The MINLP optimization in civil engineering

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Abstract: - The paper presents the optimization of structures in civil engineering, performed by the Mixed-Integer Non-linear Programming (MINLP) optimization approach. The MINLP is a combined continuous/discrete optimization technique, where a structural topology and standard sizes are optimized simultaneously with the continuous parameters (e.g. strains, stress, costs, mass, etc.). Discrete binary 0-1 variables are used to express the discrete decisions. For the solution of this non-linear, continuous/discrete and non-convex MINLP class of the optimization problem, the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm is used and a two-phase MINLP strategy is applied. The optimization is performed by a user-friendly version of the MINLP computer package MIPSYN. Two examples are presented at the end of the paper.

Key-Words: - Mixed-Integer Non-linear programming, MINLP, Topology optimization, Discrete sizing optimization

1 Introduction

The paper presents the optimization of structures in civil engineering, performed by the Mixed-Integer Non-linear Programming approach (MINLP). The MINLP is a combined continuous/discrete optimization technique. It handles with continuous and discrete binary 0-1 variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (dimensions, stresses, strains, mass, costs, etc.), discrete variables are used to express discrete decisions, i.e. usually the existence or non-existence of structural elements inside the defined structure. Different standard sizes may also be defined as discrete alternatives. Since the MINLP performs continuous and discrete optimizations simultaneously, the MINLP approach also finds optimal continuous parameters (mass, costs, stresses, etc.), a structural topology (an optimal number and a configuration of structural elements) and discrete standard sizes simultaneously.

The MINLP optimization approach is proposed to be performed through three steps. The first one includes the generation of a mechanical superstructure of different topology and standard dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The MINLP continuous/discrete optimization problems of structural optimization are in most cases comprehensive, non-convex and highly non-linear. The Outer-Approximation/Equality-Relaxation (OA/ER) algorithm [1], [2] is thus used. A two-phase MINLP optimization is proposed to accelerate the convergence of the mentioned algorithm. The optimizations are carried out by an MINLP computer package MIPSYN, the successor of PROSYN [1] and TOP [2-4].

Two examples are presented at the end of the paper. The first one shows the topology and discrete/standard sizing optimization of a simply supported steel truss and the second one presents the topology and standard sizing optimization of a single-storey industrial steel building.

2 Mechanical superstructure

The MINLP optimization approach requires the generation of an MINLP mechanical superstructure composed of various topology and design alternatives that are all candidates for a feasible and optimal solution. While topology alternatives represent different selections and interconnections of corresponding structural elements, design alternatives include different standard dimensions.

The superstructure is typically described by means of unit representation: i.e. structural elements and their interconnection nodes. Each potential topology alternative is represented by a special number and a configuration of selected structural elements and their interconnections; each structural element may in addition have different standard dimension alternatives.

The main goal is thus to find within the given superstructure a feasible structure that is optimal with respect to the topology and standard dimensions and all defined continuous parameters.

3 MINLP model formulation

It is assumed that a general non-linear and non-convex continuous/discrete optimization problem can be formulated as an MINLP problem in the form:

$$\min \quad z = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{y} + f(\boldsymbol{x})$$

$$\begin{aligned} &\text{s.t.:} \\ &\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ &\boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0} \qquad\qquad (\textbf{MINLP}) \\ &\boldsymbol{By} + \boldsymbol{Cx} \leq \boldsymbol{b} \end{aligned}$$

$$\boldsymbol{x} \in X = \{\boldsymbol{x} \in R^{\mathrm{n}}: \boldsymbol{x}^{\mathrm{lo}} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{up}}\}$$
$$\boldsymbol{y} \in Y = \{0,1\}^{\mathrm{m}}$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X and $\boldsymbol{y}$ is a vector of discrete, binary 0-1 variables. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are non-linear functions involved in the objective function z, equality and inequality constraints, respectively. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable. Finally, $\boldsymbol{By}+\boldsymbol{Cx}\leq\boldsymbol{b}$ represents a subset of mixed linear equality/inequality constraints.

The above general MINLP model formulation has been adapted for structural optimization. It is postulated that it helps us construct an MINLP mathematical optimization model for any structure.

