Dynamic Computer Simulation Techniques to Capture Multi-modal Vibration in Slender Footbridge Structures

DAVID P THAMBIRATNAM
School of Urban Development
Queensland University of Technology, Brisbane
AUSTRALIA

Abstract: - High strength materials, modern technology and demand for aesthetics have resulted in slender and vibration sensitive structures. These structures unfortunately exhibit complex multi-modal and coupled vibration which is not well understood at present and as a result there is no adequate guidance for their design. Dynamic computer simulation techniques can be used to capture these vibrations and quantify the influence of controlling parameters. The information so generated can be used to develop design guidance to enable safe and efficient slender structures. Research at the Queensland University of Technology has focussed on generating new and generic information on the vibration characteristics of these slender structures in terms of common dynamic parameters. This paper treats the first stage of this research project, in which vibration characteristics of a slender footbridge are established.

Key-Words: - Computer simulations, dynamics, finite elements, multi-modal, coupled, vibration, footbridge.

1. Introduction

Research into life time performance and safety of slender and large span bridges, roofs, floor plates and cantilever structures that are sensitive to multi-modal dynamic excitations is a very high priority as this phenomenon is not understood at present. New materials, modern technology and aesthetic requirements have resulted in slender structures with low natural frequencies and prone to amplified resonant response under service induced loads resulting in extreme discomfort, injury, loss of life and unhealthy environments to occupants.

Fig. 1: Millennium Footbridge, London

Many bridges, stadiums and other structures are known to have experienced excessive vibration of different kinds; for example the Millennium footbridge shown in Fig. 1. Past research has shown that slender suspension footbridges with shallow cable profiles often exhibit coupled vibration modes [4]. Synchronous, coupled and interactive excitations suffered by these footbridges continue to be discovered as “new phenomena”, which is not understood at present and there is no adequate guidance for their safe design, leaving a knowledge gap.

The complex and diverse action effects induced in foot bridges are similar to those in other slender structures because their responsive behaviour is dependent on the common dynamic characteristics. Large perceptible movements in these structures result in panic stricken crowd behaviour, damage and collapse, as evidenced by real events. This paper treats the vibration characteristics of a slender cable supported footbridge under human induced loads, using validated finite element models. The research information can be used to develop guidance for safe and efficient designs of slender footbridges and to develop generic procedures for evaluating performance of slender structures that exhibit multi-modal vibration.

2. Suspension Footbridge Model

The proposed suspension footbridge model has three groups of cables: top supporting cables, pre-tensioned bottom reverse profiled cables (Fig. 2) and pre-tensioned side bi-concave cables (Fig. 3). The two top suspending cables provide tension forces to support the whole structure, applied loads and extra internal forces induced by the bottom cables. The two parallel bottom cables are designed to have reverse profiles in the vertical plane and provide extra internal vertical forces to transverse bridge frames and the top supporting cables. The side cables are a pair of bi-concave cables with the same cable profiles in the horizontal plane, and their main function is to provide extra internal horizontal forces and horizontal stiffness.

Transverse bridge frames have been designed to support the deck and hold the cables (Fig 4). These frames form a set of spreaders for the cables to create the required profiles and are hung from the top cables, and further restrained by the lower reversed profile cables as well as the side cables. Two beams of rectangular section are supported on cross members of each pair of adjacent bridge frames, and the deck units are supported at the ends on these beams.

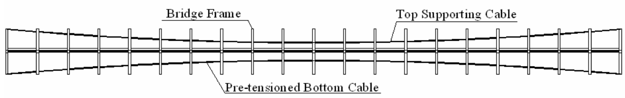


Fig. 2 Elevation

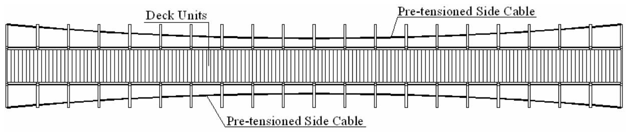


Fig. 3 Plan

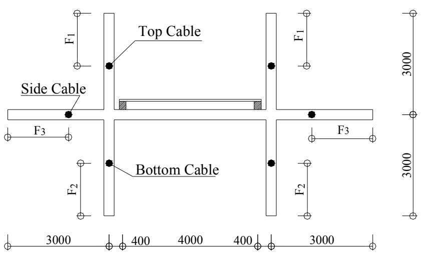


Fig. 4 Transverse bridge frame

In order to simplify the analysis, all transverse bridge frames are assumed to have the same size, and hence the weight of frame and deck acting on the cables can be considered as equal concentrated loads. All the cables are stretched by introducing initial distortions to keep the designed cable sags or cable profiles and required internal forces, and then the decks can be kept in a horizontal plane.

