Mathematical Model for Determining the Performance Characteristics of Multi-Crystalline Photovoltaic Modules

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Abstract: Mathematical models for predicting the performance characteristics of multi-crystalline photovoltaic (PV) modules have been developed. In any prediction of the electric output of PV systems, an understanding of its characteristics is of fundamental importance. These characteristics are popularly expressed in the form of I-V curves that may be presented graphically; or using equations. There exists many equations for calculating the photogenerated current I_L, saturated current I_o and a fitting-parameter a. The characteristics of PV modules using simulations and experiments on multi-crystalline modules have been conducted. To achieve this, an experimental PV system of 5 kWp capacity was set-up comprising of 40 modules, arranged in a string combination of 8 in series by 5 in parallel. The current and voltage data were recorded at various solar radiation levels and module temperatures. This paper then presents and discusses a set of new equations pertaining to a, I_L, I_0, and the maximum power point. These results were expressed using graphs and equations with constant values and compared with those obtained from the experimental set-up. This set of equations could be used to predict performances and maximum power points of a PV module using temperature and solar radiation data only.

Keywords: photovoltaic; PV characteristic; maximum power point; mathematic model.

1. Introduction

Energy systems in the foreseeable future will tend to be environmental friendly; in which renewable energy resources offer a most promising option. These renewable energy resources are not only clean but they are uniquely sustainable. One of the most promising renewable energy resources is solar energy. Solar energy can be utilized as thermal energy, direct electricity, or a combination of both. Within a variety of renewable and sustainable energy technologies in progress, photovoltaic technology appears to be one of the most promising ways meeting the future energy demands as well as environmental issues.

The Solar Energy Research Institute (SERI) of University Kebangsaan Malaysia (UKM) has installed a 5 kWp grid-interactive PV system. The PV system comprises 40 modules, each string of 8 modules connected in series by 5 modules in parallel. The grid-interactive inverter is equipped with a maximum power point tracker (MPPT) and synchronizer. In order to predict the electric output of any PV system, characteristics of the modules must be understood. The equations showing *I-V* and *P-V* characteristic of PV cells and modules have been established [1, 2, 3]. Ideally, a PV module should always operate at the maximum power point, which is achieved by using a Maximum Power Point Tracker (MPPT). So the results of output PV electric will be always a maximum. The equation to predict maximum output has been others based on interpolation model [4]. This paper discusses a new set of equations on the curve fitting parameter (a), light current (I_L), diode reserve saturation current (I_0) and the maximum power point based on *I-V* and *P-V* characteristics.

Table 1 Summary of module characteristics.

Type	Multi-crystalline
Maximum power output (P_{MP})	35 Wp
Maximum power voltage (V_{MP})	16.9 V
Maximum power current (I_{MP})	7.10 A
Open Circuit voltages (V_{OC})	21.5 V
Short Circuit current (I_{SC})	7.45 A
Voltage temperature coefficient (β_0)	-0.115 2 V per ^oC
Current temperature coefficient (α_0)	0.000 124 A per ^oC
Length	1425 mm
Width	652 mm
Depth	52 mm

2. I-V Characteristic of PV.

The PV moduleequivalent circuit diagram is shown in Fig 1 [1]. Here, the current balance can show the current–voltage (*I-V*) characteristic equation of a PV module.

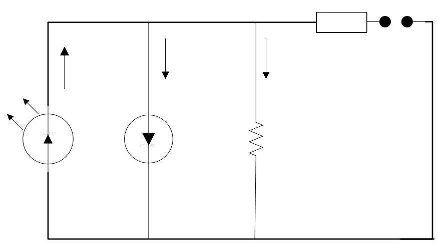

Figure 1 The equivalent circuit for a PV generator.

