A Mathematical Model for the Simulation of the Income of Chilean Universities to Aid Their Strategic Decision Making Process

CARLO A. CASORZO, MIGUEL D. ALFARO, JAVIER I. GONZÁLEZ
Department of Industrial Engineering
University of Santiago of Chile
3769 Ecuador Ave. Santiago, Chile. PO Box 10233

Abstract: - The Chilean universities that belong to the Council Board of Rectors of Chilean Universities are subject to direct and indirect fiscal money contributions, that depend on performance measures among other factors, and thus, their income is affected by these two contributions plus the student body's payment. The prediction of future income and the selection of strategic policies of these universities is a non trivial task. This paper presents a mathematical model of the Chilean universities dynamic system, used to create a simulation tool that gives these universities the capability to predict their income under different scenarios, created to aid their decision making process.

Key-Words: - Chilean university, Simulation, Mathematical model, Multi-agent systems, Difference equations.

1 Introduction

Chilean universities are classified in two main groups: the universities that belong to the Council Board of Rectors of Chilean Universities (CBR) [1], and the ones that don't. There are 25 universities that belong to the CBR, and most of them are dependent of the Chilean State. They are subject to Fiscal Money Contributions that depend on certain university performance metrics, among other factors. These contributions are the Direct Fiscal Contribution (DFC), and the Indirect Fiscal Contribution (IFC). Besides, the universities receive money through the student body's payment, which is distributed in two parts: a short term part, which has to be paid every month, and a debt of long term. Both parts depend on the University Credit Percentage assigned to the student.

The calculation and prediction of these income sources is a non trivial task, because they depend on many factors, which can increment certain source, but decrement another. Besides, the calculations have many complex mathematical and logic aspects that can be confuse to the strategic planners of the universities.

The aim of this paper is to provide a mathematical model based on difference equations and multi-agent systems, that can be used to create a simulation tool to simulate the behavior of the different sources of funds that universities have, to give them the capability of putting themselves in different hypothetical scenarios, and thus, aid their decision making process.

The multi-agent approach has been widely used as an approach for simulation in many applications. In [4], a bus-network simulation tool for evaluating the performance of bus-networks was created, using a multi-agent approach. A multi-agent qualitative simulation approach using E-Government Group Behavior Model (EGGBM) to research complex group behavior in E-Government policy decision is made in [5]. A multi-agent system model to study the interactions between the traffic and the marine mammals in an estuary of a marine park is proposed in [6]. In [7], the authors propose an agent-based micro model for grocery shopping.

The remainder of this paper is organized as follows. In Section 2, a full system description is given. The mathematical model is formulated in Section 3. Section 4 contains the simulation model description and the algorithms used in the model and simulations. Finally, conclusions and future work are presented in Section 5.

2 System Overview

As mentioned before, the Chilean universities that belong to the CBR have three main sources of funds; the Direct Fiscal Contribution (DFC), the Indirect Fiscal Contribution (IFC) and the student body's payment. These two fiscal contributions are amounts of money that the Chilean State gives to these universities for their operation every year, and both depend on different performance metrics that will be explained as follows.

The IFC depends on the scores obtained in the University Selection Test (PSU for its name in Spanish) by the students that entered the university the previous year. Every year, the best 27.500 scores are divided in 5 ranges, and a contribution value per student is assigned to each range. Then, the IFC given to the university is the sum of the values of each student that entered the university the preceding year.

The DFC is a contribution that depends on two main parts; a part with a distribution between universities based on historical weights, which corresponds to the 95% of the total contribution; and a performance based distribution that uses a university performance index, which is a function of five performance metrics, such as the number of students per career, number of students over total equivalent complete work journeys (ECW), number of investigation projects over the total ECW, etcetera.