The structural analysis package SAP2000 was adopted in the numerical study. Stainless steel (Young's modulus 2.0×10^{11} N/m^2 and density 7850 kg/m^3) was chosen for the transverse bridge frames and support beams, and Aluminium (Young's modulus 6.5×10^{10} N/m^2 and density 2700 kg/m^3) was chosen for the deck units. To reduce the weight of the bridge structure, hollow rectangular sections and extruded sections are used for the members of the transverse bridge frames, support beams and decks. 8 deck units are simply supported on the support beams which span on the cross members of the adjacent transverse bridge frames. The cable systems are of stainless steel having the same material properties as those of bridge frames.

This paper treats the free vibration characteristics and the lateral vibration response of the bridge under human loads. In the finite element modelling, bridge deck units are assumed to be simply supported on the supporting beams and are modelled as 3D beam elements with pin connections to the supporting beams. They can hence carry torques and axial forces in order to keep the structure symmetric about the bridge centre line. The supporting beams are also modelled as 3D beam elements with released ends (pin connections), but one end can not carry torque and axial force. The bridge frame members are also modelled as 3D beam elements rigidly connected together at the intersection points. All the joints on the bridge frames at the two ends of the bridge model are assumed to have fixed joint restraints and therefore these frames will have almost no effect on the structural performance and vibration properties. The cables are modelled as tension only members having large deflections by using beam/frame elements. To simulate the flexible behaviour of cables, each cable element is divided into 20 segments and the moments of inertia of section and torsional constant are modified by introducing a factor of 0.01.

In order to investigate the effect of cable configuration, two types of bridge models with different cable configurations are studied: (i) Model A with top supporting cables and pre-tensioned bottom reverse profiled cables but, with the side cables and side legs of the bridge frames removed, (ii) Model B with top supporting cables, and pre-tensioned bottom and side cables. In each bridge model it is assumed that all the cables have the same diameter D_i $(i = 1.2,3)$ and cable sag.

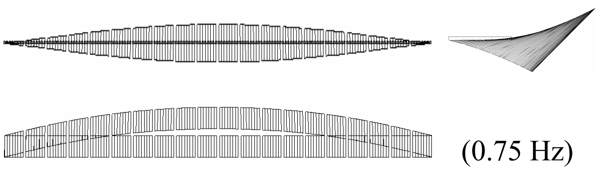
2.1 Vibration modes and coupling coefficient

Natural frequencies and corresponding vibration modes are important dynamic properties which influence the dynamic performance of structures. Suspension bridges have 4 main types of vibration modes: lateral, torsional, vertical and longitudinal. Results show that the lateral and torsional modes are often coupled and appear as (i) coupled lateral-torsional modes (LmTn) and (ii) coupled torsional-lateral modes (TmLn), where L and T represent lateral and torsional modes respectively and m and n are the number of half waves. Coupled lateral-torsional modes are dominated by lateral vibration, while coupled torsional-lateral modes are dominated by torsional vibration. Most vertical vibration modes are not coupled. The longitudinal modes are sensitive to the connection between the adjacent bridge frames and disappear from the first 20 frequencies when pre-tensions are introduced [4]. Figs. 5 and 6 show the first coupled lateral-torsional (L_1T_1) and coupled torsional-lateral (T_1L_1) modes respectively for an 80m span model B bridge with 120 mm diameter cables (Table 1).

In order to investigate the effect of coupled modes on the dynamic performance, a factor called "coupling coefficient" is introduced to describe the degree of coupling in the coupled vibration modes. The coupling coefficient ψ is defined as the ratio of vertical deflection U_v to lateral one U_l:

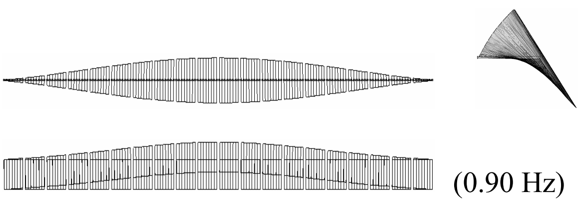
$$\psi = U_v / U_l \tag{1}$$

A typical deflected bridge frame located at an "anti-node" is shown in Fig. 7, in which the vertical and lateral deflections are picked up from the same point A. As a sign convention, the upward vertical deflection and rightward lateral deflection are defined as positive deflections. When the bridge deck and bridge frames sway about a point above the deck plane, the rightward lateral deflection is accompanied by downward vertical deflection and hence the coupling coefficient is negative according to Eqn. (1). On the other hand, a positive coupling coefficient indicates that the vibration mode is coupled torsional-lateral one and the bridge deck as well as bridge frames sway about a point below the deck plane. The values of coupling coefficients reflect the degree of coupling. For the coupled lateral-torsional modes, large coupling coefficient indicates high degree of coupling and large vertical component accompanies the lateral deflection; while for the coupled torsional-lateral vibration modes, large coupling coefficient indicates low degree of coupling and the mode tends towards a pure torsional one.



(0.75 Hz)

Fig 5. Coupled L_1T_1 mode



(0.90 Hz)

Fig. 6 Coupled T_1L mode

3. Walking Dynamic Loads

Synchronous excitation can be caused by the combination of a high density of pedestrians and low natural frequencies of bridges within the frequency range of pacing rate. When synchronisation occurs, footbridges resonate and part of the pedestrians will change their footfalls to match the vibration. To model the synchronous walking load, the following assumptions are made:

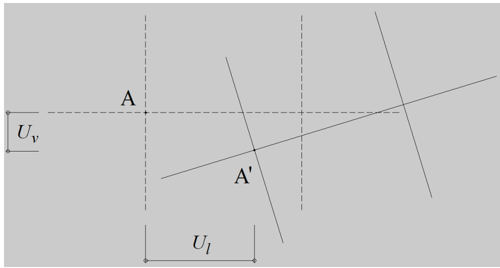


Fig. 7 Deflected bridge frame

(i) 20% of pedestrians participate fully in the synchronization process and generate vertical and lateral dynamic loads at a pacing rate coinciding with one of the natural frequencies of the footbridge. The remaining 80% pedestrians generate only static vertical load on the bridge deck as they walk with random pacing rates and phases.

(ii) force generated by a footfall has vertical, lateral and longitudinal components. The vertical and lateral components follow the force-time functions proposed by Wheeler [5] for a range of walking speeds, but the magnitude of the lateral component is 4% of the vertical component.

(iii) pedestrian load is uniformly distributed on the bridge deck, with a load density of 1.5 persons/m^2 and the average weight of a person is 700 N.

As the force functions are frequency dependant, walking activities can be classified into four types based on pacing rate. Each type of activity covers a range of frequencies and has a similar force function: slow walk (less than 1.8Hz), normal walk (1.8Hz ~ 2.2Hz), brisk walk (2.2 - 2.7Hz) and fast walk (greater than 2.7Hz).

Considering the normal walk for example, if the vertical force function of one foot is defined as $F_n[t]$, and the period and foot contact time are T_n and T_{nc} respectively, then this function, shown in Fig. 8, has the following property:

$$F_n[t] = \begin{cases} 0 & t < 0 \text{ or } t > T_{nc} \\ F_n[t] & 0 \le t \le T_{nc} \end{cases} \tag{2}$$

The continuous vertical force function $F_{nv}(t)$ and lateral force function $F_{nl}(t)$ can then be expressed in terms of the pacing rate f_p or load period T_p (T_p=1/ f_p).

$$F_{nv}(t) = \sum_{k=0}^{\infty} F_n[\alpha(t - kT_p)] \tag{3}$$

$$F_{nl}(t) = \sum_{k=0}^{\infty} \{F_n[\alpha(t - 2kT_p)] - F_n[\alpha(t-(2k+1)T_p)]\} \tag{4}$$

$$\alpha = T_n / T_p \quad (1.8\,\text{Hz} \le f_p < 2.2\,\text{Hz}) \tag{5}$$

where α is a time factor, f_n is the pacing rate and T_n the period (f_n=1/ T_n).