Using Kirchoff's Law, the current balance is

$$I = I_L - I_D - I_{sh} = I_L - I_O\left\{\exp\left(\frac{V + I.R_s}{a}\right) - 1\right\} - \frac{V + IR_S}{R_{sh}}$$

If the shunt resistance (R_{sh}) is very much larger than the series resistance (R_s), the equation can be simplified as

$$I = I_L - I_0\left\{\exp\left(\frac{V + IR_s}{a}\right) - 1\right\}$$

At short circuit condition, the photogenerated current (I_L) is equivalent to the short circuit current ($I_L=I_{SC}$). At open circuit condition, there is no current ($I=0$) and 1 is relatively smaller compared to the exponential term. Thus equation 2 becomes

$$I_0 = I_L \exp\left(-\frac{V_{OC}}{a}\right)$$

At the maximum power condition with substitution equation (5) to (2) and neglecting the 1, the result is

$$R_S = \frac{a \ln\left(1 - \frac{I_{MP}}{I_L}\right) - V_{MP} + V_{OC}}{I_{MP}}$$

The series resistance is independent of temperature and PV modules with ε_{gap} = 1.12 eV for silicon, and ε_{gap} = 1.35 eV for gallium arsenide. Subscript ref is the reference condition.

$$\frac{a}{a_{ref}} = \frac{T_C}{T_{C,ref}}$$

$$I_L = \frac{G_T}{G_{T,ref}}[I_{L,ref} + \mu_{I,SC}(T_C - T_{C,ref})]$$

$$\frac{I_O}{I_{O,ref}} = \left(\frac{T_C}{T_{C,ref}}\right)^3 \exp\left[\frac{\varepsilon_{gap}.N_S}{a_{ref}}\left(1 - \frac{T_{C.ref}}{T_C}\right)\right]$$

$$a_{ref} = \frac{\mu_{V.oc}T_{C,ref} - V_{OC,ref} + \varepsilon_{gap}.N_S}{\frac{\mu_{I,sc}.T_{C,ref}}{I_{L,ref}} - 3}$$

$$\mu_{I,sc} = \frac{d.I_{sc}}{dT} = \frac{I_{SC}|_{T_2} - I_{SC}|_{T_1}}{T_2 - T_1}$$

$$\mu_{V,OC} = \frac{d.V_{OC}}{dT} = \frac{V_{OC}|_{T_2} - V_{OC}|_{T_1}}{T_2 - T_1} \quad (1)$$

Parametric model, equation of light current (I_L) uses α and β parameters as follows [3]

$$I_L = \alpha(1 + \beta T_c)G_T S$$

And the diode saturation current (I_0) is presented with C_M parameter

$$I_0 = C_M S T_c^3 \exp\left(-\frac{\varepsilon_{gap}}{kT_c}\right)$$

where k is Boltzman's constant (J/K).

The interpolation model based on short circuit current (I_{SC}), open circuit voltage (V_{OC}), maximum power point voltage (V_{MP}) and maximum power point current (I_{MP}).

$$C_1 = \left(1-\frac{I_{MP,ref}}{I_{SC},ref}\right)\exp\left(-\frac{V_{MP,ref}}{C_2.V_{OC,ref}}\right)$$

$$C2 = \frac{\left(\frac{V_{MP,ref}}{V_{OC},ref}-1\right)}{\ln\left(1-\frac{I_{MP,ref}}{I_{SC,ref}}\right)}$$

The two parameters (D_t) and (V_R) can be calculated with temperature of PV and solar irradiation.

$$\Delta I = \alpha_0\left(\frac{G_T}{G_{T,ref}}\right)\Delta T + \left(\frac{G_T}{G_{T,ref}}-1\right)I_{SC,ref}$$

$$V_R - V = \Delta V = \beta\Delta T + R_S\Delta I$$

$$\Delta T = T_{Cell} - T_{Cell,ref}$$

So the output current can be calculated with the equation below:

$$I = I_{SC,ref}\left(1-C_1\left\{\exp\left(\frac{V-\Delta V}{C_2 V_{OC,ref}}\right)-1\right\}\right)+\Delta I$$

