Analysis of composite T beam composed of timber, concrete and carbon strip

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Abstract: This paper provides a mathematical model and numerical example of composite T-section composed of a concrete plate and a timber beam strengthened at the bottom tension side with a carbon fibre-reinforced polymer (CFRP) strip. Analysis is provided in accordance with the european standards for timber, steel and concrete structures. The tensile strength of the carbon strip as well as the compressive strength of the concrete plate are higher than the bending strength of the timber beam, therefore it is convenient to use such composition of material to gain a higher load bearing capacity. Furthermore, the CFRP strip's contribution to the bending resistance and stiffness of the element is presented as a function of the fastener's spacing.

Key-Words: Composite structures, Timber structures, Carbon strip, Load bearing capacity, Modeling

1 Introduction

Nowadays the protection and reconstruction of old and historic buildings have increasing significance in modern planning. Therefore has drawn attention an efficient type of floor system which consists of timber members in the tensile zone, a thin concrete layer in compression zone and the connection between timber and concrete. The results of such reconstruction-strengthening procedure or new composite construction (compared to timber floors) are an increase of stiffness and load bearing capacity, an improved sound insulation and a better fire resistance.

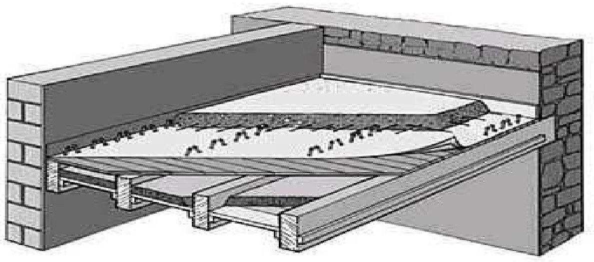


Fig. 1: Example of reconstruction of timber floor.

The structural behavior of timber-concrete composite members is governed by the shear connection between them. But nevertheless which system of fasteners or connectors we use the usage of steel fibre reinforced concrete (SFRC) shows much better characteristic then classic reinforced concrete. Holschemacher, Klotz and Weibe (2002) demonstrated their experimental studies using SFRC [9]. Ultimate load carrying capacity of fastener is 27% higher and initial slip modulus is 180% higher against classic reinforced concrete.

As the tensile strength of timber is usually not much lower than the compressive strength, the applications of fibre reinforced polymers (FRP) or carbon fibre reinforced polymers (CFRP) in timber have not been frequent as in masonry or especially in concrete structures. The potential of FRP in combination with steel and timber structures has only been explored recently. The main advantages of using FRP in particular compared to other materials (for example steel plates) are their corrosion resistance, light weight and flexibility, which allow convenient and easy transport to the place of errection.

The availability of advanced composite materials has stimulated much interest in reinforcement of timber elements, especially on glued laminated beams. Timber is an uncommon material for critical highway bridge structures, though several applications of strengthening using FRP and CFRP to gain higher ductility and bending resistance can be found in this field. Dagher and Breton (1998) reinforced laminated timber beams in the tensile area using FRP lamellas. The test results showed an essential increase in bending resistance. Stevens and Criner (2000) conducted an economic analysis of FRP glulam beams. The results showed practical applicability of FRP reinforced elements, especially for bridges of greater spans, where beam dimensions can be substantially reduced using the presented FRP solution. The test results using carbon fibers in laminated beams are presented in Bergmeister and Luggin (2001).

Use of HSF and CFRP for the repair and strengthening of timber elements opens new perspectives for timber structures design. Continuously decreasing prices of these materials make the new technology more economical and interesting. On the other hand, applying composite fibres to timber structures requires experience and higher quality of workmanship than traditional reinforcements.