In the context of structural optimization, continuous variables $\boldsymbol{x}$ define structural parameters (dimensions, strains, stresses, costs, mass...) and binary variables $\boldsymbol{y}$ represent the potential existence of structural elements within the defined superstructure. An extra binary variable y is assigned to each structural element. The element is then selected to compose the structure if its subjected binary variable takes value one (y=1), otherwise it is rejected (y=0). Binary variables also define the choice of discrete/standard sizes.

The economical (or mass) objective function z involves fixed costs (mass) in the term $\boldsymbol{c}^T\boldsymbol{y}$, while the dimension dependant costs (mass) are included in the function $f(\boldsymbol{x})$. Non-linear equality and inequality constraints $\boldsymbol{h}(\boldsymbol{x})=\boldsymbol{0}$, $\boldsymbol{g}(\boldsymbol{x})\leq\boldsymbol{0}$ and the bounds of the continuous variables represent the rigorous system of the design, loading, resistance, stress, deflection, etc. constraints known from the structural analysis. Logical constraints that must be fulfilled for discrete decisions and structure configurations, which are selected from within the superstructure, are given by $\boldsymbol{By}+\boldsymbol{Cx}\leq\boldsymbol{b}$. These constraints describe relations between binary variables and define the structure's topology and standard dimensions. It should be noted, that the comprehensive MINLP model formulation for mechanical structures may be found elsewhere [3, 5].

4 Optimization

After the MINLP model formulation is developed, the defined MINLP optimization problem is solved by the use of a suitable MINLP algorithm and strategies. A general MINLP class of optimization problem can be solved in principle by the following algorithms and their extensions:

-the *Nonlinear Branch and Bound*, NBB, proposed and used by many authors, e.g. E.M.L. Beale [6], O.K. Gupta and A. Ravindran [7];

-the *Sequential Linear Discrete Programming* method, SLDP, by G.R. Olsen and G.N. Vanderplaats [8] and M. Bremicker et al. [9];

-the *Extended Cutting Plane* method by T. Westerlund and F. Pettersson [10];

-*Generalized Benders Decomposition*, GBD, by J.F. Benders [11], A.M. Geoffrion [12];

-the *Outer-Approximation/ Equality-Relaxation algorithm*, OA/ER, by G.R. Kocis and I.E. Grossmann [13];

-the *Feasibility Technique* by H. Mawengkang and B.A. Murtagh [14]; and

-the *LP/NLP based Branch and Bound* algorithm by I. Quesada and I.E. Grossmann [15].

4.1 Modified OA/ER algorithm

The OA/ER algorithm consists of solving an alternative sequence of Non-linear Programming (NLP) optimization subproblems and Mixed-Integer Linear Programming (MILP) master problems. The former corresponds to continuous optimization of parameters for a mechanical structure with a fixed topology (and fixed discrete/standard dimensions) and yields an upper bound to the objective to be minimized. The latter involves a global approximation to the superstructure of alternatives in which a new topology, discrete/standard dimensions are identified so that its lower bound does not exceed the current best upper bound. The search of a convex problem is terminated when the predicted lower bound exceeds the upper bound, otherwise it is terminated when the NLP solution can be improved no more. The OA/ER algorithm guarantees the global optimality of solutions for convex and quasi-convex optimization problems.

The OA/ER algorithm as well as all other mentioned MINLP algorithms do not generally guarantee that the solution found is the global optimum. This is due to the presence of nonconvex functions in the models that may cut off the global optimum. In order to reduce undesirable effects of nonconvexities, the Modified OA/ER algorithm was proposed by Z. Kravanja and I.E. Grossmann [1], see also S. Kravanja et al. [2], by which the following modifications are applied for the master problem: the deactivation of linearizations, the decomposition and the deactivation of the objective function linearization, the use of the penalty function, the use of the upper bound on the objective function to be minimized as well as the global convexity test and the validation of the outer approximations.

4.2 Two-phase MINLP optimization

The optimal solution of a complex, non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to obtain. The optimization is thus proposed to be performed sequentially in two different phases to accelerate the convergence of the OA/ER algorithm. The optimization starts with the topology optimization of a structure, while discrete sizes are relaxed temporary into continuous parameters. When the optimal topology is found, standard sizes are in the second phase re-established and the discrete dimension optimization of the structure is then continued until the optimal solution is found.