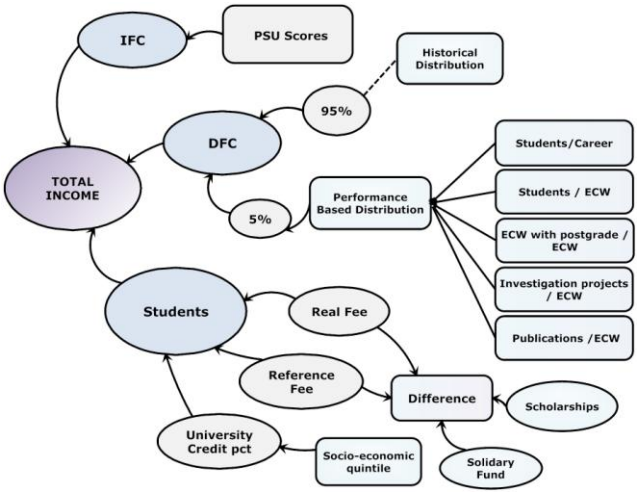

Fig. 1. *System Overview*

The third main source of income is the student body's payment, which works as follows. A University Credit Percentage is assigned to every student, according to their socio-economic quintile. The students from the quintiles I, II and III are given a 100% credit; the students from the fourth quintile are given from 90% to 10%, and 0% is given to quintile V. Then, the student has to pay in cash the percentage of the fee of his/her career not covered by the University Credit. The part covered by the credit is loaned by the State of Chile, but with a maximum amount given by the reference fee of the career. This difference between the real fee and the reference fee can be covered by scholarships, which also have a maximum value of coverage, and the rest is covered by the called Solidary Fund of the university, which is a fund used to support this difference.

When the student finishes the career, the cumulated debt must be paid, with two years of grace and an interest rate of 2%. Fig. 2 shows a diagram with a mental map of the structure of the income dynamic system.

3 Mathematical Model

This section contains the full formulation of the mathematical model, including the definition of the *University* system, which includes the *Student Body* system, the mechanisms of calculation of the IFC and the DFC, and the equations that rule the reference fee fixation.

3.1 University System

Each university has four main groups of variables. Let's define U as the set of Chilean universities that belong to the CBR. Then, each university $u \in U$ has a set of variables composed as follows.

The set of careers of university $u \in U$ in the year k is C_u, and the set of socio-economic quintiles is $Q = \{I, II, III, IV_1, IV_2, ..., IV_{10}, V\}$. As mentioned before, quintile IV is divided into ten deciles, from IV_1 to IV_{10}.

The student body of university $u \in U$ in the year k is defined by $x_{cq}^u(k)$, where c and q represent the career and the quintile of the student, respectively. So, the number of students in the year k in university u, from the career c, and from quintile q is given by $x_{cq}^u(k)$. The total number of students is given by,

$$x_u(k) = \sum_{c \in C_u} \sum_{q \in Q} x_{cq}^u(k). \qquad (1)$$

The set of difference equations that rule the behavior of the number of students are,

$$x_{cq}^u(k) = x_{cq}^u(k-1) + x_{ucq}^{(in)}(k) - x_{ucq}^{(out)}(k), \quad \forall u \in U, c \in C_u, q \in Q. \qquad (2)$$

The values $x_{ucq}^{(in)}(k)$ and $x_{ucq}^{(out)}(k)$ represent the students that enter and leave the university, respectively.

The number of total equivalent complete work journeys (ECW) of university $u \in U$ in the year k is given by,

$$y_u(k) = y_{Dr}^u(k) + y_{Mg}^u(k) + y_0^u(k). \qquad (3)$$

Here, $y_{Dr}^{u}(k)$, $y_{Mg}^{u}(k)$ and $y_{0}^{u}(k)$ are the ECW of academics with doctorates, masters and with no post grades, respectively.

There are two kinds of publications considered: ISI publications and Scielo publications. Scielo (Scientific Electronic Library Online) [2] is a selected collection of Chilean scientific journals. The publications of university $u \in U$ in the year k are given by $P_{ISI}^{u}(k)$ and $P_{Scielo}^{u}(k)$. The number of investigation projects of university $u \in U$ in the year k is given by $V_u(k)$.

The variable $S_u(k)$ represents the money collected from the student body's payment in the year k. Also, the level of money in the Solidary Fund of university $u \in U$ in the year k is $F_u(k)$.

