Flow of a Casson fluid through a stenosed artery subject to periodic body acceleration

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Abstract: - The pulsatile flow of blood through a stenosed artery under the influence of external periodic body acceleration is studied. The effect of non-Newtonian nature of blood in small blood vessels has been taken into account by modeling blood as a Casson fluid. The non-linear coupled equations governing the flow are solved using perturbation analysis assuming that the Womersley frequency parameter is small which is valid for physiological situations in small blood vessels. The effect of pulsatility, stenosis, body acceleration, yield stress of the fluid and pressure gradient on the yield plane locations, velocity distribution, flow rate, shear stress and frictional resistance are investigated. It is noticed that the effect of yield stress and stenosis is to reduce flow rate and increase flow resistance. The impact of body acceleration is to enhance the flow rate and reduces resistance to flow.

Keywords: Body acceleration, Casson fluid, Stenosed artery, Pulsatile Flow, non-Newtonian fluids, Blood rheology.

Introduction

External accelerations cause disturbance quite often in human life. In situations like traveling in vehicles or aircraft, operating jackhammer or the sudden movements of the body during sports activities, the human body experiences external body acceleration. Prolonged exposure to such external body acceleration may cause serious health problem such as headache, increase in pulse rate and loss of vision on account of disturbances in blood flow [1-2]. It is therefore desirable to set a standard for short and long term exposures of human being to such acceleration. If the response of the human system to such accelerations is understood properly, the controlled accelerations can be used for therapeutic treatments, development of new diagnostic tools and for better designing of protective pads [3-4]. It is quite common to find localized narrowing, commonly called stenosis, caused by intravascular plaques in the arterial system of humans or animals. This stenosis disturbs the normal pattern of blood flow through the artery. Recognizing of the flow characteristics in the vicinity of stenosis may help to further understanding of some major complications which can be arise such as, an ingrowths of tissue in the artery, the development of a coronary thrombosis etc. The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis.

Due to physiological importance of body acceleration many theoretical investigations have been carried out for the flow of blood under the influence of body acceleration with and with out stenosis. Sud and Sekhon [5] studied the pulsatile flow of blood through a rigid circular tube subject to body acceleration, treating blood as a Newtonian fluid. Misra and Sahu [6] analysed the flow of blood through large arteries under the action of periodic body acceleration. Belardinelli *et al.* [7] proposed mathematical models for various forms of body acceleration. Usha and Prema [8] studied the pulsatile flow of particle-fluid suspension model of blood under the presence of periodic body acceleration. Using Laplace and Hankel transforms Elshehawey *et al.* [9] studied the effect of body acceleration on pulsatile flow of blood through a porous medium by treating blood as a Newtonian fluid. Later El-Shahed [10] extended this study for a stenosed porous medium.

In all these investigations blood is modelled as a Newtonian fluid. It is reported that the rheological properties of blood and its flow behaviour through tubes of varying cross section play an important role in understanding the diagnosis and treatment of many cardiovascular diseases [11, 12,13]. It is well known that blood being a suspension of cells, behaves as a non-Newtonian fluid at low shear rates and during its flow through small blood vessels, especially in diseased states when clotting effects in small arteries are present. Experiments conducted on blood [14,15,16] with varying heamatocrits, anticoagulants, temperature etc suggested that the behaviour of blood at low shear rates can best be described by Casson model [17,18]. Aroesty and Gross [19, 20] used Casson theory in their mathematical analysis to study the pulsatile flow of blood vessels with application to microcirculation. Chaturani and Palanisamy [21,22] analysed the pulsatile flow of blood under the influence of periodic body acceleration by assuming blood as a Casson fluid and also a Power law fluid by using finite difference scheme. Majhi and Nair [23] studied the pulsatile flow of blood under the influence of body acceleration treating blood as a third grade fluid. Sarojamma and Nagarani [24] studied the flow of a Casson fluid in a tube filled with porous medium under periodic body acceleration with applications to artificial organs. In recent paper Mandal *et al.*, [25] developed a two dimensional mathematical model to study the effect of externally imposed periodic body acceleration on non-Newtonian blood flow through an elastic stenosed artery where the blood is characterized by the generalized power-law model.