2 Analytic solution for derivation of load carrying capacity and stiffness

For design purposes a simplified design method for mechanically jointed elements according to Annex B of Eurocode 5 [3] is used. Expression of the so called »γ-method« is used in equations with the following fundamental assumptions:

a) Bernoulli's hypothesis is valid for each sub-component,
b) material behavior of all sub-components is linear elastic,
c) the distances between the fasteners are constant along the beam,
d) slip modulus is taken in plastic area for ultimate limit state or elastic area for serviceability limit state
e) bending moment varying sinusoidally or parabolically

2.1 Basic determinations

Determination of the neutral axe of the composed section is analogical to Eurocode 5 [3] for mechanically jointed elements if we are proceed from center of gravity of timber beam:

$$z_0 = \frac{\sum S_{y,i}^{\gamma_{ct}}}{\sum A_i} = \frac{\left(n_f \cdot A_f \cdot \left(h_t + h_f\right) - n_c \cdot \gamma_{y,ct} \cdot A_c \cdot \left(h_t + h_c\right)\right)}{2 \cdot \left(n_c \cdot \gamma_{ct} \cdot A_c + A_t + n_f \cdot A_f\right)} \tag{1}$$

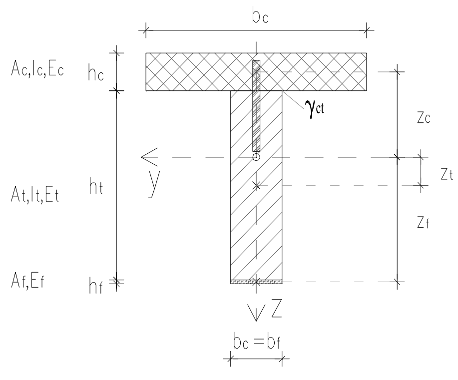


Fig. 2: Composite cross-section.

Where relations between modulus of elasticity are:

$$n_c = \frac{E_c}{E_t} \text{ for concrete; and } n_f = \frac{E_f}{E_t} \text{ for CFRP} \tag{2}$$

There on we can obtain distances from main neutral axe to center of gravity of each sub-component as written below:

$$z_c = \left(-\frac{h_t}{2} - z_0\right) - \frac{h_c}{2} \quad \text{for concrete slab} \tag{3}$$

$$z_f = \left(\frac{h_t}{2} - z_0\right) + \frac{h_f}{2} \quad \text{for carbon strip} \tag{4}$$

$$z_t = -z_0 \quad \text{for timber beam} \tag{5}$$

In upper forms the stiffness coefficient of the fasteners in plane between concrete and timber ($\gamma_{y,ct}$) can be defined using Eurocode 5 [3] in the form of:

$$\gamma_{y,ct} = \frac{1}{1+k}; \quad k_y = \frac{\pi^2 \cdot E_{cm} \cdot A_c \cdot s_i}{K \cdot l_{eff}^{\;2}} \tag{6}$$

The stiffness coefficient in plane between carbon strip and timber which are glued together is considered 1.0.
Upper value of the modulus (K) is taken by Eurocode 5 [3] for dowels type of fasteners where for concrete to timber connections K_{ser} should be based on ρ_m for timber member and may be multiplied by 2,0. So the final form for K is:

$$K_{ser} = 2.0 \cdot \frac{\rho_m^{1.5} \cdot d}{23} \quad \text{for serviceability limit state} \tag{7}$$

$$K_u = \frac{2}{3} K_{ser} \quad \text{for ultimate limit state} \tag{8}$$

The effective bending stiffness $(EI_y)_{ef}$ of mechanically jointed elements taken from Eurocode 5 [3], can be analogical written in the form of:

$$\left(EI_y\right)_{ef} = E_c \cdot \left(\frac{h_c^3 \cdot b_c}{12} + \gamma_{y,ct} \cdot A_c \cdot z_c^2\right) + E_t \cdot \left(\frac{h_t^3 \cdot b_t}{12} + A_t \cdot z_t^2\right) + E_f \cdot \left(A_f \cdot z_f^2\right) \tag{9}$$

With use of relations between modulus of elasticity from equations (2) we get the effective bending stiffness as:

$$E_t \cdot I_{y,ef} = E_t \cdot \left(\begin{array}{l} n_c \cdot \left(\dfrac{h_c^3 \cdot b_c}{12} + \gamma_{y,ct} \cdot A_c \cdot z_c^2\right) + \\ \left(\dfrac{h_t^3 \cdot b_t}{12} + A_t \cdot z_t^2\right) + n_f \cdot \left(A_f \cdot z_f^2\right) \end{array}\right) \tag{10}$$