3.2 Indirect Fiscal Contribution (IFC)

As mentioned before, the Indirect Fiscal Contribution (IFC) of every year is an amount of money given to each university that depends on the PSU scores obtained by the students that entered the university the preceding year.

The 27.500 best scores of the PSU are divided in five ranges, and a weight is assigned to each range. Then, a contribution value per student is assigned to each range. And so, the IFC for each university is the sum of the contributions per student of each student that entered the university the previous year. The total IFC for the year k is given by,

$$IFC(k) = \sum_{u \in U} IFC_u(k). \tag{4}$$

Here, $IFC_u(k)$, $u \in U$, is the IFC amount given to university u. The ranges that the best 27.500 scores are divided in, are numbered from 0 to 5, where 0 is the range of the rest of the students, whose contribution value is zero. Range 5 is the range of the best scores. The weights assigned to each range are $p = (p_0, p_1, ..., p_5) = (0,1,3,6,9,12)$ [3]. Then, the contribution value per student is given by the expression,

$$A_r(k) = \frac{p_r \cdot IFC(k)}{\sum_{s \in R} p_s \cdot m_s(k)}, r \in R, \tag{5}$$

where $R = \{0,...,5\}$ is the set of ranges of PSU scores, and $m_r(k)$ is the number of students in range i in the year k. In general, $\sum_{r \in R} m_r(k) = 27,500$.

Then, each university will have two sets of control variables, related to the number of students admitted each year. These variables are $\alpha_{ru}(k)$ and $\beta_{ru}(k)$, which represent the fraction and the number of students of each PSU range that enter the university each year. They are considered control variables because they can be manipulated by giving incentives to the students with good PSU scores, like scholarships. These two variables are related to each other by the number of entering students, $x_{ucq}^{(in)}(k)$, through the expression,

$$\beta_{ru}(k) = \alpha_{ru}(k) \cdot x_{ucq}^{(in)}(k). \tag{6}$$

And so, the IFC of university $u \in U$ in year k is given by these two expressions,

$$IFC_u(k) = x_{ucq}^{(in)}(k-1) \sum_{r \in R} \alpha_{ru}(k-1) A_r(k-1), \tag{7}$$

$$IFC_u(k) = \sum_{r \in R} \beta_{ru}(k-1) A_r(k-1). \tag{8}$$

3.3 Direct Fiscal Contribution (DFC)

The Direct Fiscal Contribution (DFC) is the second money contribution that the State of Chile gives to the CBR universities every year. This contribution is composed by a historical weights distribution, which corresponds to the 95% of the total DFC, and a performance based distribution, that represents the 5% of the DFC, and depends on university performance metrics. Then, as IFC,

$$DFC(k) = \sum_{u \in U} DFC_u(k). \tag{9}$$

The decomposition of the DFC of each university is given by,

$$DFC_u(k) = DFC_u^{(h)}(k) + DFC_u^{(v)}(k), \tag{10}$$

where $DFC_u^{(h)}(k)$ represents the historical, 95% part, and $DFC_u^{(v)}(k)$ is the performance based 5% variable part of the DFC. It's easy to see that,

$$DFC_u^{(h)}(k) = 0.95 \cdot DFC(k) \cdot h_u, \tag{11}$$

where h_u is the historical weight of university $u \in U$. These weights satisfy $\sum_{u \in U} h_u = 1$.

On the other hand, in the calculation of the variable 5% of the DFC, a general performance index

$I_u(k)$, is used, which is a function of five performance metrics. The university weights of the variable 5% DFC are calculated with the expression [4],

$$q_u(k) = \frac{I_u(k)}{\sum_{s \in U} I_s(k)}, u \in U. \quad (12)$$

For the calculation of the general performance index, the five performance metrics, $\delta_{u\ell}(k)$, shown in Table 1 are used. The procedure is as follows. The metrics are normalized with the expression [9],

$$z_{u\ell}(k) = \frac{\delta_{u\ell}(k) - \mu_u(k)}{\sigma_u(k)}, \quad (13)$$

for each $\ell = 1,...,5$ and $u \in U$. Here, $\mu_u(k)$ and $\sigma_u(k)$ are the mean and standard deviation of the five metrics.Then, the Chilean law dictates that the general performance index is calculated with the expression [9],

$$I_u(k) = \sum_{\ell=1}^{5} r_\ell \cdot e^{\left(\frac{z_{u\ell}(k)+2}{4} - 1.9\right)^3}, u \in U, \quad (14)$$

where $r = [r_1,...,r_5] = [.01,.15,.24,.25,.35]$ are the weights of each performance metric.