In view of the above, a mathematical model is developed to study the pulsatile flow behaviour of blood in an artery under stenotic condition subject to both the pulsatile pressure gradient due to normal heart action and of periodic body acceleration. Blood is modelled as a Casson fluid by properly accounting for yield stress of blood. The combined effect of pulsatility, stenosis, body acceleration, yield stress on the flow parameters is investigated.

2 Mathematical Formulation

Consider the pulsatile flow of blood in presence of externally imposed periodic body acceleration in an artery with mild stenosis. We consider the flow is axially symmetric, laminar, fully developed where the flowing blood is modelled as a Casson fluid. Following Young [26] the stenotic protuberance is assumed to be an axisymmetric surface generated by a cosine curve. The geometry of the stenosis is as shown in Fig.1 and is given by

$$\overline{R}(\bar z) = R_0 - \bar\delta \left(1 + \cos\frac{\pi \bar z}{2\bar z_0}\right) \quad \text{for} \quad -2\bar z_0 \le \bar z \le 2\bar z_0$$
$$= R_0 \quad \text{otherwise} \qquad \text{----(1)}$$

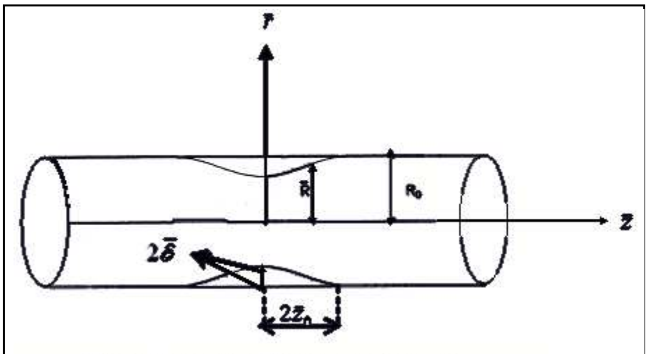

Fig.1 Geometry of the stenosed artery

where $4\bar z_0$ is the length of the stenotic region, $2\bar\delta$ is the maximum protuberance of the stenotic form of the artery wall and R_0 is the radius of the normal artery. The periodic body acceleration $F(\bar t)$ in the axial direction is given by

$$F(\bar t) = a_0 \cos(\omega_b \bar t + \phi) \qquad \text{----(2a)}$$

where a_0 is its amplitude, $\omega_b = 2\pi f_b$, f_b is its the frequency in Hz, ϕ the lead angle of $F(\bar t)$ with respect to the heart action. The frequency of body acceleration f_b is assumed to be small, so that wave effects can be neglected. The pressure gradient at any $\bar z$ may be represented as follows

$$-\frac{\partial \bar p}{\partial \bar z} = A_0 + A_1 \cos(\omega_p \bar t) \qquad \text{----(2b)}$$

where A_0 is steady component of the pressure gradient, A_1 is amplitude of the fluctuating component and $\omega_p = 2\pi f_p$, f_p is the pulse frequency. Both A_0 and A_1 are functions of $\bar z$. It can be shown that the radial velocity is very small in magnitude so that it may be neglected for problem with mild stenosis. The specified momentum equation for the flow in cylindrical coordinate system is given by

$$\bar\rho \frac{\partial \bar u}{\partial \bar t} = -\frac{\partial \bar p}{\partial \bar z} + \frac{1}{\bar r}\frac{\partial}{\partial \bar r}(\bar r \bar\tau_{\bar r \bar z}) + F(\bar t) \qquad \text{----(3a)}$$

$$0 = \frac{\partial \bar p}{\partial \bar r} \qquad \text{-----(3b)}$$

Where $\bar r$ and $\bar z$ denote the radial and axial coordinates respectively and $\bar\rho$ denotes density, $\bar u$ axial velocity of blood, $\bar t$ time, $\bar p$ pressure and $\bar\tau$ the shear stress. For Casson fluid the relation between shear stress and shear rate is given by [27] ,

$$\bar\tau^{\frac{1}{2}} = \bar\tau_y^{\frac{1}{2}} + \left[\mu\left(\frac{-\partial \bar u}{\partial \bar r}\right)\right]^{\frac{1}{2}} \quad \text{if } \bar\tau \ge \bar\tau_y$$

$$\frac{\partial \bar u}{\partial \bar r} = 0 \quad \text{if } \bar\tau \le \bar\tau_y \qquad \text{-----(4)}$$

where $\bar{\tau}_y$ denotes yield stress and μ, the Casson's viscosity. These relations correspond to vanishing of velocity gradients in regions where the shear stress $\bar{\tau}$ is less than the yield stress $\bar{\tau}_y$, this in turn implies a plug flow whenever $\bar{\tau} \le \bar{\tau}_y$.