5 Computer package MIPSYN

The optimization was carried out by a user-friendly version of the MINLP computer package MIPSYN, the successor of PROSYN [1] and TOP [2-4, 16]. MIPSYN is the implementation of many advanced optimization techniques, where the most important are the Modified OA/ER algorithm and the MINLP strategies. In terms of complexity, the MIPSYN's synthesis problems can range from a simple NLP optimization problem of a single structure up to the MINLP optimization of a complex superstructure problem. MIPSYN runs automatically or in an interactive mode and thus provides the user with a good control and supervision of the calculations. GAMS/CONOPT2 (Generalized reduced-gradient method) [17] is used to solve the NLP subproblems and GAMS/Cplex 7.0 (Branch and Bound) [18] is used to solve the MILP master problems.

5.1 Optimization models

For each type of structure, a special optimization model must be developed. Each model is constructed on the basis of the mentioned general MINLP-G model formulation. As an interface for mathematical modelling and data inputs/outputs GAMS (General Algebraic Modelling System), a high level language, is used [19].

6 Numerical examples

MINLP optimization approach is illustrated by two examples. The first one presents the simultaneous topology and discrete/standard sizing optimization of a simply supported truss girder and the second one shows the topology and standard sizing optimization of a single-storey industrial building.

6.1 Topology, shape and discrete/standard sizing optimization of a 40 m long steel truss

The first example introduces the simultaneous topology, shape and discrete/standard sizing optimization of a simply supported truss girder over the span of 40 m. The proposed superstructure includes 18 nodes and 41 alternative elements. The truss is subjected to a design uniformly distributed load q_{Sd} of 30 kN/m, acting on the bottom chord. The uniform load is approximated to the nodal forces acting on the joints of the bottom chord. Truss elements are designed from standard circular hollow sections made of S 235 steel. The input data also include Young's modulus E = 210 GPa and the unit mass of steel material 7850 kg/m^3.

The MINLP optimization model STEELTRUSS was developed for the optimization. The objective function of the structure's mass was subjected to the set of constraints known from structural analysis. The finite element equations were defined for the calculation of internal forces and displacements. Constraints for the dimensionig were defined in accordance with Eurocode 3 [20]. Buckling lengths of the truss elements are considered as being equal to the system lengths of the elements for both in-plane and out-of-plane buckling. The vertical coordinates y_i of top chord joints are defined as shape variables with their lower/upper bounds $y_i^{\mathrm{LO}}/\,y_i^{\mathrm{UP}}$ = 200/2000 cm.

The vectors of discrete/standard alternative values for the diameter d and wall thickness t of cross-sections are given as follows:

$\boldsymbol{d}$ = {42.4, 48.3, 60.3, 76.1, 88.9, 108.0, 114.3, 133.0, 139.7, 159.0, 168.3, 177.8, 193.7, 219.1, 244.5 273.0} [mm] and

$\boldsymbol{t}$ = {2.0, 2.9, 3.2, 4.0, 5.0, 6.3, 7.1, 8.0, 10.0, 12.5, 14.2, 16.0} [mm].

With respect to the available standard cross sections the lower and upper bound on wall thickness for each section diameter are defined. In this way, 76 alternative standard cross-sections are defined, which can be attributed to each element of the truss. The cross sections of the chords are forced into being constant through the entire span. Since the loading of the defined truss is symmetric, symmetry of topology with respect to the vertical axis through the midspan of the structure is required. The MINLP optimization of the structure's mass was performed by the computer package MIPSYN (GAMS/CONOPT2 and GAMS/Cplex 7.0). The Modified OA/ER algorithm and the two-phase MINLP optimization were applied. The optimization model contained 885 mainly nonlinear (in)equality constraints, 1634 continuous and 1189 discrete/binary variables. Fig. 1 shows the calculated optimal truss. The optimal mass yields 1925.549 kg, obtained at the 5th main MINLP iteration. The obtained optimal shape variables amout to: $y_4 = y_{16}$=659.02 cm, $y_6 = y_{14}$= 912.92 cm, $y_8 = y_{12}$=1032.06 cm and y_{10}=1031.29 cm. The calculated optimal standard cross-section dimensions are listed in Table 1.