2.2 Bending bearing capacity

Normal stresses in composite section for each of sub-components are defined in form of:

$$\sigma = \frac{M_y}{W_{y,ef}} = \frac{M_y \cdot E_i}{\left(EI_y\right)_{ef}} \cdot \left(\gamma_{y,i} \cdot z_i \pm \Delta z_i\right) \tag{11}$$

With usage of equation (10) and (11) we obtain normal stresses in the edges of the concrete slab in form of:

$$\sigma_c = \frac{M_y \cdot n_c}{I_{y,ef}} \cdot \left(\gamma_{y,ct} \cdot z_c \pm \frac{h_c}{2} \right) \leq f_{c,k} \tag{12}$$

Where $f_{i,k}$ represent characteristic strength of sub-component material. For normal stresses in the edge fibres of the timber beam in form of:

$$\sigma_t = \frac{M_y}{I_{y,ef}} \cdot \left(z_t \pm \frac{h_t}{2} \right) \leq f_{m,k} \tag{13}$$

For normal stresses in the edge fibres of CFRP:

$$\sigma_f = \frac{M_y \cdot n_f}{I_{y,ef}} \cdot \left(z_f \pm \frac{h_f}{2} \right) \leq f_{f,k} \qquad \text{or} \tag{14}$$

$$* \sigma_f \doteq \sigma_t \cdot n_f \leq f_{f,k}$$

*simplified variant, because of small thickness of strip

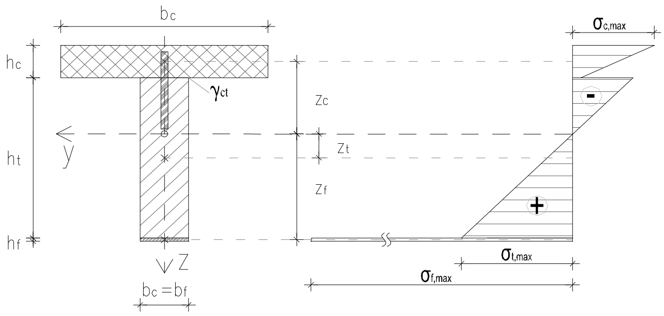


Fig. 3: Flow of normal stress in cross-section.

According with equations (12) - (14) design bending moment (design bending capacity) can be evaluated:

$$M_{y,d} = \sigma_d \cdot W_{y,ef} = f_{i,d} \cdot \frac{I_{y,ef}}{n_i \cdot \left(\gamma y_{,i} \cdot z_i \pm \Delta z_i \right)} \tag{15}$$

For concrete slab we obtain equation (16):

$$M_{y,d,c} = \frac{\alpha}{\gamma_c} \cdot M_{y,k,c} = \frac{\alpha}{\gamma_c} \cdot \left(f_{c,k} \cdot \frac{I_{y,ef}}{n_c \cdot \left(\gamma_{y,ct} \cdot z_c - \frac{h_c}{2} \right)} \right) \tag{16}$$

For timber beam we obtain equation (17):

$$M_{y,d,t} = \frac{k_{\text{mod}}}{\gamma_m} \cdot M_{y,k} = \frac{k_{\text{mod}}}{\gamma_m} \cdot \left(f_{m,y,k} \cdot \frac{I_{y,ef}}{z_t + \frac{h_t}{2}} \right) \tag{17}$$

In case when the normal stress in the center of timber beam is high we must use the equation (18) and that will be only in case if equation (19) is fulfilled:

$$M_{y,d,t} = \frac{k_{\text{mod}}}{\gamma_m} \cdot M_{y,k} = \frac{k_{\text{mod}}}{\gamma_m} \cdot \left(f_{t,0,k} \cdot \frac{I_{y,ef}}{z_t} \right) \tag{18}$$

$$1 + \frac{h_t}{2 \cdot z_t} > \frac{f_{m,k}}{f_{t,0,k}} \tag{19}$$

For carbon strip we obtain equation (18):

$$M_{y,d,f} = \frac{1}{\gamma_f} \cdot M_{y,k,f} = \frac{1}{\gamma_f} \cdot \left(f_{f,k} \cdot \frac{I_{y,ef}}{n_f \cdot \left(z_f + \frac{h_f}{2} \right)} \right) \tag{20}$$