The boundary conditions appropriate to the problem under study are the no slip condition

(i) u = 0 at $\bar{r}$ =R($\bar{z}$) -----(5a)

(ii) $\bar{\tau}$ is finite at $\bar{r}$ = 0 -----(5b)

Introducing the non-dimensional variables

$$u = \frac{\bar{u}}{A_0 R_0{}^2 / 4\mu},\ z = \frac{\bar{z}}{R_0},\ z_0 = \frac{\bar{z}_0}{R_0},\ t = \omega_p \bar{t},$$
$$\delta = \frac{\bar{\delta}}{R_0}, \tau = \frac{\bar{\tau}}{A_0 R_0 / 2},\ \theta = \frac{2\bar{\tau}_y}{A_0 R_0},\ R(z) = \frac{\bar{R}(\bar{z})}{R_0},$$
$$r = \bar{r} / R_0,\ e = \frac{A_1}{A_0},\ B = \frac{a_0}{A_0},\ \omega = \frac{\omega_b}{\omega_p} \qquad \text{----(6)}$$

The non-dimensional momentum equation (3a) becomes

$$\alpha^2 \frac{\partial u}{\partial t} = 4(1 + e\cos t) + 4B\cos(\omega t + \phi) + \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \qquad \text{-----(7)}$$

where $\alpha^2 = \dfrac{\omega_p R_0{}^2}{(\mu/\rho)}$, α is called Womersley frequency parameter.

Equation (4) can be written as

$$\tau^{\frac{1}{2}} = \theta^{\frac{1}{2}} + \frac{1}{\sqrt{2}}\left(\frac{-\partial u}{\partial r}\right)^{\frac{1}{2}} \quad \text{if} \quad \tau \ge \theta \qquad \text{----(8a)}$$

$$\text{and} \quad \frac{\partial u}{\partial r} = 0 \quad \text{if} \quad \tau \le \theta \qquad \text{----(8b)}$$

The boundary conditions (5a, b) reduce to

(i) u = 0 at r = R(z) ----(9a)

(ii) τ is finite at r = 0 ----(9b)

The geometry of the stenosis in non-dimensional form is given as

$$R(z) = 1 - \delta\left(1 + \cos\frac{\pi z}{2z_0}\right) \text{ for } -2z_0 \le z \le 2z_0$$
$$= 1 \quad \text{otherwise.} \qquad \text{----(10)}$$

3. Method of Solution

On using perturbation method, the velocity u, shear stress τ, plug core radius R_p and plug core velocity u_p are expanded as follows in terms of α^2 (where $\alpha^2 \ll 1$)

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \ldots\ldots \quad \text{---(11a)}$$
$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2 \tau_1(z,r,t) + \ldots\ldots \quad \text{--(11b)}$$
$$R_p(z,r,t) = R_{0p}(z,t) + \alpha^2 R_{1p}(z,t) + \ldots\ldots \quad \text{(11c)}$$
$$u_p(z,r,t) = u_{0p}(z,t) + \alpha^2 u_{1p}(z,t) + \ldots\ldots \quad \text{-(11d)}$$

Substituting (11a) and (11b) in equation (7) and equating the constant term and α^2 term we get

$$\frac{\partial}{\partial r}(r\tau_0) = -2r[(1 + e\cos t) + B\cos(\omega t + \phi)] \quad \text{---(12)}$$

$$\frac{\partial u_0}{\partial t} = \frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \qquad \text{----(13)}$$

Integrating equation (12) and using the boundary condition (9b) we obtain

$$\tau_0 = -f(t)r \qquad \text{----(14)}$$

where

$$f(t) = [(1 + e\cos t) + B\cos(\omega t + \phi)] \ . \qquad \text{----(15)}$$

Substituting (11a) and (11b) in (8) we get

$$-\frac{\partial u_0}{\partial r} = 2[\ \theta + |\tau_0| + -2\sqrt{\theta / |\tau_0|}\ ] \qquad \text{----(16)}$$

$$-\frac{\partial u_1}{\partial r} = 2|\tau_1|\ [1 - \sqrt{\theta / |\tau_0|}\ ] \qquad \text{----(17)}$$