Many researchers have used equation (2) as standard equation, but for equations of a, I_L, I_0 there are many variations, as such Kau, *et al*. (1998); Beckman and Duffie (1991), have used the equations of Townsend (equations 5 to 9), Khouzam and Hoffman (1996), have used the equation of parametric model (equation 10-11). Parameter values, reference conditions and photovoltaic area are fixed, so those equations can be simplified as below:

$$a = k_1 T_c$$

$$I_L = k_2(1+k_3T_c)G_T \qquad (11)$$

The result of the substitution of equations (17 to 19) into equation (2) is

$$I = k_2(1+k_3T_C)G_T - k_4 T_C{}^3 \exp\left(-\frac{k_5}{T_C}\right)\left\{\exp\left(\frac{V+I.R_s}{k_1T_C}\right)-1\right\}$$

3. Maximum Power Point of PV

Ideally, a PV module should always operate at the maximum power point. In this condition the power is P_{MP}, the current is I_{MP} and the voltage is V_{MP}. The maximum power that can be obtained corresponds to the rectangle of maximum area under the *I-V* curve, or the maximum *P-V* curve.

(12)

Ai, *et al*. (2003) uses interpolation model to evaluate PV array, operated by maximum power point tracker (MPPT). The equations of ΔV and V in maximum power conditions are

(13)

$$V_{MP} = V_{MP,ref}\left[1+0.0539\log\left(\frac{G_T}{G_{T,ref}}\right)+\beta_0\Delta T\right] \qquad (14)$$

(15)

$$I_{MP} = I_{SC,ref}\left(1-C_1\left\{\exp\left(\frac{V_{MP}-\Delta V}{C_2V_{OC,ref}}\right)-1\right\}\right)+\Delta I$$

$$\Delta V = V - V_{MP,ref} \qquad (16)$$

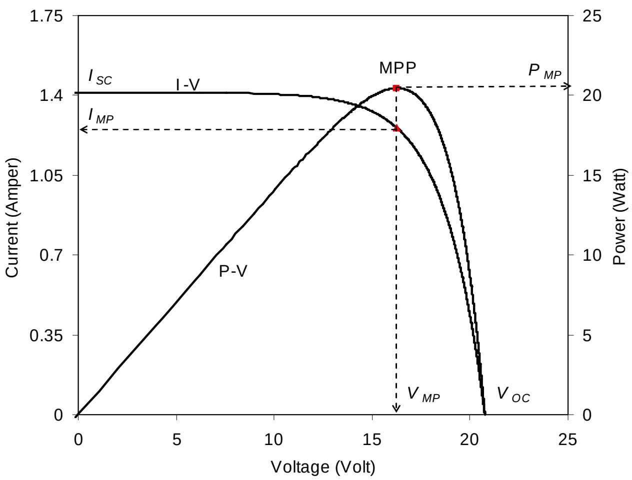

Figure 2 Typical I-V and P-V curves for a PV module.

In the maximum power point condition occurs maximum value of $I_{MP}*V_{MP}$, or at *P-V* curve the P_{MP} is the zenith or the reverse point, so the differential ($dP/dV=0$):

$$\frac{dP}{dV}=\frac{d(VI)}{dV}=I\frac{dV}{dV}+V\frac{dI}{dV}=0$$

$$I_{MP}+V_{MP}\frac{dI_{MP}}{dV_{MP}}=0$$

Based on equation (2) and the 1 is small compared to the exponential term, the *I-V* characteristic of the PV at maximum power condition is

$$V_{MP}=a\ln\left(\frac{I_L-I_{MP}}{I_0}\right)-I_{MP}R_S=$$

$$a\ln\left(\frac{\Delta I}{I_0}\right)-I_{MP}R_S$$

$$\Delta I=I_L-I_{MP} \tag{26}$$

$$\frac{dV_{MP}}{dI_{MP}}=\frac{-a}{I_L-I_{MP}}-R_S=\frac{-\Delta IR_S-a}{\Delta I} \tag{27}$$

$$\frac{dI_{MP}}{dV_{MP}}=\frac{\Delta I}{-\Delta IR_S-a} \tag{28}$$

The result of the substitution equations (26 and 28) into equation (25) is

$$I_{MP}+\left\{a\ln\left(\frac{\Delta I}{I_0}\right)-I_{MP}R_S\right\}\left(\frac{\Delta I}{-\Delta IR_S-a}\right)=0$$

