A quasi-one-dimensional Riemann problem for the isentropic gas dynamics equations

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Abstract: We consider a Riemann problem for the isentropic gas dynamics equations in two space dimensions modeling shock reflection. We write the problem in self-similar coordinates and, by analyzing a quasi-one-dimensional Riemann problem at the reflection point, we derive regimes, in terms of initial data, for which solutions model the phenomenon of regular shock reflection. This sets the stage for existence analysis of solutions to this Riemann problem using the approach by Čanić, Keyfitz, Kim and Lieberman on the study of two-dimensional Riemann problems for systems of conservation laws.

Key–Words: isentropic gas dynamics, two-dimensional Riemann problems, shock reflection

1 Introduction

In this paper we are interested in understanding of shock reflection phenomenon by considering the isentropic gas dynamics equations. For an overview of shock reflection from the experimental point of view, we refer to [1] by Ben-Dor, and for some examples of numerical studies, see [13, 14, 15, 19, 21, 22, 24].

The mathematical study of shock reflection via analysis of special problems for systems of conservation laws was employed in the case of steady and unsteady transonic small disturbance equations, nonlinear wave system, pressure-gradient system, Euler equations for potential fluids and gas dynamics equations (for example, see [4, 5, 6, 7, 8, 9, 10, 11, 16, 17, 18, 25, 26]). In these studies, the system of conservation laws is written in self-similar coordinates which results in a mixed type system and a free boundary problem for the reflected shock and the subsonic flow. As the starting point, the initial data has to be carefully chosen so that the solutions model a desired type of reflection. This is done by resolving the interactions in the hyperbolic region and by solving quasi-one-dimensional Riemann problems (for introduction to quasi-one-dimensional Riemann problems see [2, 3] by Čanić and Keyfitz).

We consider a Riemann problem for the isentropic gas dynamics equations and we derive regimes for which regular reflection occurs. This sets the stage for existence analysis of solutions to Riemann problems for the isentropic gas dynamics equations modeling regular reflection using the ideas by Čanić, Keyfitz, Kim and Lieberman (for more details about this approach, see [20] by Keyfitz). The main tools in this approach are the theory of second order elliptic equations with mixed boundary conditions by Gilbarg, Trudinger and Lieberman [12, 23] and various fixed point arguments.

2 Isentropic gas dynamics equations

We consider the system of isentropic gas dynamics equations in two space dimensions given by

$$\begin{aligned} &\rho_t + (\rho u)_x + (\rho v)_y = 0, \\ &(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0, \\ &(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0, \end{aligned} \tag{1}$$

where $(x, y, t) \in \mathbb{R}\times\mathbb{R}\times[0,\infty)$, $\rho : \mathbb{R}\times\mathbb{R}\times[0,\infty) \to (0,\infty)$ stands for density, $u, v : \mathbb{R}\times\mathbb{R}\times[0,\infty) \to \mathbb{R}$ are velocities in x- and y-directions, respectively, and $p = p(\rho) : (0,\infty) \to \mathbb{R}$ is pressure. We assume that $c^2(\rho) := p'(\rho)$ is a positive and increasing function of ρ. Following ideas in [4, 5, 6, 7, 8, 16, 17, 18], we write system (1) in self-similar coordinates, $\xi = x/t$ and $\eta = y/t$, and we obtain

$$\begin{aligned} &U\rho_\xi + \rho U_\xi + V\rho_\eta + \rho V_\eta + 2\rho = 0, \\ &UU_\xi + p_\xi/\rho + VU_\eta + U = 0, \\ &UV_\xi + VV_\eta + p_\eta/\rho + V = 0, \end{aligned} \tag{2}$$

where $U := u - \xi$ and $V := v - \eta$ stand for pseudo-velocities. Moreover, from (2) we obtain a second order equation for density

$$\begin{aligned}&\big((U^2-c^2)\rho_\xi + UV\rho_\eta + \rho U\big)_\xi\\ &+\big((V^2-c^2)\rho_\eta + UV\rho_\xi + \rho V\big)_\eta\\ &+(UV_\eta - VU_\eta)\rho_\xi \,+\, (VU_\xi - UV_\xi)\rho_\eta\\ &+2(U_\xi V_\eta - V_\xi U_\eta)\rho = 0.\end{aligned}$$

Note that given a constant state $\overline{U}_0 = (\rho_0, u_0, v_0)$, the above equation is hyperbolic outside of the sonic circle $C_0 : (u_0-\xi)^2 + (v_0-\eta)^2 = c^2(\rho_0)$.