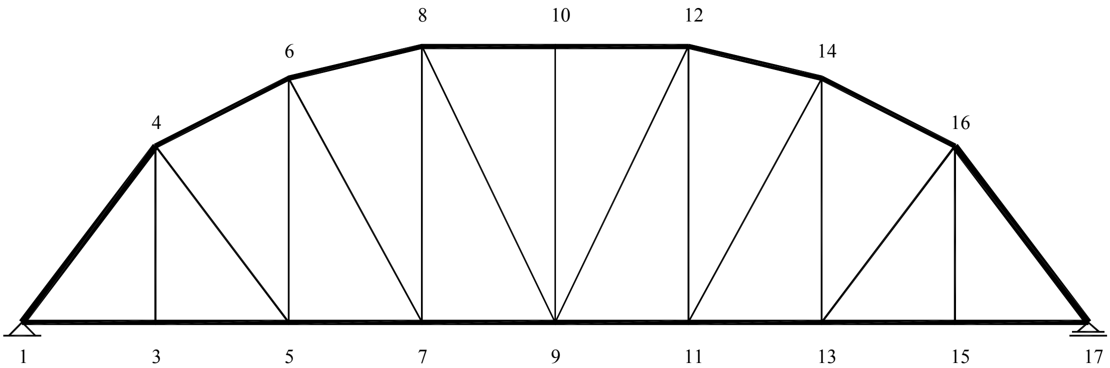

Fig. 1: Optimal steel truss

Elemet	Diameter d [mm]	Wall thickness t [mm]
Bottom chord (1-17)	177.8	4.0
Top chord (4-16)	177.8	4.0
1-4, 16-17	244.5	4.0
3-4, 4-5, 13-16, 15-16	60.3	2.9
5-6, 6-7, 7-8, 8-9, 9-10, 9-12, 11-12, 11,14, 13,14	42.4	2.0

Table 1: Optimal standard cross-sections

6.2 Topology and standard sizing optimization of an industrial building

The second example presents the topology and standard dimension optimization of a single-storey industrial building. The building is 22 meters wide, 38 meters long and 5.5 meters heigh. The structure is consisted from equal non-sway steel portal frames which are mutually connected with the purlins.

The portal frame is subjected to self-weight g, uniformly distributed surface variable load q (snow s and wind w), concentrated variable load P (wind F and initial frame imperfection $F\varphi$). Variable imposed load s = 2,0 kN/m^2 (snow), w_v = 0,125 kN/m^2 (vertical wind) and w_h = 0,5 kN/m^2 (total horizontal wind) are defined as the uniformly distributed surface load in the model input data. Both, the horizontal concentrated load at the top of the columns and the vertical uniformly distributed line load on the beams are calculated considering the intermediate distance betwen the portal frames.

Design/dimensioning was performed in accordance with Eurocode 3 [20] for the conditions of both the ultimate and serviceability limit states. While internal forces were calculated by the elastic first-order analysis, the deformation of the frame members were calculated by the force method.

The portal frame superstructure was generated in which all possible structures were embended by 30 portal alternatives, 20 purlin alternatives and different standard size variation. The superstructure also comprised 24 different standard hot rolled European wide flange I beams, i.e. HEA sections (from HEA 100 to HEA 1000) for each column, beam and purlin seperately. The material used was steel S 355.