$$\left(-2R_S-\frac{a}{\Delta I}\right)I_{MP}+a\ln\left(\frac{\Delta I}{I_0}\right)=0$$

$$I_{MP}=\frac{a\ln\left(\frac{\Delta I}{I_0}\right)}{\left(2R_S+\frac{a}{\Delta I}\right)}$$

$$P_{MP}=I_{MP}V_{MP}$$

4. Result and Discussions

Data of the correlation between current (I) and voltage (V) of a photovoltaic module, the result of experiments in some variations of temperatures and solar radiations, is used for the suitable values evaluation of a, I_0 and I_L. The optimization values a, I_0 and I_L used the Hooke-Jeeves method, with minimization of sum square of error from *I-V* experiment and calculation with equation (2) data by trial and error of a, I_0 and I_L values. The comparison of experiment and calculation data of the PV module characteristic (Figure 3), shows that experiment data not so different with calculation data.

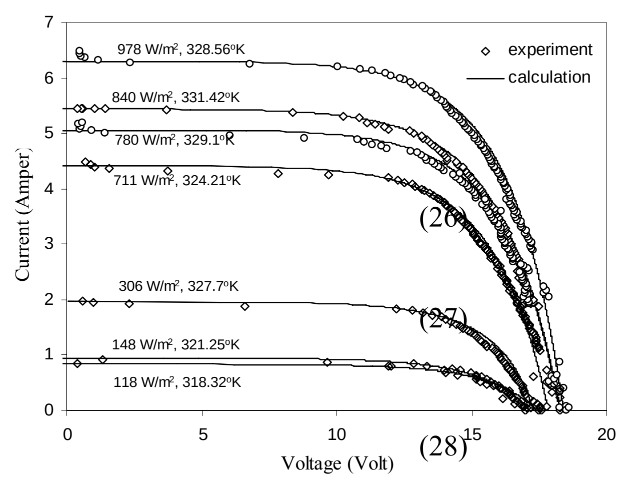

Figure 3 *I-V* characteristic of PV module, experiment and calculation data.

The optimizations of k_1 to k_5 used Hooke-Jeeves and Golden-Section methods with minimization of SSE among *a*, I_0 and I_L experiments and calculation with equation (17-19). The result of these optimizations are $k_1=0.0065$, $k_2=0.0066$, $k3 = 4.4477E\text{-}8$, $k4 = 2858173.2394$ and $k_5 = 12959.7288$.

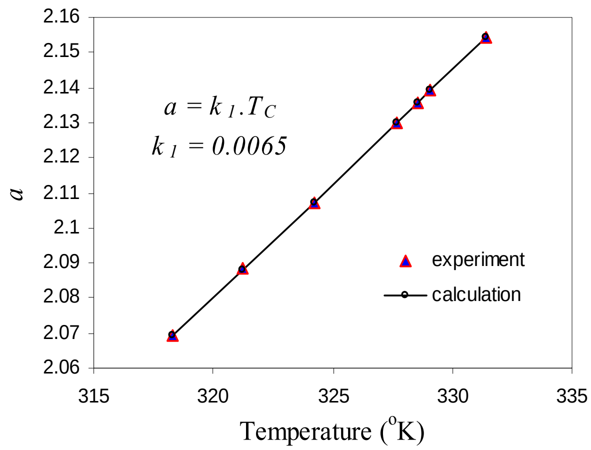

Figure 4 Experimental and calculation of *a* (equation12) at varying temperature.

Figures 3, 4, 5 and 6 show that the experimental data are not so different with the calculated data. The *I-V* curve as the result of predictions with equation (20) at fixed temperature and several radiation levels is shown at Figure 7; and at fixed radiation level and several temperature is shown at Figure 8.