Walking dynamic load will have 3 parts: vertical dynamic force $q_{nv}(t)$, lateral dynamic force $q_{nl}(t)$ and vertical static force $q_{sv}(t)$. In the numerical analysis, the static load is modelled as a ramp function in order to reduce the fluctuation of dynamic response at the beginning of time history analysis. Therefore the loads for normal walk can be modelled as:

$$q_{nv}(t) = 210 F_{nv}(t) \quad (\mathrm{N/m^2}) \tag{6a}$$

$$q_{nl}(t) = 8.4 F_{nl}(t) \quad (\mathrm{N/m^2}) \tag{6b}$$

$$q_{sv}(t) = \begin{cases} 840t/(10\alpha) & (\mathrm{N/m^2}) \quad 0 < t < 10\alpha \\ 840 & (\mathrm{N/m^2}) \quad t \geq 10\alpha \end{cases} \tag{6c}$$

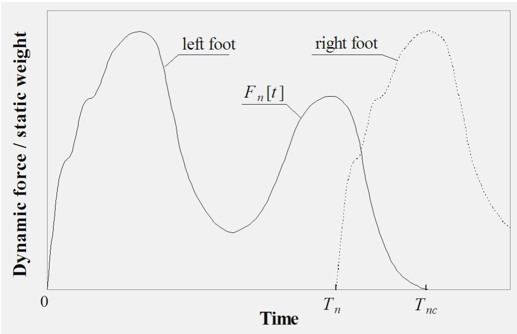


Fig. 8 Force – time function for normal walk

The loads models for other walking activities at other pacing rates can be similarly developed.

4. Lateral Response

It is known that lateral synchronous excitation can occur on any footbridge with lateral natural frequency below 1.3 Hz when the footbridge is subject to high density pedestrian load [3]. Although it is believed that the normal pacing rate varies from 1.4 Hz to 2.4 Hz, corresponding to lateral frequencies of 0.7 Hz to 1.2 Hz [1], lateral synchronous excitation can occur at a much lower lateral frequency; for example 0.5 Hz in the case of the Millennium Bridge. As slender long span footbridges often have low natural frequencies, low frequency vibration is more important than high frequency one. In order to investigate the dynamic performance of slender suspension footbridges with coupled vibration modes and the effect of some structural parameters, the bridge models are tuned to have fundamental lateral frequency of 0.75 Hz corresponding to the first coupled lateral-torsional mode (L1T1). In the numerical analysis, Hilber-Hughes-Taylor method [2] is used for the non-linear time history analysis, and it is assumed that the first and second modes (T1L1 and L2T2) have the same damping ratio of 0.01. In this paper, only the resonant lateral vibration at the first coupled lateral-torsional mode (0.75 Hz) is treated as this one half-wave vibration mode is easy to be excited by crowd pedestrians uniformly distributed on the entire bridge deck. The dynamic responses discussed in the following sections are picked up from point A (Fig. 7) of the bridge frame at or close to mid span, as the maximum deflections occur at this location for the one half-wave coupled mode.

4. 1 Resonant vibration

Bridge models A and B with span L = 80m, cable sag = 1.8m and cable diameters of D_i = 90mm, 120mm and 180 mm, were treated. Results for D_i = 180 mm are not presented for want of space. Table 1 shows the main structural parameters and vibration properties. Here the mass density m is obtained by dividing the total structural mass by the span length and deck width, and the tension force T_1, T_2 and T_3 are the maximum tension forces at the end segments of the top supporting cables, reverse profiled bottom and side cables respectively. From this Table, it can be seen that for bridge model A, the cable section has only slight effect on the frequencies of coupled lateral-torsional modes and higher vertical modes. The frequencies of the first coupled torsional-lateral mode T1L1 and fundamental vertical mode V1 increase while frequencies of higher coupled torsional-lateral modes decrease with the increase in cable diameter. For bridge model B, as the natural frequency of the first coupled lateral-torsional mode is kept at 0.75 Hz, all other natural frequencies except the fundamental vertical mode reduce significantly when the cable section (diameter) increases. Only the fundamental vertical frequency increases with the cable section.

It was also found that for bridge model A, the coupling coefficient of the coupled mode L1T1 decreases while the coupling coefficients of other coupled lateral-torsional modes increase when the cable diameter increases. The coupling coefficient of mode T1L1 increases, but those of the other coupled torsional-lateral modes decrease.