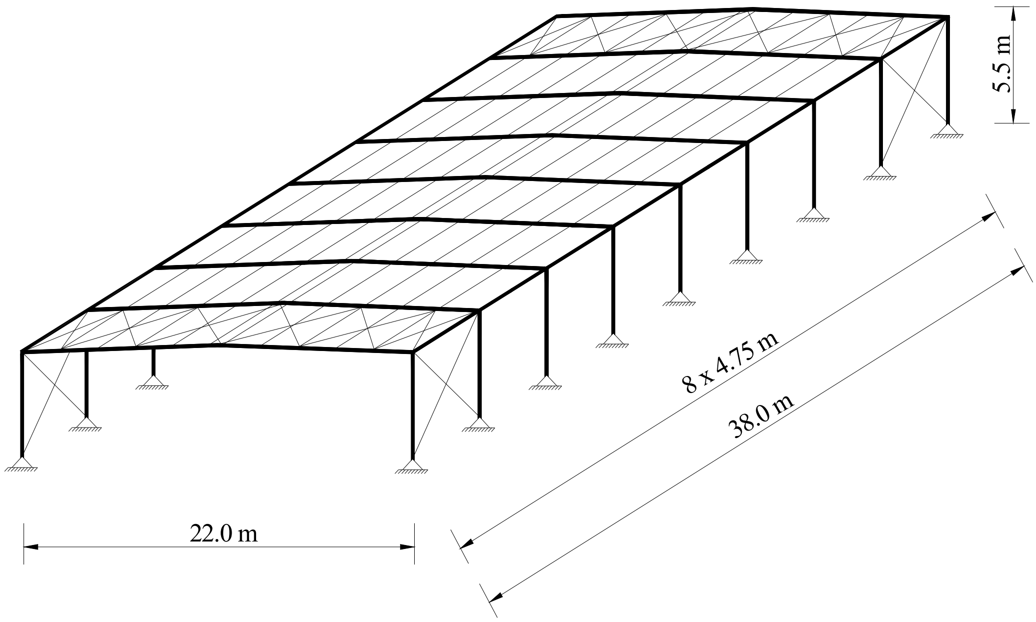

Fig. 2: Optimal topology of the single-storey industrial building

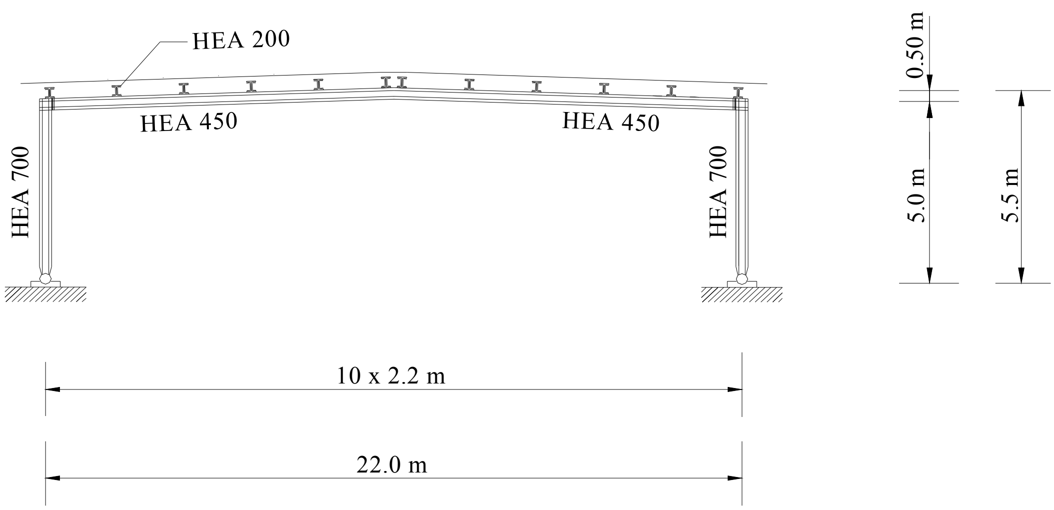

Fig. 3: Optimal steel sections

The optimization was carried out by the user-friendly version of the MINLP computer package MIPSYN. As an interface for mathematical modelling and data inputs/outputs GAMS (General Algebraic Modelling System), a high level language, was used. The Modified OA/ER algorithm and the two-phased optimization were applied, where GAMS/CONOPT2 (Generalized reduced-gradient method) was used to solve NLP subproblems and GAMS/Cplex 7.0 (Branch and Bound) was used to solve MILP master problems.

The optimization model contained 120 (in)equality constraints, 173 continuous and 122 binary variables. The final optimal solution of 65,32 tons was obtained in the 5th main MINLP iteration.

The optimal result represents the mentioned optimal structure mass of 65,32 tons, the obtained optimal topology of 9 portal frames an 12 purlins, see Fig. 2 and the calculated optimal standard sizes of columns, beams and purlins, see Fig. 3.

7 Conclusion

The paper presents the Mixed-Integer Non-linear Programming approach (MINLP) to structural optimization. The Modified OA/ER algorithm and the two-phase MINLP optimization strategy were applied. The optimization is performed by a user-friendly version of the MINLP computer package MIPSYN. Beside the optimal structure costs or mass, the optimal topology with the optimal number of structural elements, the optimal discrete/standard cross-sectional sizes can be obtained simultaneously. Two examples, presented at the end of the paper, clearly show the efficiency of the proposed MINLP approach.

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