Metric	Expression	Description
$\delta_{u1}(k)$	$\frac{x_u(k)}{\|C_u\|}$	Students per career
$\delta_{u2}(k)$	$\frac{x_u(k)}{y_u(k)}$	Students per ECW
$\delta_{u3}(k)$	$\frac{y^u_{Dr}(k) + 0.33 \cdot y^u_{Mg}(k)}{y_u(k)}$	ECW with post grade per ECW
$\delta_{u4}(k)$	$\frac{V_u(k)}{y_u(k)}$	Investigation projects per ECW
$\delta_{u5}(k)$	$\frac{P^u_{ISI}(k) + 0.33 \cdot P^u_{Scielo}(k)}{y_u(k)}$	Publications per ECW

Table 1. *Performance metrics for the DFC*

Finally, the fraction of the variable 5% of the DFC that corresponds to university $u \in U$ is given by,

$$DFC^{(v)}_u(k) = 0.05 \cdot DFC(k) \cdot q_u. \quad (15)$$

Then, it can be said that the DFC that corresponds to each university is given by,

$$DFC_u(k) = DFC(k)\left[0.95 \cdot h_u + 0.05 \cdot q_u(k)\right]. \quad (16)$$

3.4 Student Body's Payment

The third source of funds of each university comes from the student body's payment of the fee of their career. As mentioned before, each student j is assigned a University Credit Percentage ρ_j, which depends of the socio-economic quintile of the student. Besides, each career $c \in C_u$ of university $u \in U$ has a real fee, $f^{(real)}_{uc}(k)$, set by the university; and a reference fee, $f^{(ref)}_{uc}(k)$, set by the Chilean State. The credit assigned to the student is covered by the State, but with a maximum value of $f^{(ref)}_{uc}(k)$. The difference between $f^{(ref)}_{uc}(k)$ and $f^{(real)}_{uc}(k)$ must be covered by the Solidary Fund. Also, each student can have a scholarship, s_j, which is also used to finance the fee, with a maximum value of s_{max}.

The payment dynamics of student j in the year k is given by the following difference equations,

$$d_j(k) = \rho_j \cdot \max(0, f^{(real)}_{uc_j}(k) - \min(s_j, s_{\max})), \quad (17)$$

$$d^{(S)}_j(k) = \max(f^{(ref)}_{uc_j}(k), d_j(k)), \quad (18)$$

$$d^{(F)}_j(k) = d_j(k) - d^{(S)}_j(k). \quad (19)$$

Here, $d^{(S)}_j(k)$ and $d^{(F)}_j(k)$ represent the increment of the debt with the Chilean State and the Solidary Fund of the university in the year k. Also, c_j represents the career of student j. So, the cumulated debt with the State and the Fund is given by,

$$D^{(S)}_j(k) = (1+i)D_j(k-1) + d^{(S)}_j(k), \quad (20)$$

$$D^{(F)}_j(k) = (1+i)D_j(k-1) + d^{(F)}_j(k). \quad (21)$$

The amount of money that the student must pay in cash is,

$$p_j(k) = (1-\rho_j) \cdot \max(0, f^{(real)}_{uc_j}(k) - \min(s_j, s_{\max})). \quad (22)$$