Integrating equation (16), using the relation (14) and the boundary condition (9a) we obtain

$$u_0 = f(t)R^2\left\{1 - (r/R)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}[1 - (r/R)^{\frac{3}{2}}] + \frac{2k^2}{R}[1 - (r/R)]\right\} \qquad \text{----(18)}$$

where $k^2 = \theta / f(t)$

The plug core velocity u_{0p} can be obtained from equation (18) as

$$u_{0p} = f(t)R^2\left\{1 - (R_{0p}/R)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}[1 - (R_{0p}/R)^{\frac{3}{2}}] + \frac{2k^2}{R}[(1 - (R_{0p}/R)]\right\} \qquad \text{-----(19)}$$

where R_{0p} is the first approximation plug core radius.

Neglecting the terms of o(α^2) and higher powers of α in equation (11c) R_{0p} can be obtained from (14) as $R_{0p} = \theta / f(t) \equiv k^2$ -----(20)

Similarly the solution for τ_1, u_1, u_{1p} can be obtained using equations (13), (17) and (18) as

$$\tau_1 = \frac{f'(t)\,R^3}{8}\{2\frac{r}{R}-(\frac{r}{R})^3-\frac{8}{21}\frac{k}{\sqrt{R}}[7\frac{r}{R}-4(\frac{r}{R})^{\frac{5}{2}}]\}$$
----(21)

$$u_1 = \frac{f'(t)\,R^4}{16}\left\{(\frac{r}{R})^4-4(\frac{r}{R})^2+3+\frac{k}{\sqrt{R}}\left[\frac{16}{3}(\frac{r}{R})^2-\frac{424}{147}(\frac{r}{R})^{\frac{7}{2}}+\frac{16}{3}(\frac{r}{R})^{\frac{3}{2}}-\frac{1144}{147}\right]+\frac{k^2}{R}\left[\frac{128}{63}(\frac{r}{R})^3-\frac{64}{9}(\frac{r}{R})^{\frac{3}{2}}+\frac{320}{63}\right]\right\}$$ ----(22)

$$u_{1p} = \frac{f'(t)\,R^4}{16}\left\{(\frac{R_{0p}}{R})^4-4(\frac{R_{0p}}{R})^2+3+\frac{k}{\sqrt{R}}\left[\frac{16}{3}(\frac{R_{0p}}{R})^2-\frac{424}{147}(\frac{R_{0p}}{R})^{\frac{7}{2}}+\frac{16}{3}(\frac{R_{0p}}{R})^{\frac{3}{2}}-\frac{1144}{147}\right]+\frac{k^2}{R}\left[\frac{128}{63}(\frac{R_{0p}}{R})^3-\frac{64}{9}(\frac{R_{0p}}{R})^{\frac{3}{2}}+\frac{320}{63}\right]\right\}$$ ----(23)

Using equation (11), the total velocity distribution and shear stress can be written as

$$u = f(t)\,R^2\left\{1-(\frac{r}{R})^2-\frac{8}{3}\frac{k}{\sqrt{R}}[1-(\frac{r}{R})^{\frac{3}{2}}]+\frac{2k^2}{R}[1-(r/R)]+\frac{\alpha^2 R^2 C}{16}\left[(\frac{r}{R})^4-4(\frac{r}{R})^2+3+\frac{k}{\sqrt{R}}[\frac{16}{3}(r/R)^2-\frac{424}{147}(r/R)^{\frac{7}{2}}+\frac{16}{3}(r/R)^{\frac{3}{2}}-\frac{1144}{147}]+\frac{k^2}{R}[\frac{128}{63}(r/R)^3-\frac{64}{9}(r/R)^{\frac{3}{2}}+\frac{320}{63}]\right]\right\}$$

$$|\tau|_w = f(t)r\{1+\frac{C\alpha^2R^2}{8}(1-\frac{8k}{7\sqrt{R}})\}$$ ------(25)

where C $= f'(t)/f(t)$

The second approximation plug core radius R_{1p} can be obtained by neglecting terms of o(α^4)and higher powers of α in equation (11c) as

$$R_{1p} = \frac{-|\tau_1(R_{0p})|}{f(t)}$$ -----(26)