3 Riemann initial data

We consider system (1) with initial data posed in two sectors (Figure 1)

$$\overline{U}(x,y,0) := \begin{cases} \overline{U}_1 = (\rho_1, 0, 0), & x > \text{sign}(y)\,ky, \\ \overline{U}_0 = (\rho_0, u_0, 0), & \text{otherwise}. \end{cases} \tag{3}$$

We assume that $\rho_0 > \rho_1$ are given arbitrarily, $k > 0$ will be specified later in terms of ρ_0 and ρ_1, and we take

$$u_0 = \sqrt{1+k^2}\sqrt{\frac{[\rho][p]}{\rho_0\rho_1}}.$$

Here, $[\,\cdot\,]$ denotes the jump between states $\overline{U}_0$ and $\overline{U}_1$. With this choice of u_0, each of the two initial discon-

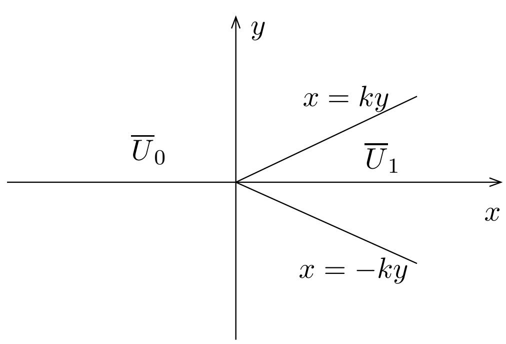

Figure 1: The Riemann initial data.

tinuities $x = \pm ky$, $x \geq 0$, results in a shock and a linear wave in the far field (Figure 2). More precisely, the one dimensional Riemann problem along the discontinuity $x = ky$, $y \geq 0$, results in a shock $S_1 : x = ky + wt$, with $\overline{U}_1$ on the left and an intermediate state $\overline{U}_a = (\rho_0, u_a, v_a)$ on the right, and a linear wave $l_1 : x = ky + At$ with $\overline{U}_a$ on the left and $\overline{U}_0$ on the right. Using the Rankine-Hugoniot jump relations we find

$$u_a = \frac{1}{1+k^2}\sqrt{\frac{[\rho][p]}{\rho_0\rho_1}}, \quad v_a = -ku_a,$$

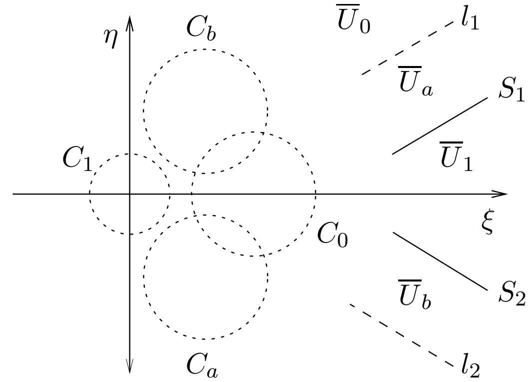

Figure 2: The solution in the far field.

$$w = \sqrt{1+k^2}\sqrt{\frac{\rho_0[p]}{\rho_1[\rho]}}, \quad \text{and} \quad A = u_0,$$

where the jump $[\,\cdot\,]$ is between states $\overline{U}_0$ and $\overline{U}_1$. It is easy to check $u_0 > u_a$ and $w > A$. On the other hand, the one-dimensional solution along $x = -ky$, $y \leq 0$, consists of a linear wave $l_2 : x = -ky + At$ connecting $\overline{U}_0$ to a state $\overline{U}_b = (\rho_0, u_a, -v_a)$ and a shock $S_2 : x = -ky + wt$ connecting $\overline{U}_b$ to $\overline{U}_1$.