The, the collected money of the university is in the year k is,

$$S_u(k) = \sum_{j \in Z_u} \left[p_j(k) + d^{(S)}_j(k) \right]. \quad (23)$$

where z_u is the set of students of the university. When the student leaves the university, it starts paying the debt with the State and with the Solidary Fund, using the equations,

$$p_j^{(S)}(k) = p_j(k)\left[\frac{D_j^{(S)}(k)}{D_j^{(S)}(k)+D_j^{(F)}(k)}\right], \quad (24)$$

$$p_j^{(F)}(k) = p_j(k)\left[\frac{D_j^{(F)}(k)}{D_j^{(S)}(k)+D_j^{(F)}(k)}\right], \quad (25)$$

$$D_j^{(S)}(k) = (1+i)D_j^{(S)}(k-1) - p_j^{(S)}(k), \quad (26)$$

$$D_j^{(F)}(k) = (1+i)D_j^{(F)}(k-1) - p_j^{(F)}(k). \quad (27)$$

where $p_j(k)$ represents the total amount of money paid by the student in the year k, once he/she has left the university. This value may depend on the salary of the ex-student. Then, the Solidary Fund is ruled by the following difference equation,

$$F_u(k) = F_u(k-1) - \sum_{j\in Z_u} p_j^{(F)}(k) + \sum_{t\in T_u} p_t^{(F)}(k). \quad (28)$$

Here, T_u is the set of ex-students of the university that are paying their cumulated debt.

Metric	Expression	Description
$\delta_{u6}(k)$	$\frac{y_{Dr}^u(k)+0.33\cdot y_{Mg}^u(k)}{x_u(k)}$	ECW with post grade per Student
$\delta_{u7}(k)$	$\frac{V_u(k)}{y_{Dr}^u(k)+0.33\cdot y_{Mg}^u(k)}$	Investigation projects per ECW with post grade
$\delta_{u8}(k)$	$\frac{P_{ISI}^u(k)+0.33\cdot P_{Scielo}^u(k)}{y_{Dr}^u(k)+0.33\cdot y_{Mg}^u(k)}$	Publications per ECW with post grade
$\delta_{u9}(k)$	$\frac{x_u^{(r)}(k)}{x_u(k)}$	Rate of opportune graduation
$\delta_{u,10}(k)$	$\frac{x_u^{(out,k-1)}(k)}{x_u(k)}$	Retention rate in first year

Table 2. *Performance metrics for the reference fee fixation*

3.5 Reference Fee Fixation

As mentioned before, the reference fee of each career is used as a maximum state coverage for the student's credit. To determine it, in the first place, universities are ordered into 4 groups every year, according to the reference fee index, $R_u(k)$, that is calculated with the following expression [9],

$$R_u(k) = \sum_{\ell=6}^{10} \frac{\delta_{u\ell}(k)}{5}, \quad (29)$$

where the performance indices, from 6 to 10, are described in Table 2. In Table 2, $x_u^{(r)}(k)$ is the number of students that graduate within the duration time of the career plus one year, and $x_u^{(out,k-1)}(k)$ is the number of students that leave the university in year k, and entered in year $k-1$.The criterion to organize the groups is the following,

$$\text{if } \frac{R_u(k)}{\max_{s\in U} R_s(k)} \geq 0.75 \text{ then } u \in G_1,$$

$$\text{if } 0.5 \leq \frac{R_u(k)}{\max_{s\in U} R_s(k)} < 0.75 \text{ then } u \in G_2,$$

$$\text{if } 0.25 \leq \frac{R_u(k)}{\max_{s\in U} R_s(k)} < 0.5 \text{ then } u \in G_3,$$

$$\text{if } \frac{R_u(k)}{\max_{s\in U} R_s(k)} < 0.25 \text{ then } u \in G_4.$$

where $G_1,\ldots,G_4$ are the four groups of universities. Then, inside each group, the reference fee for each career is the one of the university with the most years of accreditation.

Main Algorithm

```
ask universities [
  for each career c do:
   for each quintile q do:
    add new students to q and c
    remove students with [ttg = 0], from q and c
   end
  end

  calculate performance indices
  calculate DFC using indices and DFC equations
  calculate IFC using indices and IFC equations
]

ask students [
  if ttg = 0[
    pay part of the debt, if any
  ]
  else [
    cumulate debt
    ttg = ttg - 1
  ]
]
```

Fig. 2. *Main agent algorithm of the model*

4 Simulation Model

The simulation model tool was developed in Java language, and the programming had an agent-based orientation. There were created two types of agents: the universities and the students of each university. The students have the following fields: *University, Quintile, Career, PSU range, Time left to graduate (TTG), State debt* and *Solidary Fund debt.* Universities have the fields *Careers, Solidary Fund, ECW Dr., ECW Mg., ECW with no post grade, Years*

of accreditation, Investigation projects, ISI and Scielo publications, along with the fields of the performance indices. One of the universities is set to be the *Main University*, which has the role to be the university in study, i.e. the university for which the simulation is made. The main algorithm of simulation is detailed in Fig 2. One step in this algorithm represents a year, and it is repeated until the simulation horizon is met.