With the help of equations (26), (20) and (11c), R_p can be given by

$$R_p = k^2-\alpha^2\frac{f'(t)}{f(t)}\frac{R^3}{8}\left\{2\left(\frac{k^2}{R}\right)-\left(\frac{k^2}{R}\right)^3-\frac{8}{21}\frac{k}{\sqrt{R}}\left[7\left(\frac{k^2}{R}\right)-4\left(\frac{k^2}{R}\right)^{\frac{5}{2}}\right]\right\}$$ ----(27)

The volumetric flow rate Q is given by

$$Q(t) = 4\int_0^{R(z)} r\,u(z,r,t)\,dr = f(t)\,R^4\,\{\frac{1}{4}-\frac{4}{7}\frac{k}{\sqrt{R}}+\frac{1}{3}(\frac{k}{\sqrt{R}})^2+\frac{\alpha^2 R^2 C}{16}[\frac{2}{3}+\frac{120}{77}\frac{k}{\sqrt{R}}+\frac{32}{35}(\frac{k}{\sqrt{R}})^2]\}$$ ----(28)

4 Results and Discussion

The objective of the present investigation is to study the combined effect of body acceleration, stenosis and yield stress of the fluid on the pulsatile flow of blood through a circular cylinder by modeling blood as a Casson fluid. The governing equations of the flow are solved using perturbation analysis assuming that the Womersley frequency parameter is small which is valid for physiological situations in small blood vessels. The effect of pulsatility, stenosis, body acceleration, yield stress of the fluid and pressure gradient on velocity distribution, plug radius, plug flow velocity, shear stress, flow rate, and frictional resistance are investigated. The results are discussed by computing the flow variables at different values of yield stress of the fluid θ , body acceleration parameter *B*, stenotic radius δ , pressure gradient *e* and for different values of time *t* by fixing the other parameters occurred in the flow.

Axial velocity profiles at the peak of the stenosis ($z = 0$) for a fixed value of pressure gradient and for different values of *B*, θ, δ and *t* are shown in Fig.2. It is observed that the body acceleration parameter *B* brings in quantitative and as well as qualitative changes in velocity profiles (Fig. 2a). In the presence of body acceleration velocity is more and with increase in body acceleration the plug region shrinks and hence more flow takes place. For the same values of pressure gradient and yield stress when the body acceleration is 2, the magnitude of velocity is almost doubled to the case when body acceleration is absent. In the absence of yield stress (Fig.2b) i.e. when the fluid is Newtonian (valid in large vessels) velocity rises sharply with point of maximum on the axis of the tube. The presence of yield stress reduces velocity and the velocity profile is blunt in the mid region of the tube indicating plug flow. As yield stress increases, the magnitude of velocity is very much reduced and thus the plug flow becomes prominent. In the absence of body acceleration and yield stress the velocity is lesser than the case when body acceleration is present. For a fixed value of yield stress and body acceleration the axial velocity decreases with time in a rigid tube as well as in a stenosed tube and also observed that the presence of stenosis qualitatively decreases the velocity (Fig. 2c). In a stenosed tube (when $\delta = 0.2$) the magnitude of velocity is reduced four times to the magnitude of velocity in a rigid tube. The combined effect of stenosis and yield stress is to enhance the plug flow region.

The plug radius pattern is depicted in Fig.3 for different variations of various flow parameters. The effect of pulsatility on yield plane is that the locations of yield plane are changed and hence vary during the course of motion. In the absence of body acceleration plug radius is minimum at t = 0° and starts increasing in the first half of the cycle attaining maximum value at t = 180° and then starts decreasing in the second half cycle. In the presence of body