2.3 Shear bearing capacity

Shear stresses in section for any fiber is given as:

$$\tau = \frac{V \cdot S_i \cdot E_i}{\left(EI_y \right)_{ef} \cdot b_i} \qquad \tau^{i-i} = \frac{V_z \cdot \sum S_{y,i}^{(\gamma)} \cdot n_i}{I_{y,ef} \cdot b_i} \tag{21}$$

With usage of equation (21) we obtain shear stress in the bottom edge of concrete slab in form of:

$$\tau_c^{\max} = \frac{V_z \cdot n_c \cdot \left(\gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| \right)}{I_{y,ef} \cdot b_c} \leq \tau_{R,k,1} \tag{22}$$

For maximal shear stress in neutral axis of the cross section:

$$\tau_t^{\max} = \frac{V_z \cdot \left(n_c \cdot \gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| + b_t \cdot \left(\frac{h_t}{2} - z_t \right) \cdot \left(\frac{h_t}{4} - \frac{z_t}{2} \right) \right)}{I_{y,ef} \cdot b_t} \leq f_{v,k} \tag{23}$$

For shear stress in upper fibres of CFRP:

$$\tau_f = \frac{V_z \cdot S_{y,i}^{(\gamma)} \cdot n_i}{I_{y,ef} \cdot b_t} = \frac{V_z \cdot \left(n_f \cdot \left(h_f \cdot b_f \right) \cdot z_f \right)}{I_{y,ef} \cdot b_f} \leq \tau_{Rf,k} \tag{24}$$

Where $f_{v,k}$, and $\tau_{i,k}$ represent characteristic strength of sub-component material accordingly to Eurocode 5 [3]. For concrete plate accordingly to Eurocode 2 [4] as:

$$\tau_{R,k,1} = \tau_{R,k} \cdot k \cdot \left(1.2 + 40 \cdot \rho_1 \right) \tag{25}$$

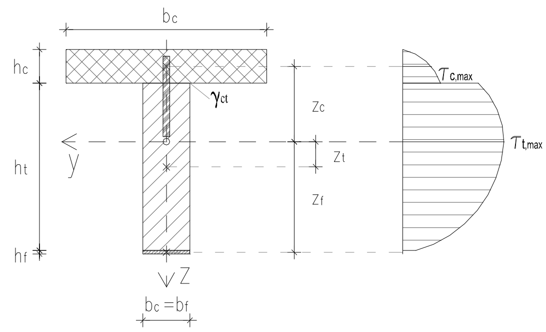


Fig. 4: Flow of shear stress in cross-section.

According with equations (22) - (24) design shear force (design shear capacity) can be evaluated:

$$V_{z,d,i} = f_{i,d} \cdot \frac{I_{y,ef} \cdot b_i}{\sum S_{y,i}^{(\gamma)} \cdot n_i} \tag{26}$$

For concrete slab we obtain equation (27):

$$V_{z,d,c} = \frac{1}{\gamma_c} \cdot V_{z,k,c} = \frac{1}{\gamma_c} \cdot \left(\tau_{R,k,1} \cdot \frac{I_{y,ef} \cdot b_c}{n_c \cdot \left(\gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| \right)} \right) \tag{27}$$

For timber beam we obtain equation (28):

$$V_{z,d,t} = \frac{k_{\text{mod}}}{\gamma_m} \cdot V_{z,k,t} =$$

$$\frac{k_{\text{mod}}}{\gamma_m} \cdot \left(f_{v,k} \cdot \frac{I_{y,ef} \cdot b_t}{\left(n_c \cdot \gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| + b_t \cdot \left(\frac{h_t}{2} - z_t \right) \cdot \left(\frac{h_t}{4} - \frac{z_t}{2} \right) \right)} \right) \tag{28}$$

