the random pattern signal, and this method is known as the *S*-transform [2]. This transform is also included as the time-frequency domain, which is an extension of the ideas of the continuous wavelet transforms (CWT). Using the time-frequency localisation, the time and frequency of an oscillating signal can be detected. As continuity to the CWT development, the *S*-transform was introduced for the simplification to the data analysis.

The objective of this paper is to observe the applicability of the *S*-transform in determining fatigue features or fatigue damaging events in a variable amplitude loading. These features were identified and extracted using a wavelet-based algorithm, called the Wavelet Bump Extraction (WBE) [3,4]. Since WBE uses the orthogonal wavelet transform (by means of 12^{th} order of the Daubechies wavelet), the *S*-transform may give a better indication to help in the identification of fatigue damaging segments.

2 Literature Background

Before looking to a detail application of the *S*-transform with variable amplitude loadings, it is a good idea to describe a literature background of the signal analysis. Thus, information related to the Fourier Transform, the short-time Fourier transform (STFT), the wavelet transform and the *S*-transform is necessary.

2.1 Frequency Domain Signal Analysis

Frequency analysis is performed in order to convert a time domain signal into the frequency domain. The results of a frequency analysis are most commonly presented by means of graph having frequency on the *x*-axis and amplitude on the *y*-axis. The algorithm that is used to split the time history into its constituent sinusoidal components is the Fourier transform. This transform was expressed as the summation of sinusoidal waves of varying frequency, amplitude and phase. The most common algorithm used for the Fourier transform is the fast Fourier transform (FFT) algorithm which was introduced in order to have a faster DFT calculation of the time series [5]. Various FFT algorithms were developed and the algorithm introduced by Cooley and Tukey [6] is the most commonly used because of its simplicity and fast computing time.

Using the Fourier transform the frequency components of an entire signal can be analysed, but it is not possible to locate at what point in time that a frequency component occurred or its duration. This is not problematic when a stationary signal is analysed. However, Fourier analysis is not suitable for non-stationary signals. If there is a time localisation due to a particular feature in a signal such as impulse, this will only contribute to the overall mean valued frequency distribution and feature location on the time axis is lost [1]. Thus, the short-time Fourier transform (STFT) was developed introduce in order to solve the irregularities behaviour of FFT in analysing nonstationary signals,

2.2 Time-Frequency Signal Analysis

The STFT is a method of time-frequency analysis which aims to produce frequency information which has a localisation in time. It provides information about when and at what frequencies a signal event occurs [7]. The STFT approach assumes that if a time-varying signal is divided into several segments, each can be assumed stationary for analysis purposes. The Fourier transform is applied to each of the segments using a window function, which is typically nonzero in the analysed segment and is set to zero outside [8]. The most important parameter in the analysis is the window length, which is chosen to isolate the signal in time without any distortions.

The STFT was developed from the Fourier transform, and it is mathematically defined as

$$X(\tau,\omega) = \int_{-\infty}^{\infty} \omega(t-\tau)e^{-i\omega t}h(t)dt \tag{1}$$

where the Fourier transform of the windowed signal is $h(t)e^{-i\omega t}$, ω is the frequency and τ is the time position of the window [9]. The result of this transformation is a number of spectra, each localised in a windowed segment.

While a useful tool, the STFT has a resolution problem, i.e. short windows provide good time resolution but poor frequency resolution. On the other hand, long windows provide good frequency resolution, but poor time resolution. The wavelet transform, which is described in the next section, is one of the most recent solutions to overcome the shortcomings of STFT [10].

A wavelet is a small wave with a signal energy concentrated in time, on the condition of admissibility condition. The wavelet transform is defined in the time-scale domain and is a significant tool for analysing time-localised features of a signal. It represents a windowing technique with variable-sized region. The harmonic form of the wavelet transform can be derived from the Fourier transform, which gave

$$X(\omega) = \int_{-\infty}^{\infty} h(t)\sin\left(\frac{t-d}{\tau}\right)dt \tag{2}$$

where a is a scale parameter which controls the frequency by dilating or scaling the time t. The parameter d translates the basic sine wave up and down the time axis and it is known as the translation parameter. A wavelet transform can be classified as either a CWT or a discrete wavelet transform (DWT) depending on the discretisation of the scale parameter of the analysing wavelet. The CWT is given by

$$W(\tau,d) = \int_{-\infty}^{\infty} h(t)\omega(t-\tau,d)\,\mathrm{d}t \tag{3}$$

As continuity to the CWT development, the *S*-transform was introduced for the simplification to the data analysis. The *S*-transform is a time–frequency representation whose analyzing function is the product of a fixed Fourier sinusoid with a scalable, translatable window [11]. It combines elements of wavelet transform and windowed Fourier transform. The *S*-transform can also be generalized to include windows that have frequency-dependent functional form, and frequency-dependent complex phase modulation, essentially giving phase-shifted wavelets, which have no semblance at different scales.