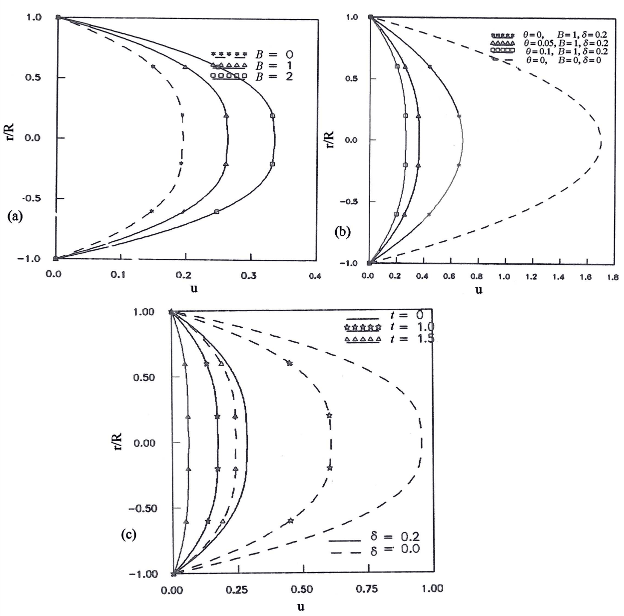

Fig. 2 Axial velocity profiles for $e = 1$, $\omega = 1$, $z = 0$, $\phi = 0.2$, $\alpha = 0.5$
(a) $t = 1$, $\delta = 0.2$, $\theta = 0.1$ (b) $t = 1$ (c) B = 1, $\theta = 0.1$