4 Parameter k

Suppose that densities $\rho_0 > \rho_1$ are fixed. In this section we investigate values of the parameter $k > 0$ for which the Riemann problem (1), (3) leads to regular reflection. In our examples we assume that the pressure is given by the γ-law relation, $p(\rho) = \rho^\gamma/\gamma$, with $\gamma = 2$, and we take $\rho_0 = 4$ and $\rho_1 = 1$. Let us denote the projected intersection point of shocks S_1 and S_2 by

$$\Xi_s := (\xi_s, 0) = (w, 0),$$

and the sonic circles for states $\overline{U}_0$, $\overline{U}_1$, $\overline{U}_a$ and $\overline{U}_b$ by C_0, C_1, C_a and C_b, respectively. We distinguish the following three regions depending on the value of k.

- **Region A :** The value of k is such that the incident shock S_1 (equivalently, S_2) intersects either the circle C_1 or the circle C_a (equivalently, C_b). In this case regular reflection cannot occur.

We claim that, given densities $\rho_0 > \rho_1$, there exists a value $k_A(\rho_0, \rho_1) > 0$, depending on ρ_0 and ρ_1, such that for $k \in (0, k_A)$, the shocks S_1 and S_2 do not intersect at the ξ-axis.

- **Region B :** The value of k is such that $\Xi_s \notin C_1 \cup C_a \cup C_b$. In other words, the point Ξ_s is hyperbolic with respect to both $\overline{U}_a$ and $\overline{U}_b$.

However, for these values of k, we assume that the quasi-one-dimension Riemann problem at the point Ξ_s with states $\overline{U}_b$ and $\overline{U}_a$, on the left and on the right, does not have a solution consisting of two shocks. Therefore, regular reflection cannot happen.

In order to solve the quasi-one-dimensional Riemann problem at the point Ξ_s with states $\overline{U}_b$ and $\overline{U}_a$, on the left and on the right, respectively, we first find the shock polars for these two states. In general, using (2), the Rankine-Hugoniot jump relations along the shock $\eta = \eta(\xi)$ between a fixed state $\overline{U}_0 = (\rho_0, u_0, v_0)$ and a state $\overline{U} = (\rho, u, v)$ are given by

$$\begin{aligned} &[\rho U]d\eta = [\rho V]d\xi, \\ &[\rho U^2 + p]d\eta = [\rho UV]d\xi \\ &[\rho UV]d\eta = [\rho V^2 + p]d\xi, \end{aligned} \tag{4}$$

where $[\,\cdot\,]$ stands for the jump between the two states. We obtain

$$\frac{d\eta}{d\xi} = \frac{U_0V_0 \pm \sqrt{\bar{c}^2(U_0^2 + V_0^2 - \bar{c}^2)}}{U_0^2 - \bar{c}^2} =: \sigma_\pm(\rho), \tag{5}$$

and

$$\begin{aligned} v(\rho) &= v_0 + \tfrac{p(\rho)-p(\rho_0)}{\rho_0((u_0-\xi)\sigma_\pm(\rho)-v_0)}, \\ u(\rho) &= u_0 - \sigma_\pm(\rho)(v(\rho) - v_0), \end{aligned} \tag{6}$$

where

$$\bar{c}^2(\rho) := \rho[p]/(\rho_0[\rho]).$$

(It is easy to show that $\bar{c}^2(\rho)$ is an increasing function of ρ). Therefore, given a state $\overline{U}_0 = (\rho_0, u_0, v_0)$, the states $\overline{U} = (\rho, u(\rho), v(\rho))$, which can be connected to $\overline{U}_0$ by a shock, satisfy (6), where $\sigma_\pm(\rho)$ is given by (5). The signs $+$ and $-$ correspond to a $+$-shock and a $-$-shock, respectively.

We consider states $\overline{U}_a$ and $\overline{U}_b$ and we plot the shock polars $S^\pm(\overline{U}_a)$ and $S^\pm(\overline{U}_b)$. The quasi-one-dimensional Riemann problem at the point Ξ_s with states $\overline{U}_b$ and $\overline{U}_a$, on the left and on the right, respectively, has a solution consisting of two shocks and an intermediate state if $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ intersect. (These two shocks are called the "reflected shocks".) Clearly, from the relation for $u(\rho)$ in (6), the intersections of projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in the (ρ, v)-plane correspond to intersections of $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in the (ρ, u, v)-space.