The *S*-transform, which is introduced in this correspondence, is an extension of the ideas of the continuous wavelet transform (CWT), and is based on a moving and scalable localizing Gaussian window. The *S*-transform is unique which provides frequency-dependent resolution while maintaining a direct relationship with the Fourier spectrum. These advantages of the *S*-transform are due to the fact that the modulating sinusoids are fixed with respect to the time axis, whereas the localizing scalable Gaussian window dilates and translates.

According to the related literature [2], the *S*-transform of a function $h(t)$ in Eq. (3) is defined as a

CWT with a specific mother wavelet multiplied by the phase factor, i.e.

$$S(\tau,d) = e^{i2\pi f\tau} W(\tau,d) \tag{4}$$

where the mother wavelet is defined as

$$\omega(t,f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-i2\pi f\tau} \tag{5}$$

where f is the frequency of the samples. Written out explicitly, the S-transform is

$$S(\tau,d) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{-i2\pi f\tau} dt \tag{6}$$

If the S-transform is a representation of the local spectrum, a simple operation of averaging the local spectra over time can be used to give the Fourier spectrum, i.e.

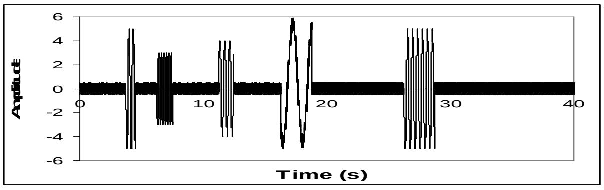
$$\int_{-\infty}^{\infty} S(\tau,f) d\tau = H(f) \tag{7}$$

3 Computational Simulation: Results and Discussions

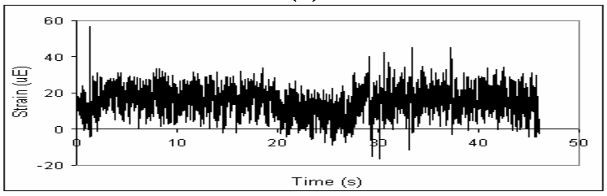
3.1 Application of Fatigue Data Editing Algorithm

The accuracy of the fatigue damaging event identification process was evaluated by the application to two types of the variable amplitude fatigue loadings. The T1 signal (Fig. 1a), was an artificial signal and it was defined to have 16,000 data points which are sampled at 400 Hz. The using T1 was to show the effectiveness of the S-transform in dealing with any signal containing large transients in an otherwise small amplitude background. T1 consists of a combination of sinusoidal and random segments of different amplitude or frequency. The signal was intentionally defined to be a mixture of both high amplitude bump events and low amplitude harmonic backgrounds. This signal T2 (Fig. 1b) was measured on a van while driving over a pavé test track and it was sampled at 500 Hz with a record length 46 seconds.

Both fatigue loadings were then processed using the Wavelet Bump Extraction (WBE) algorithm which was developed using FORTRAN by Abdullah et al. [3,4]. Generally, WBE is a wavelet-based fatigue data editing technique which is used to identify and extract fatigue damaging events, and to produce a shortened mission signal of similar behaviour. The WBE algorithm uses the 12th order Daubechies wavelets [12] as the basis functions. The unique of the WBE algorithm compared to other methods is that the retention of the most original fatigue damage potential in the WBE shortened (output) loading. In addition, the original vibrational signal energy, the phase and amplitude were also preserved in the WBE shortened loading.



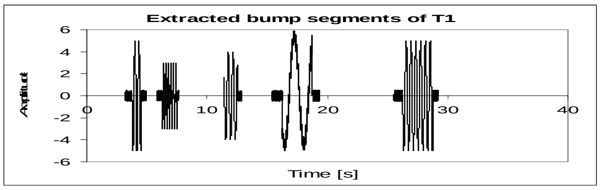
(a)



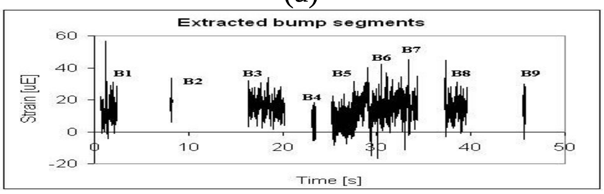
(b)

Fig. 1. Variable amplitude fatigue loadings used for this study: (a) T1, (b) T2

Using the WBE (with the implementation of the orthogonal wavelet transform (OWT)) with these two signals, the fatigue damaging events were identified based on the statistical properties of the signal. The identified fatigue damaging events are shown in Fig. 2. These segments for both signals were combined in order to produce the shortened signals and they are shown in Fig. 3. Based on the similar results presented in the related papers [3,4] by the main author, almost all fatigue damage was retained when the shortened loading is 40% of the original loading time length.