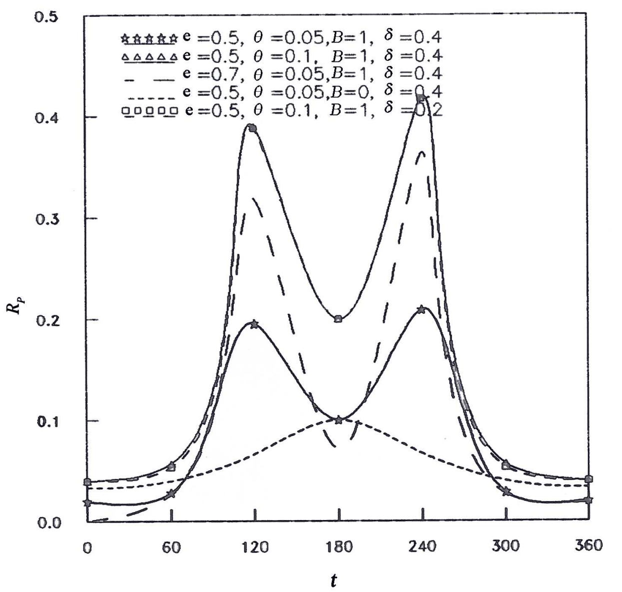

Fig. 3 Variation of plug core radius R_p with time at the centre of the stenosis

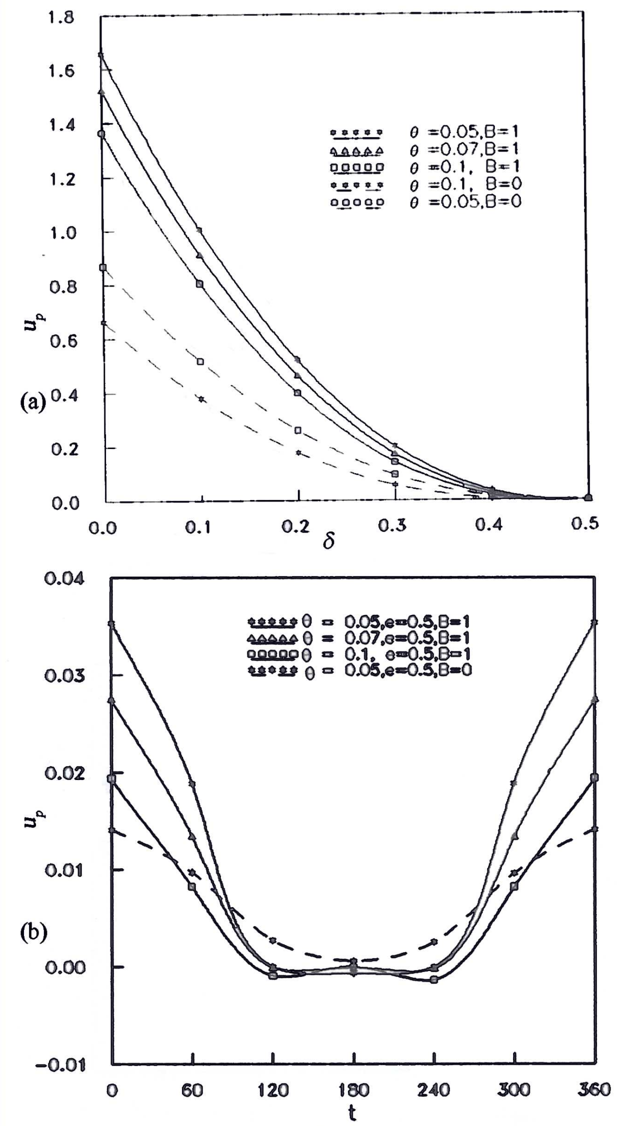

Fig. 4 Variation of plug velocity u_p
(a) with stenotic radius δ (b) with time t

acceleration it is interesting to note that there are two points of maximum. In first half cycle the plug radius rises from a minimum value and reaches a maximum at t = 120° and starts decreasing with point of minimum at t = 180° and the same behaviour is repeated in the second half. When the value of yield stress is more the width of the plug flow region is more and hence the flow is significantly reduced. The effect of stenotic radius is negligibly small on Rp. It is noticed that plug flow region increases with pressure gradient.

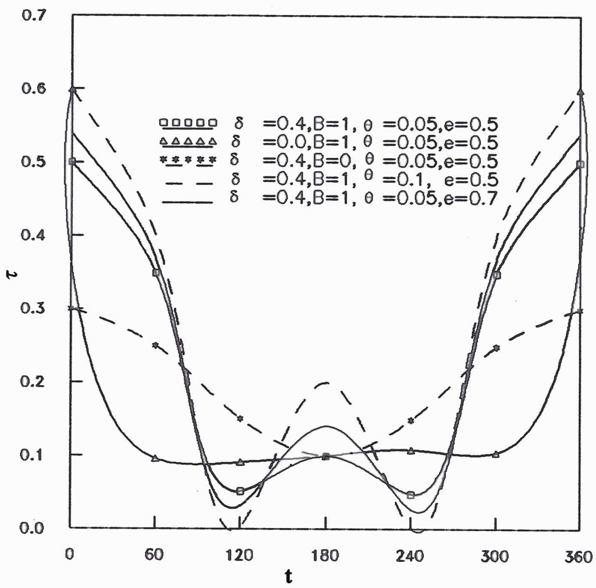

Fig. 5 Wall shear stress distribution

Plug flow velocity for different values of yield stress is presented in Fig.4a. It is noticed that the plug flow velocity decreases with δ and it approaches zero when $\delta = 0.42$ in the absence of body acceleration and in the presence of body acceleration when $\delta = 0.45$. This indicates that for this set of values the whole flow region is almost plugged. The plug velocity (Fig.4b) is symmetrical about the time t =180°. In the absence of body acceleration plug velocity is less when t ≤ 45° and during the interval 90° < t < 120° it is more than the corresponding case when body acceleration is present. For higher values of yield stress, plug velocity reduces.

Fig.5 shows shear stress variation. The behaviour of shear stress is symmetrical about t = 180°. In a rigid tube the wall shear stress is maximum initially and decreases sharply attaining a minimum value at t = 60° and increases steadily in the interval 60° ≤ t ≤ 180°. It is noticed that the effect of yield stress is small and enhances shear stress. In the absence of body acceleration, wall shear stress is less compared to the case when body acceleration is present and it steadily decreases with time with point of minimum at t =180°.

The variation of flow rate with pressure gradient is presented in Fig. 6a. For $\theta = 0$, the curves are linear. For positive values of θ, the curves are slightly non-linear. Flow rate in a normal tube is more than that in the stenosed tube. It is noticed that body acceleration enhances flow rate. Fig.6b represents variation of flow rate with yield stress. When θ increases there is a substantial decrease in flow rate which is due to increase in the width of the plug region. An increase in δ results in the reduction of flow rate which is due to the reduced lumen size.