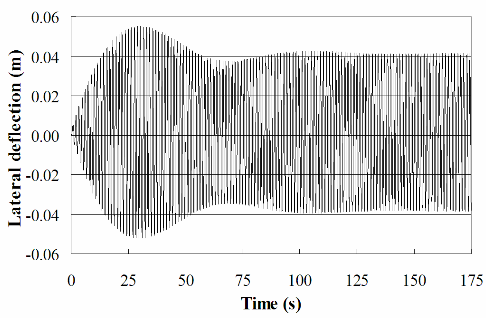


Fig. 9 Resonant lateral deflection

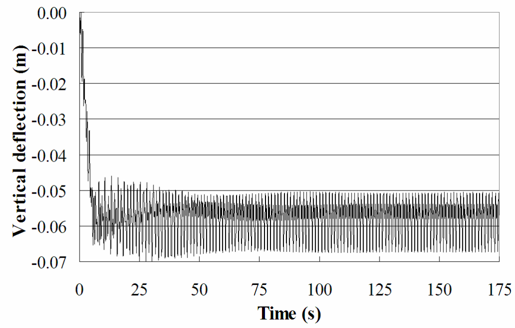


Fig. 10 Resonant vertical deflection

These results indicates that the mode L1T1 changes gradually to become a pure lateral mode and the mode T1L1 changes to be a pure torsional mode when the structural stiffness is improved by increasing the cable sections. However, results differed for the coupling coefficients of the first coupled modes L1T1 and T1L1 for bridge model B. When the cable diameter increases, it was found that the coupling coefficient of the coupled mode L1T1 increases and that of the mode T1L1 decreases. This result illustrates that the degree of coupling increases with increase in cable section.

When pedestrians walk across the footbridge with the pacing rate of 1.5 Hz, the first coupled lateral-torsional mode L1T1 will be excited at the frequency of 0.75 Hz which is half of the pacing rate, and resonant vibration in the lateral direction is expected. Figs. 9 and 10 show typical lateral and vertical dynamic deflections of bridge model A with cable diameters of 120 mm when the bridge resonates under the walking dynamic loads (slow walk) at the pacing rate of 1.5 Hz. It can be seen that the amplitude of the lateral deflection increases to the maximum value, then fluctuates and finally becomes steady after several decays. In the vertical direction, the vibration amplitude is much smaller than the lateral one, though the vibration also trends to be steady after several fluctuations. The vertical vibration is contributed by three loads: static load, vertical dynamic load and the lateral sway of bridge frame under lateral dynamic load, with the static load being most dominant. It is evident that the bridge does not resonate in the vertical direction when subjected to the vertical dynamic load and the maximum vertical dynamic deflection is mainly produced by the resonant lateral sway. The same feature was found in the vibration of bridge models A and B with different cable diameters.

Table 2 shows the statistics of steady state lateral deflections of different footbridge models excited by the crowd pedestrians walking at the pacing rate of 1.5 Hz. Here, the maximum and minimum steady deflections are the maximum and minimum peak values of the steady vibrations within a period of fifteen seconds.

Table 1 Vibration properties of footbridges

Model	A	A	B	B
D_1 (mm)	90	120	90	120
m (kg/m^2)	326.6	363.8	410.6	465.8
T_1 (N)	6324574	6987428	5395303	5536132
T_2 (N)	3399147	3722268	1725983	1356765
T_3 (N)	--	--	1724983	1110712
Modes	Natural frequencies (Hz)			
L1T1	0.7500	0.7500	0.7500	0.7500
L2T2	1.4645	1.4585	1.2549	1.0980
L3T3	2.1727	2.1634	1.8159	1.5602
L4T4	2.8775	2.8656	2.3827	2.0340
L5T5	3.5769	3.5654	2.9567	2.5246
L6T6	4.2665	4.2572	3.5250	3.0111
T1L1	1.1259	1.1949	0.9205	0.8982
T2L2	1.8999	1.8718	1.5770	1.4158
T3L3	2.7618	2.7238	2.2991	2.0593
T4L4	3.6301	3.5793	3.0226	2.7023
V1	0.9853	1.0943	0.8569	0.9062
V2	1.5175	1.5151	1.2945	1.1633
V3	2.2831	2.2866	1.9476	1.7597
V4	3.0263	3.0239	2.5798	2.3203
V5	3.7769	3.7785	3.2187	2.8998
Modes	Coupling coefficient			
L1T1	-0.1374	-0.1092	-0.2829	-0.3372
L2T2	-0.3161	-0.3322	-0.4594	-0.6123
L3T3	-0.3901	-0.4066	-0.5415	-0.6966
T1L1	6.6242	8.1716	2.5706	2.1232
T2L2	2.7916	2.5716	1.4698	1.0620
T3L3	2.3488	2.1244	1.4100	1.0752