2.4 Fastener bearing capacity

On this point we must also considered construction parameters given by Eurocode 5 [3] like minimum spacings and edge and end distances for different types of fasteners. Force on one fastener from equation (21) in function of distances between fasteners (s_i) is given as:

$$F_i = \frac{V_z \cdot S_i \cdot E_i}{\left(EI_y \right)_{ef}} \cdot s_i = \frac{V_z \cdot \sum S_{y,i}^{(\gamma)} \cdot n_i}{I_{y,ef}} \cdot s_i \tag{29}$$

In our case we obtain design force on one fastener for connection between concrete slab and timber beam in form of (see Fig. 4):

$$F_{v,d} = \frac{V_{z,d} \cdot n_c \cdot \left(\gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| \right)}{I_{y,ef}} \cdot s_i \leq F_{v,Rd} \tag{30}$$

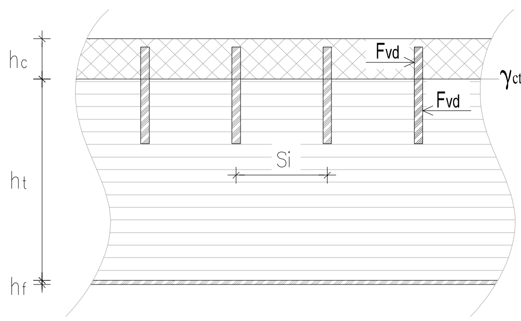


Fig. 5: Fasteners and forces on it.

Consequently design shear force is given as:

$$V_{z,d} = \frac{k_{\text{mod}}}{\gamma_m} \cdot V_{z,k} = \frac{k_{\text{mod}}}{\gamma_m} \cdot \left(\frac{F_{v,Rk} \cdot I_{y,ef}}{n_c \cdot \left(\gamma_{y,ct} \cdot \left(h_c \cdot b_c \right) \cdot \left| z_c \right| \right) \cdot s_i} \right) \tag{31}$$

Where the competent F_{vRk} is given in Eurocode 5 part 1-1 [3] characteristic load carrying capacity per shear plane per fastener (according to the Johansen yield theory) and increased for 20% according to Eurocode 5 part 2 [5]. The load carrying capacity of fastener can be additionally controlled according to Eurocode 4 [6].

2.5 Bending stiffness for serviceability limit state

In these section we represent determination of bending stiffness for composite section that is needed for determination deflections and vibrations according to Eurocode 5 [3] and Eurocode 2 [4].

2.5.1 Bending stiffness at t = 0

The equations (2) - (5) are the same as written, equations (1), (8) and (10) are modified with $\gamma_{y,ct}$ where usage of K_{ser} instead of K_u is only difference in equation (6).

2.5.2 Bending stiffness at t = ∞

The time dependent effects (domination of creep) we can associated with the modulus of elasticity of each material of sub-component. In equations (2) we use modified modulus of elasticity according to Eurocode 5 [3] and Eurocode 2 [4] as:

$$E_{t,fin} = \frac{E_t}{1 + \Psi_2 \cdot k_{def}} \quad \text{for timber sub-section} \tag{32}$$

$$E_{c,eff} = \frac{E_c}{1 + \phi_{(\infty,t_0)}} \quad \text{for concrete sub-section} \tag{33}$$

Time dependent effects present in connection are associated with modulus K_{ser} as (modified) equation from Eurocode 5 [3]:

$$K_{ser,fin} = \frac{K_{ser}}{1 + \left(\frac{\Psi_2 \cdot k_{def} + \phi_{(\infty,t_0)}}{2} \right)} \tag{34}$$

That have direct influence on ($\gamma_{y,ct}$) equation (6) and consequently on equation (1) and on bending stiffness given in equation (9) – (10).

There are still some uncertainty about suitable modeling time dependent effects on composition of different materials together.

3 Numerical example

3.1 Geometrical and material properties

Numerical analysis is performed for composite T section of actual dimensions shown on (Fig.6). For fasteners between concrete plate and timber beam we use dowels according to Eurocode 5 [3] and Eurocode 3 [8] of *Φ20 mm* and length $l = 24\ cm$ at an constant spacing of $s = 10\ cm$. Dowel (bolt) grade that we use according to Eurocode 3 [2] is 8.8 ($f_{yb} = 640N/mm^2$, $f_{ub} = 800N/mm^2$).