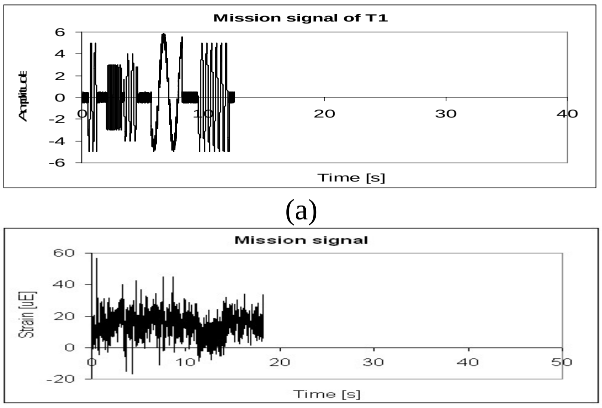


(a)



(b) B1 – B9 are the segment numbers

Fig. 2. The extracted fatigue damaging events of the input signals: (a) T1, (b) T2



(a)

(b)

Fig. 3. The shortened loadings produced using the WBE algorithm: (a) The 12.5-second T1 loading, (b) The 19-second T2 loading

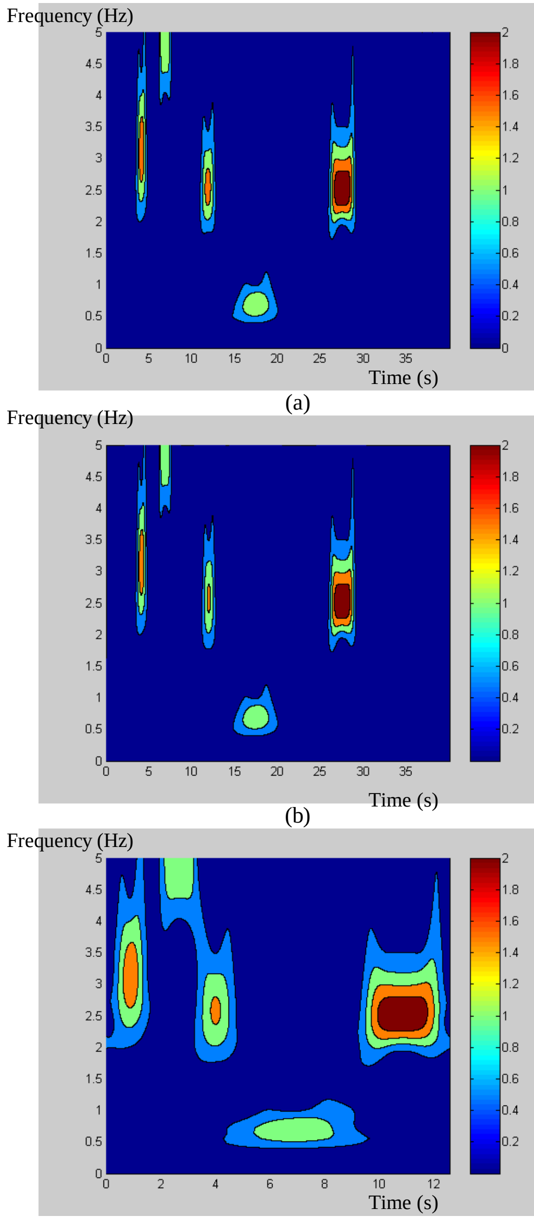
3.2 Application of *S*-Transform

The signals of Fig. 1 to 3 were simulated using the MatLab package, and the flowchart for the simulation is shown as in Fig. 5. Since the WBE was used to identify fatigue damaging events based on the implementation of the OWT, the *S*-transform is then be used to verify the appropriate identified fatigue damaging events.

Fig. 4 shows the pattern of signal energy distribution in the time-frequency axis, which is applicable for T1. Fig. 4a shows the pattern of high amplitude events detected from the original T1 signal of Fig. 1a, for which these events were distributed between 0.5 to 5 Hz. These events were extracted using WBE (Fig. 1b), and the segments were then simulated in MatLab in order to observe their signal pattern in the time-frequency localisation. The result in Fig. 4b indicates the localisation pattern is similar for both the original T1 and the segments of fatigue damaging events. The findings showed that the WBE algorithm is able to extract the right fatigue features, which contribute the damaging effects to mechanical components.