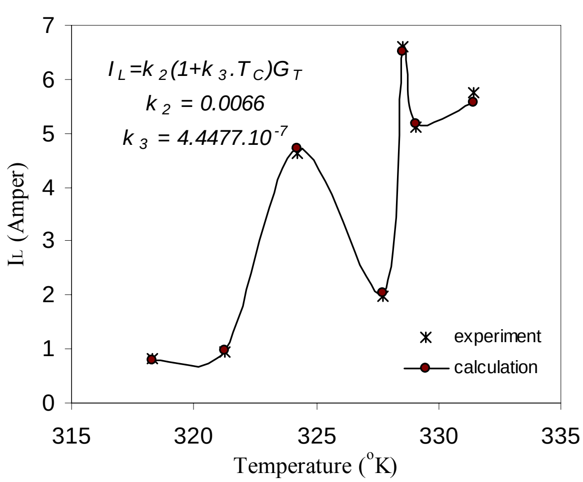

Figure 5 Experimental and calculation of I_L (equation 13) at varying temperature.

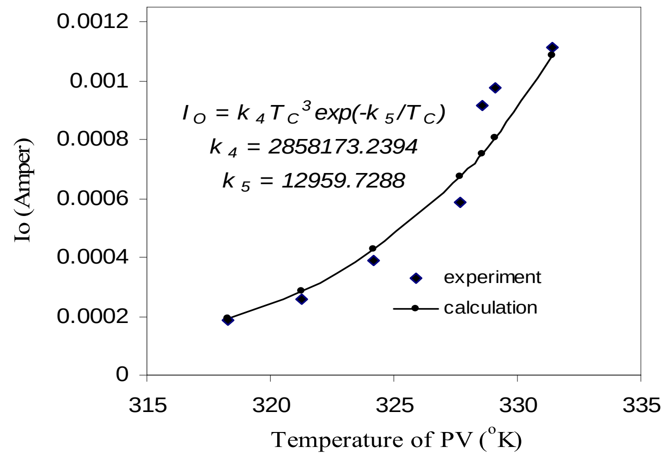

Figure 6 Experimental and calculation of I_0 (equation14) at varying temperature.

These findings agree with Beckman and Duffie (1991), in that a module operating at fixed temperature and several radiation levels, the short circuit voltage (V_{OC}) increases logarithmically and the short circuit current (I_{SC}) is nearly proportional to the incident radiation. At fixed radiation level, increasing temperature leads to decreased open circuit voltage and slightly increased short circuit current. The conclusion, that equations of mathematic model in this research are qualified. The constants in this research are special for multi-crystalline silicon.

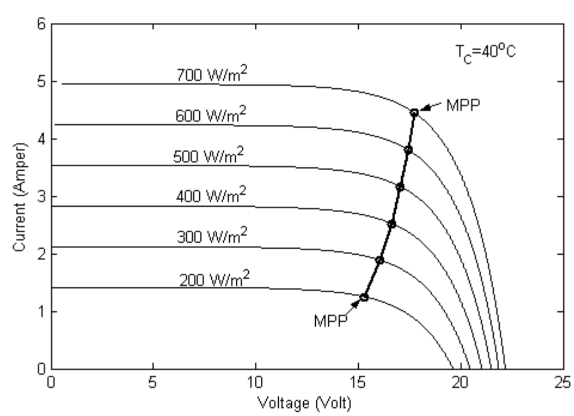

Figure 7 *I-V* characteristic and maximum power point of a PV module at temperature of 40°C and solar radiations of 200, 300, 400, 500, 600 and 700 W/m².

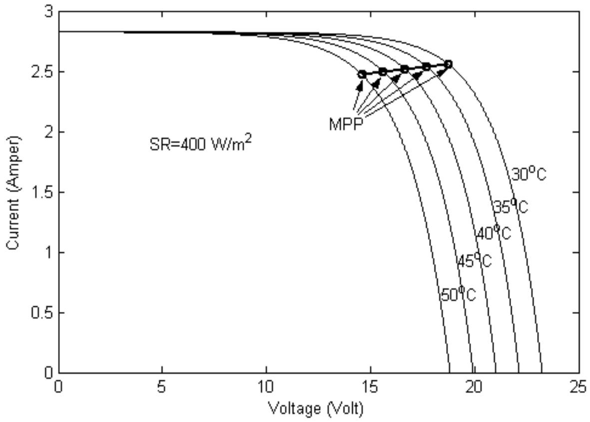

Figure 8 *I-V* characteristic and maximum power point of a PV module at temperatures of 30, 35, 40, 45 and 50°C, with solar radiation of 400 W/m^2.