Material properties for the timber quality GL24h are taken from EN 1194 [7], for the concrete slab from Eurocode 2 [4] and the CFRP – SikaWrap-230C from [11]. For all materials safety factors we use appropriate European standards. All material properties are listed in Table 1.

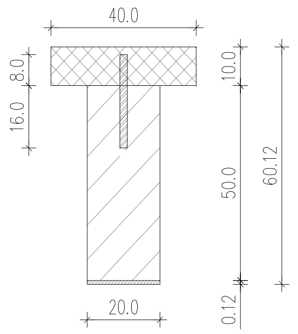


Fig. 6: Cross-section (dimension in cm).

	Concrete C30/37	Timber GL24h	SikaWrap 230C
$\mathbf{E_{0,m}}$ [kN/cm^2]	3193.9	1160	23100
$\mathbf{f_{m,k}}$ [kN/cm^2]	/	2.444	/
$\mathbf{f_{t,0,k}}$ [kN/cm^2]	0.2028	1.65	410.0
$\mathbf{f_{c,0,k}}$ [kN/cm^2]	3.0	2.4	/
$\boldsymbol{\rho_k}$ [kg/m^3]	/	380	/
$\boldsymbol{\rho_m}$ [kg/m^3]	2400	456	183.33

Table 1: Properties of used materials.

For load we predicted 50% of permanent and 50% of long term (storage) action. On that assumption we can determine proper safety factors for ultimate limit state and modification factors for long term effects for serviceability limit state. We also predict effective length of beam ($l_{eff} = 800cm$) for determination of the stiffness coefficient.

3.2 Results of numerical analysis

	ULS	SLS t = 0	SLS t =∞	Units
K	112.90	169.35	64.946	kN/cm
γ_{ct}	0.3643	0.4622	0.2479	/
$\mathbf{n_c}$ $\mathbf{n_f}$	2.753 19.914	2.753 19.914	1.091 29.472	/
$\mathbf{z_o}$	-7.48	-9.040	-1.250	cm
$\mathbf{z_c}$	-22.52	-20.960	-28.750	cm
$\mathbf{z_f}$	32.54	34.100	26.310	cm
$\mathbf{z_t}$	7.48	9.040	1.250	cm
$\mathbf{I_{y,ef}}$	$5.2755\cdot10^5$	$5.7846\cdot10^5$	$3.5192\cdot10^5$	cm^4
$\mathbf{(EI)_{y,ef}}$	61194.61	67101.18	27583.40	kNm^2
$\mathbf{M_{y,d,t}}$	222.31	bending capacity		kNm
$\mathbf{V_{z,d,t}}$	131.79	shear capacity		kN
$\mathbf{V_{z,d}}$	89.51	accordingly to fasteners		kN

Table 2: Results of numerical analysis.

Corresponding normal stresses as consequence of My,d,t are given with equations (11) to (14) and they are represented on Fig. 7.

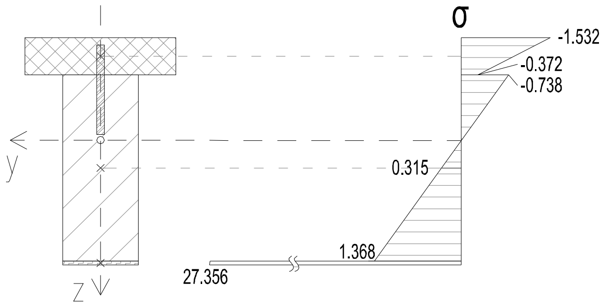


Fig. 7: Corresponding normal stresses (in kN/cm^2).

Corresponding shear stresses as consequence of Vz,d,t are given with equations (22) to (24) and they are represented on Fig. 8.