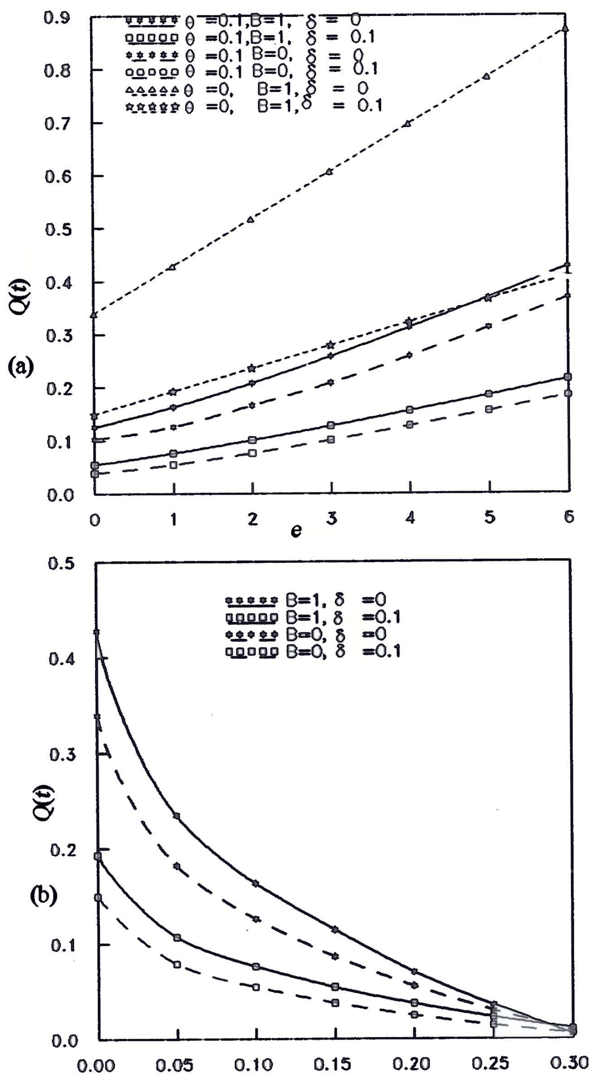

Fig. 6 Variation of flow rate (a) with e (b) with θ

The resistance to flow is calculated by using the formula $\lambda = \dfrac{\Delta p}{Q}$. Fig 7 represents the variation of frictional resistance with δ for different values of yield stress and body acceleration and a unit pressure gradient. It is noticed that the flow resistance is small when $\theta = 0$ i.e. when the fluid is Newtonian i.e.in large vessels. In small blood vessels where the non-Newtonian nature of blood is significant, the yield stress of blood creates more resistance to flow. It is also noticed that the flow resistance increases with the size of stenotic protuberance. Hence, the combined effect of stenosis and yield stress is to enhance the flow resistance and thus obstructing the fluid movement. It is interesting to note that the body acceleration reduces the flow resistance.

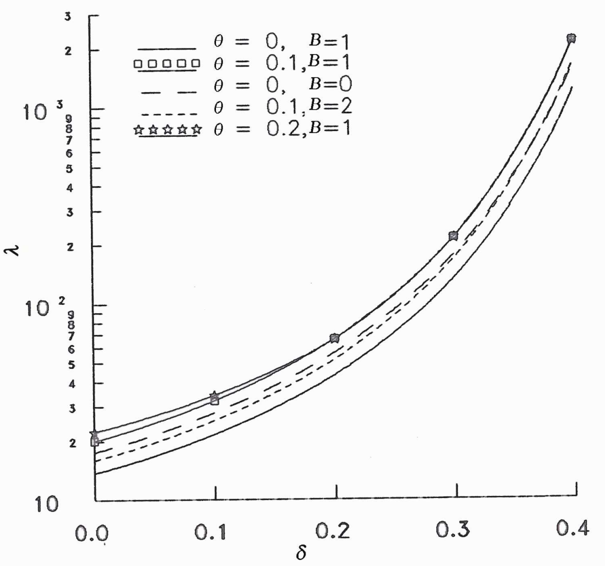

Fig.7 Variation of flow resistance λ with stenotic radius δ

5 Conclusions

By using perturbation analysis assuming that the Womersley frequency parameter is small, the pulsatile flow of blood with periodic body acceleration under the presence of stenosis is studied by modeling blood as a Casson fluid. It is observed that the body acceleration parameter, radius of stenosis and yield stress of the fluid are the strong parameters influencing the flow qualitatively and quantitatively. It is observed that, in the presence of yield stress (θ = 0.1) the magnitude of velocity is decreased 3 times of the value corresponding to the Newtonian case. It is seen that the body acceleration (when B = 2) doubled the magnitude of velocity when compared to its magnitude in the absence of the body acceleration. The presence and increase of the protuberance is found to reduce the magnitude of the velocity. The effect of yield stress and stenosis is to reduce the flow rate and the presence of body acceleration is to increase the flow rate. The flow resistance is seen to be increased substantially due to the presence of stenosis and yield stress. The body acceleration is found to reduce the flow resistance.

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