The steady dynamic amplitude A_{ulstd} and mean value M_{ulstd} of lateral deflection are calculated based on their maximum value $U_{lstdmax}$ and minimum one $U_{lstdmin}$:

$$A_{ulstd} = \left|(U_{lstd\max} - U_{lstd\min})/2\right| \tag{7}$$

$$M_{ulstd} = (U_{lstd\max} + U_{lstd\min})/2 \tag{8}$$

The dynamic amplification factor (DAF) of lateral deflection DAF_{ulstd} is calculated as:

$$DAF_{ulstd} = \left|A_{ulstd} / U_{lstatic}\right| \tag{9}$$

Here the static deflection $U_{lstatic}$ is caused by the quasi-static lateral force corresponding to the lateral force defined by Eqn. (6c) with the load density equal to the amplitude of lateral dynamic force.

Table 2 Resonant lateral deflections

	Model	A	A	B	B
Cable diameter	D_1 (mm)	90	120	90	120
Static displacement	$U_{lstatic}$ (mm)	1.41	1.26	1.1	0.95
Steady vibration	$U_{lstdmax}$ (mm)	38.40	41.45	24.0	20.14
	$U_{lstdmin}$ (mm)	-35.10	-38.53	-21.5	-18.0
	A_{ulstd} (mm)	36.75	39.99	22.7	19.07
	M_{ulstd} (mm)	1.65	1.46	1.3	1.07
	DAF_{ulstd}	26.0	31.6	20.7	20.1

From Table 2, it can be seen that for bridge model A, the dynamic amplification factor (DAF) and amplitude of the steady lateral deflection increase unexpectedly while the mean value decreases when the cable diameter increases. For the bridge model B, both the amplitude and mean value of the lateral deflection decrease when the cable diameter increases. However, the dynamic amplification factor changes slightly as the cable diameter increases from 90 mm to 120 mm, (and dropped dramatically as the cable diameter increased to 180 mm). Relating the coupling coefficient of the mode L1T1 to the steady DAF of the lateral deflection, it is found that for footbridge with the same cable configuration, the larger the coupling coefficient, the smaller the DAF. This phenomenon illustrates that for slender suspension footbridge structures with coupled vibration modes, the degree of coupling has significant effect on the dynamic response, and sometimes this effect is greater than the effect of static structural stiffness. Influence of cable sag and span length on the bridge vibration were also investigated, but for want of space the results are not presented in this paper.

5. Conclusion

Suspension footbridge is an important and popular structural form of modern footbridges. Modern suspension footbridges are often slender and flexible with low mass and low stiffness. They are prone to excessive multi-modal vibration induced by pedestrians and have risk of suffering serious vibration serviceability problems.

In this paper a suspension footbridge model with reverse profiled cables was proposed to investigate the vibration characteristics of shallow suspension pedestrian bridge structures. It was found that these footbridges often have coupled vibration modes and the coupling can be described by "coupling coefficient" defined by the ratio of vertical component to the horizontal component of displacement. Though this paper treated only the influence of cable section (for want of space), research shows that cable section, cable sag and span length have significant effects on the vibration modes, and the coupling coefficient has great effect on the lateral dynamic amplification factor. For suspension footbridges with only reverse profiled cables in vertical plane (bridge model A), the coupling coefficient of the symmetric half-wave coupled lateral-torsional mode decreases significantly with increase in cable section (and cable sag, but increases with the span length). In suspension footbridges with reverse profiled cables in both vertical and horizontal planes (bridge model B), the variation of the coupling coefficient with the structural parameters is more complicated. The coupling coefficient of the symmetric half-wave coupled lateral-torsional mode increases with increase in cable section, (but decreases with cable sag). The effect of the span length was complex and this was combined with the effect of cable section. Under crowd walking dynamic loads, larger DAF of lateral vibration is expected for the vibration mode with smaller coupling coefficient. This phenomenon indicates that resonant vibration with a pure lateral vibration mode would often have large DAF or DAF of lateral vibration could be reduced by increasing the coupling coefficient.

The information from this conceptual study is useful in understanding the complex behaviour of this type of slender footbridges which exhibit multi-modal and coupled vibration. The findings will be helpful in developing design guidance for safe and efficient performance of these bridges and in developing generic formulations for structures which exhibit such multi-modal vibration.

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