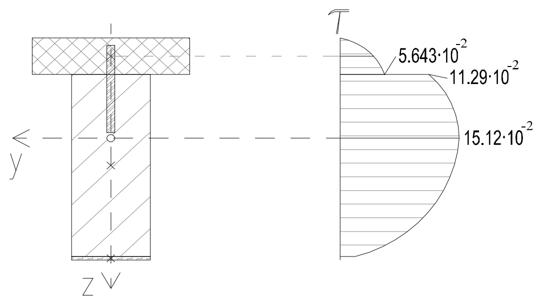
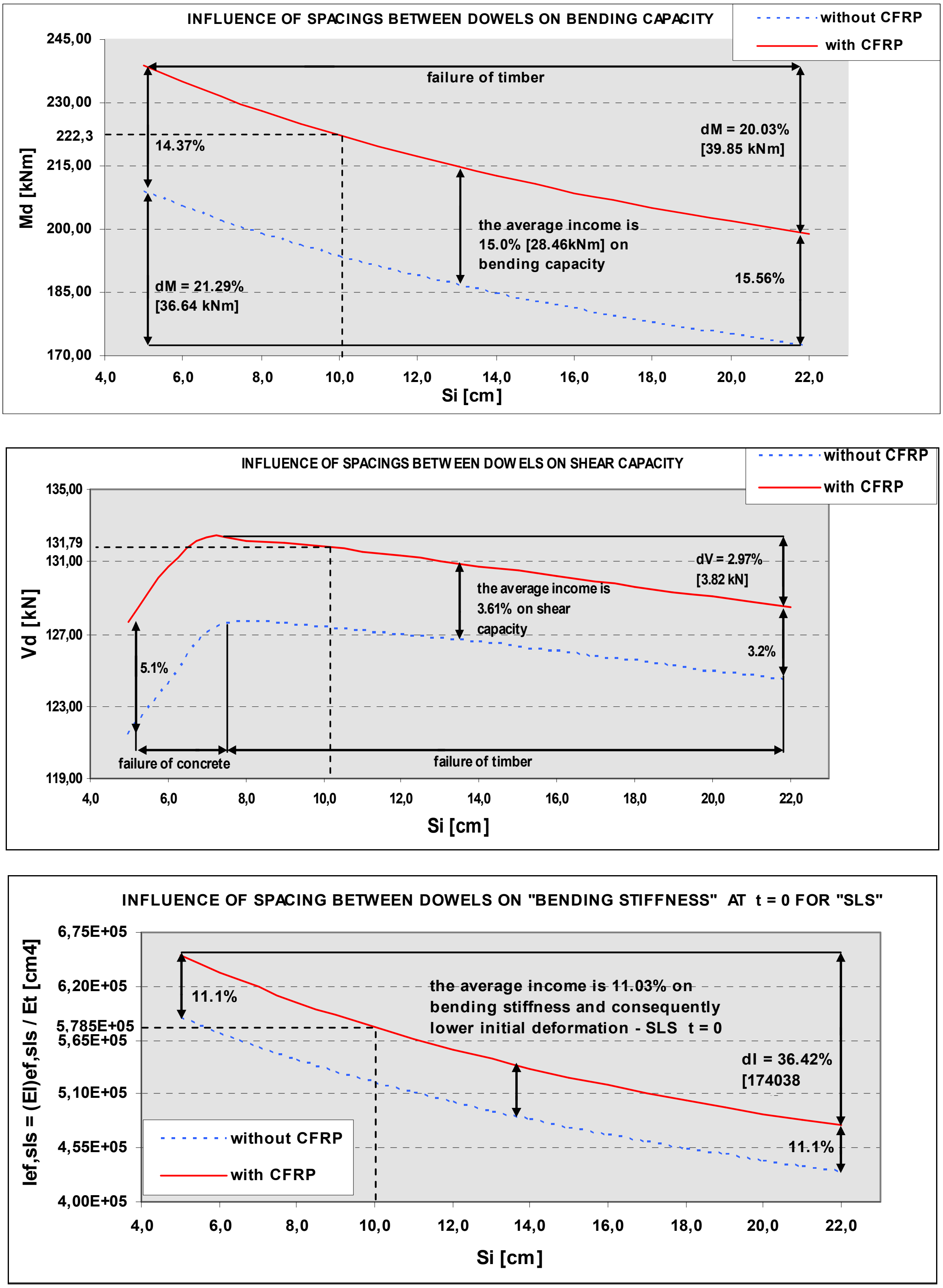


Fig. 8: Corresponding shear stresses (in kN/cm^2).