Example 1. *Let $k = 1.3$. We find $\Xi_s \notin C_1 \cup C_a \cup C_b$ and we plot the projections of the shock polars for $\overline{U}_a$ and $\overline{U}_b$ in the (ρ, v)-plane in Figure 3. The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ do not intersect, implying that the considered quasi-one-dimensional Riemann problem at the reflection point Ξ_s does not have a solution consisting of two shocks and an intermediate state.*

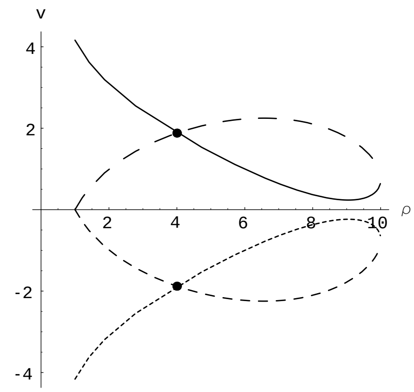

Figure 3: The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ do not intersect.

- **Region C :** The value of k is such that shocks S_1 and S_2 intersect at the point Ξ_s and, moreover, the quasi-one-dimensional Riemann problem at Ξ_s has a solution consisting of two shocks and an intermediate state.

Example 2. *Let $k = 1.455$. We find $\Xi_s \notin C_1 \cup C_a \cup C_b$. We plot the projections of shock polars $S^\pm(\overline{U}_b)$ and $S^\pm(\overline{U}_a)$ in the (ρ, v)-plane and we note that $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ intersect at exactly one point (Figure 4)*

$$\overline{U}_R := (\rho_R, u_R, 0) = (10.1, 2.97802, 0).$$

Hence, a quasi-one-dimensional Riemann problem at the point Ξ_s has a unique solution consisting of a shock connecting the states $\overline{U}_b$ and $\overline{U}_R$ and a shock connecting the states $\overline{U}_R$ and $\overline{U}_a$.

Given densities $\rho_0 > \rho_1$, we claim that there exists a value $k_C(\rho_0, \rho_1)$ such that for $k > k_C$, the shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ intersect at two points

$$\overline{U}_R = (\rho_R, u_R, 0) \quad \text{and} \quad \overline{U}_F = (\rho_F, u_F, 0).$$

Let us assume $\rho_R < \rho_F$. Note that we also have $\rho_R, \rho_F > \rho_0$, by the geometry of shock polars. Therefore, for $k > k_C$, the considered quasi-one-dimensional Riemann problem has two solutions. Each solution consists of a shock connecting $\overline{U}_b$ to an intermediate state ($\overline{U}_R$ or $\overline{U}_F$) and a shock connecting this intermediate state to $\overline{U}_a$.

The next question is whether the reflection point Ξ_s is subsonic (i.e., inside the sonic circle) or supersonic (i.e., outside of the sonic circle) with respect to the states $\overline{U}_R$ and $\overline{U}_F$. We claim that Ξ_s is subsonic with respect to the state $\overline{U}_F$ for all $k > k_C$, and that

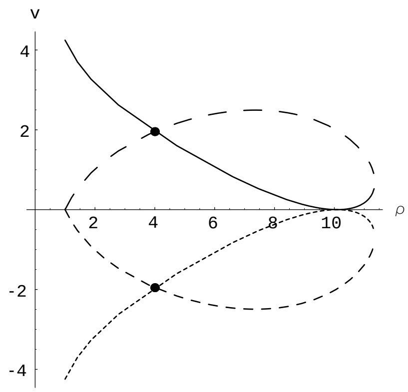

Figure 4: The projected shock polars $S^{+}(\overline{U}_b)$ and $S^{-}(\overline{U}_a)$ are tangent.

there exists a value $k_*(\rho_0, \rho_1)$ such that Ξ_s is subsonic with respect to the state $\overline{U}_R$ if $k \in (k_C, k_*)$.