4.1 Implementation

The generic nature of the design of the simulation model makes it capable to work for any university that belongs to the Council Board of Rectors of Chilean Universities. In particular, it was applied in the University of Santiago of Chile (USACH) [8]. Fig. 3 has a screenshot of the developed simulation tool for this university.

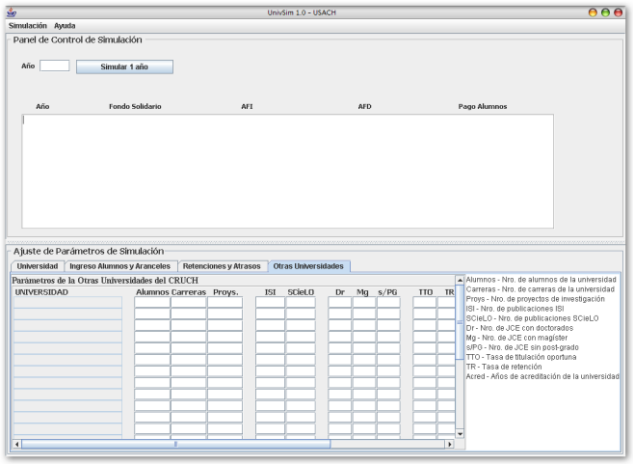

Fig. 3. *Screenshot of the simulation tool developed for USACH*

5 Conclusions and Future Work

The Chilean universities that belong to the Council Board of Rectors of Chilean Universities are subject to direct and indirect fiscal money contributions, that depend on performance measures among other factors, and thus, their income is affected by these two contributions plus the student body's payment.

The calculation and prediction of these income sources is a non trivial task, because they depend on many factors, which can increment certain source, but decrement another. Besides, the calculations have many complex mathematical and logic aspects that can be confuse to the strategic planners of the universities.

The mathematical model presented was used to create a simulation tool, and was applied to the strategic decision making process of the University of Santiago of Chile, having good acceptance by the users of it. At the time, they are working in the creation of proposals for new mechanisms for the reference fee calculation using this tool.

The generic nature of the design of the simulation model makes it capable to work for any university that belongs to the Council Board of Rectors of Chilean Universities.

Future work will include new modules for the reference fee calculation, in order to simulate and evaluate the proposals created by the University of Santiago of Chile.

References:

[1] Council Board of Rectors of Chilean Universities. http://www.cruch.cl/

[2] Scientific Electronic Library Online. http://www.scielo.cl/

[3] Law Nº 18681, Article 20, Chile.

[4] D. Meignan, O. Simonin and A. Koukam, Simulation and evaluation of urban bus-networks using a multiagent approach, *Simulation Modelling Practice and Theory*, Vol. 15, No. 6 2007, pp 659-671.

[5] J. Wu, B. Hu, J. Zhang, Multi-agent Simulation of Group Behavior in E-Government Policy Decision, *Simulation Modelling Practice and Theory (2007), doi: 10.1016/j.simpat.2007.07.007*

[6] M. Anwar, C. Jeanneret, L. Parrott and D. Marceau, Conceptualization and implementation of a multi-agent model to simulate whale-watching tours in the St. Lawrence Estuary in Quebec, Canada, *Environmental Modelling & Software*, Vol. 22, No. 12, 2007, pp. 1775-1787.

[7] T. Schenk, G. Löffler and J. Rauh, Agent-based simulation of consumer behavior in grocery shopping on a regional level, *Journal of Business Research*, Vol. 60, No. 8, 2007, pp. 894-903.

[8] University of Santiago of Chile. http://www.usach.cl.

[9] Law Nº 18768, Article 20, Chile.