Fig. 4c shows the pattern of time-frequency localization for the shortened loading, and showing high-energy distribution can be observed for the whole length of the signal. Since the shortened loading is able to preserve 100% of the original fatigue damage [10], the high-energy localization in this *S*-transform indicates the loading segments with high fatigue damage values.

Using the experimental data set of T2, Fig. 5 shows the pattern of signal T2 energy distribution in the time-frequency axis. Fig. 5a shows the pattern of high amplitude events detected from the original T2 signal (Fig. 1b), which has been simulated in the MatLab environment. Since T2 exhibits almost a random pattern with several transient effects (however, the signal is nonstationary based on the statistical analysis in [3]), low energy pattern has been observed in the time-frequency localisation background of the random signal.



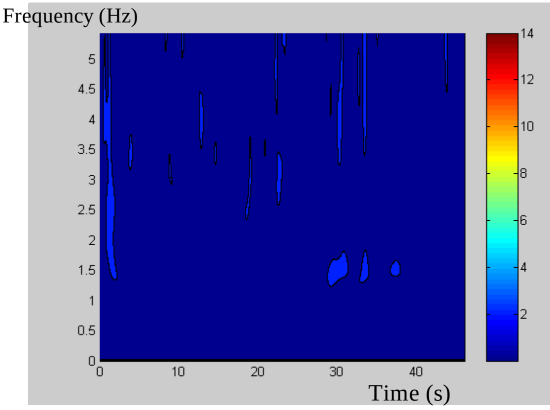
(a)

(b)

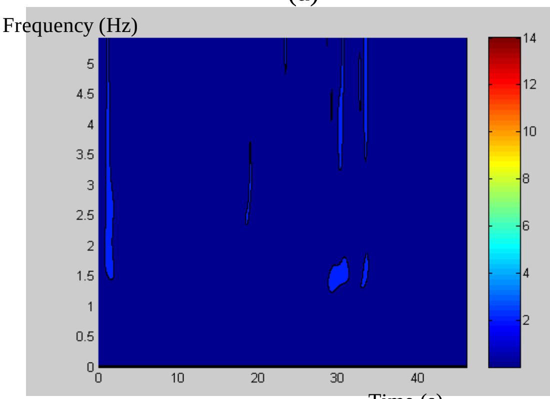
(c)

Fig. 4. The pattern of high amplitude events produced by the *S*-transform for T1: (a) The original signal, (b) The identified fatigue damaging events in WBE, (c) The shortened loading

The simulation result using the fatigue damaging event segments extracted (Fig. 2b) from the OWT approach of the WBE algorithm has been presented in Fig.5b. It shows some of the significant high amplitude events have been detected from the colour difference of the time-frequency localisation. Since the fatigue damage summation of the extracted fatigue damaging events is similar to the fatigue damage of the original signal [12], it is suggested that the *S*-transform is able to detect fatigue damaging events from a fatigue loading with random background. Even though several events were identified to have different colour pattern from the pattern of the random background (in Fig. 5b), however, it is assumed that these events have minimal fatigue damage, which need further investigation.



(a)



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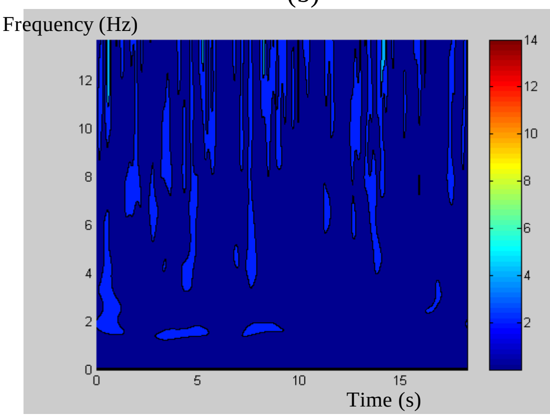

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Fig. 5. The pattern of high amplitude events produced by the *S*-transform for T2: (a) The original signal, (b) The identified fatigue damaging events in WBE, (c) The shortened loading

4 Conclusion

The objective of this paper is to investigate the applicability of the *S*-transform to determine fatigue damaging events in a variable amplitude fatigue loading. Since these events were successfully identified in WBE by means of the 12th order of the Daubechies wavelet, the *S*-transform (having a basic wavelet mathematical function) should also gave a potential approach in identifying these fatigue features. The arguments are supported by the results presented in Fig. 5 and 6.

The simulation results showed that the high amplitude events were detected in the variable amplitude loading based on the difference pattern of the time-frequency localisation. Thus, it is suggested that the *S*-transform is able to detect fatigue damaging events from a random fatigue loading. Finally, further development in the *S*-transform will help to find some applications in a broad range of disciplines, particularly in the fatigue life assessment.

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