Equations (26) and (31) are used to decide maximum power point, with values of a, I_0 , I_L from equations (17), (18), (19) and constants (k_1 to k_5) that have been known. The solution of equation (31) can be solved by numerical methods (Newton-Raphson, for example) so it will be understood maximum current (I_{MP}) value, then maximum voltage can be calculated with equation (26), and maximum power whith equation (32). From this, it can be concluded too that the equations model in this research will be able to predict maximum power point of PV module, only by temperature of PV and solar radiation data. The prediction results of maximum power point (MPP) of PV module in several temperature and solar radiation levels are shown in Figures 7 and 8.

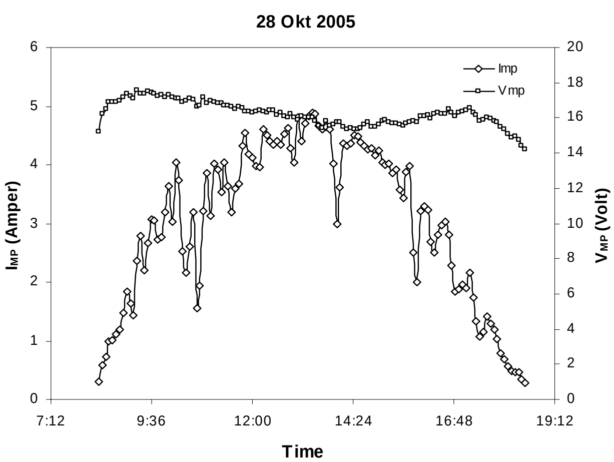

Figure 9 I_{MP} and V_{MP} of a PV module on 28 October 2005.

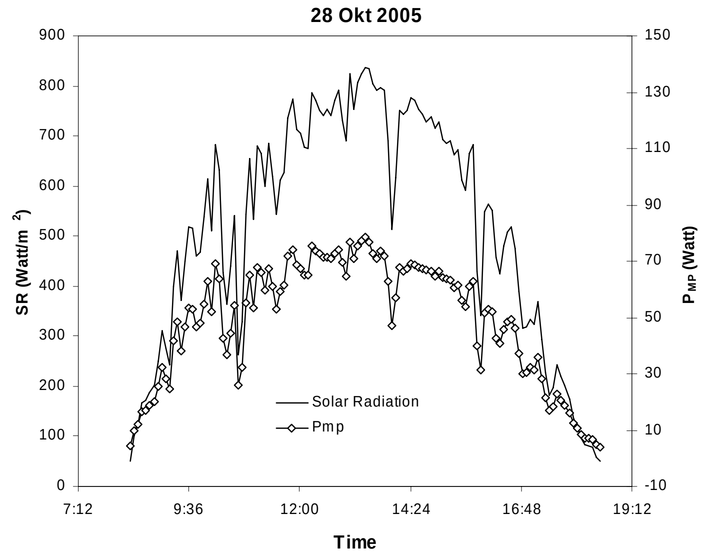

Figure 10 Maximum power of a PV module on 28 October 2005.

5. Conclusions

The equations of curve fitting parameter (a), photogenerated current (I_L), and diode reserve saturation current (I_0) have been simplified. This paper serves mathematic equations a, I_0, I_L and equation to calculate maximum power point of PV module, with constants k_1 to k_5. The equations of mathematic model in this research are simple and qualified, and can predict performance and maximum power point of PV module, only by temperature of PV and solar radiation data. The constants in this research are only for multi-crystalline silicon.

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