4 Comparison between section with and without carbon strip

Comparison of load bearing capacity and bending stiffness of composite beam made of concrete plate and timber beam with and without carbon strip with variable distance between dowels (material and geometrical properties are the same as in upper example) is shown on the following diagrams (Fig. 9):



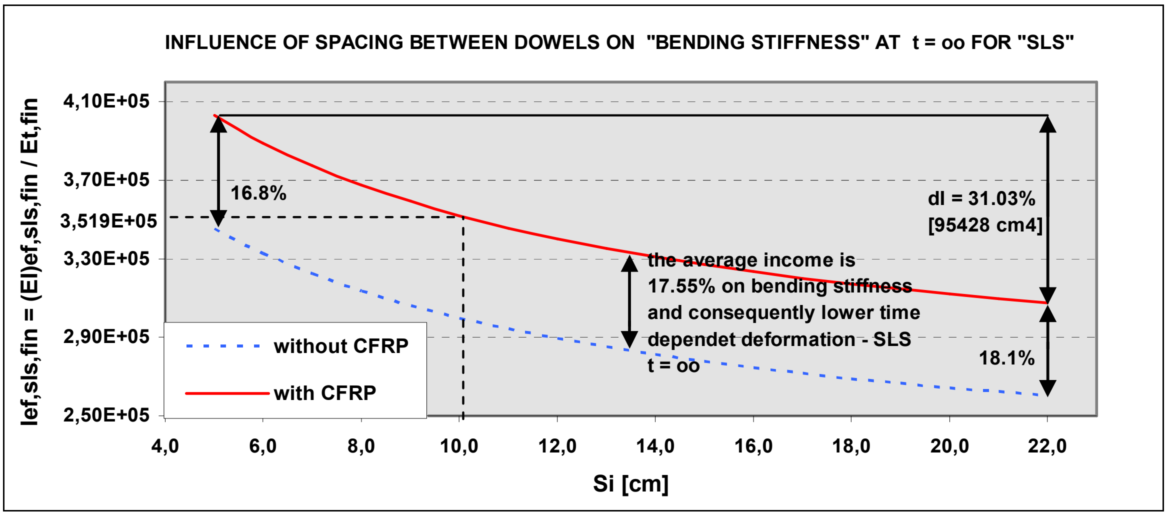


Fig. 9: Variability characteristic of section in function of variable distance between dowels

5 Conclusion

In the article was provided the analytical and numerical analysis (bearing capacity and bending stiffness) of composed T section, composed of concrete slab and timber beam without and with carbon strip glued on the bottom fiber of timber. Analytical analysis and numerical calculations by Eurocode 5 Annex B.2, have been done. The comparison shows that the bearing capacity (Md) of the structure with carbon strip is 15% higher as the bearing capacity without it. Normally it depends of the dowel's spacing also. But nevertheless the better combination of sub-components can significantly improves effects of inclusion of CFRP strip. For example greater bearing capacity and bending stiffness can be achieved with usage of carbon strip with higher Young's modulus.

In the Eurocode 5 Annex B is strictly declared usage of Young modulus as mean value of secant modulus. But for ULS computation would be necessary use the values E_i/γ_i because different materials have different safety partial factors for material properties. For instance:

$$\bar{n}_c = n_c \cdot \frac{\gamma_t}{\gamma_c} = \frac{E_{cm}}{E_{mean}} \cdot \frac{\gamma_t}{\gamma_c}; \quad \bar{n}_f = n_f \cdot \frac{\gamma_t}{\gamma_f} = \frac{E_f}{E_{mean}} \cdot \frac{\gamma_t}{\gamma_f}$$

$$\bar{k} = \frac{\pi^2 \cdot E_{cm} \cdot A_c \cdot s_i}{K_u \cdot \gamma_c \cdot l_{eff}^{\,2}}$$

In the article these propositions have not been taken into account.

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