Example 3. *Let $k = 1.48$. We depict the projections of the shock polars $S^{\pm}(\overline{U}_b)$ and $S^{\pm}(\overline{U}_a)$ in the (ρ, v)-plane in Figure 5 and find that $S^{+}(\overline{U}_b)$ and $S^{-}(\overline{U}_a)$ intersect at two points*

$$\overline{U}_R = (9.669, 2.676, 0)$$

and

$$\overline{U}_F = (10.766, 3.324, 0).$$

We remark that the reflection point $\Xi_s = (5.648, 0)$ is subsonic with respect to both $\overline{U}_R$ and $\overline{U}_F$.

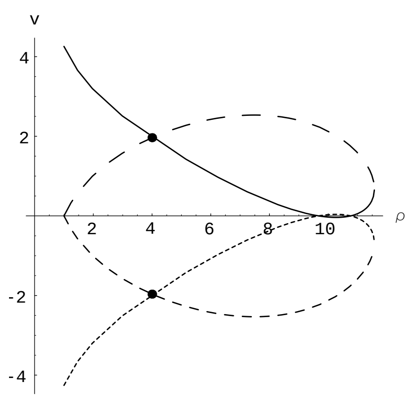

Figure 5: The projected shock polars $S^{+}(\overline{U}_b)$ and $S^{-}(\overline{U}_a)$ intersect at two points.

Example 4. *Let us consider $k = 2$. The reflection point Ξ_s is supersonic with respect to both states $\overline{U}_a$ and $\overline{U}_b$ and, moreover, the shock polars $S^{+}(\overline{U}_b)$ and $S^{-}(\overline{U}_a)$ intersect (Figure 6) at two points*

$$\overline{U}_R = (9.294, 1.776, 0)$$

and

$$\overline{U}_F = (16, 5.303, 0).$$

We further find that the reflection point $\Xi_s = (7.071, 0)$ is subsonic with respect to the state $\overline{U}_F$ and supersonic with respect to the state $\overline{U}_R$.

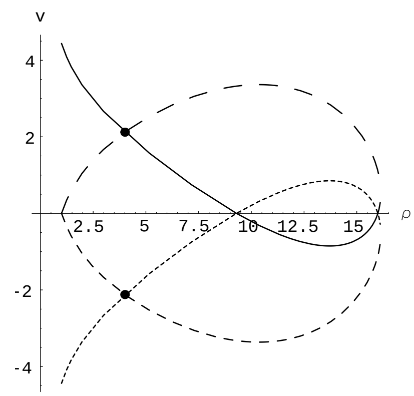

Figure 6: The projected shock polars $S^{+}(\overline{U}_b)$ and $S^{-}(\overline{U}_a)$ intersect at two points.

In Figure 7 we depict the curves k_A, k_C and k_* in the case of γ-law pressure with $\gamma = 2$, and for densities ρ_0 and ρ_1 such that $\rho_0/\rho_1 \in (1, 8]$. The region **A** is below the curve k_A, the region **B** is between the curves k_A and k_C, and the region **C** is above the curve k_C. Hence, regular reflection occurs for the values of k above k_C. When k is between k_C and k_*, then both states $\overline{U}_R$ and $\overline{U}_F$ are subsonic with respect to the point Ξ_s, and when k is above k_*, then the state $\overline{U}_R$ is supersonic and the state $\overline{U}_F$ is subsonic.

Therefore, we distinguish between the following two types of solutions.

- Given densities $\rho_0 > \rho_1$ and the parameter $k \in (k_C, k_*)$, the Riemann problem (1), (3) results in *strong (or transonic) regular reflection*. The solution at the point Ξ_s is either $\overline{U}_R$ or $\overline{U}_F$, and since the point Ξ_s is inside the sonic circles C_R and C_F, the two reflected shocks are transonic and curved. They do not exit the sonic circles C_R and C_F due to causality and they also do not enter the sonic circle C_0 by the Lax entropy condition. Hence, by writing the original Riemann problem in (ξ, η)-coordinates, by resolving the interactions in the hyperbolic region and

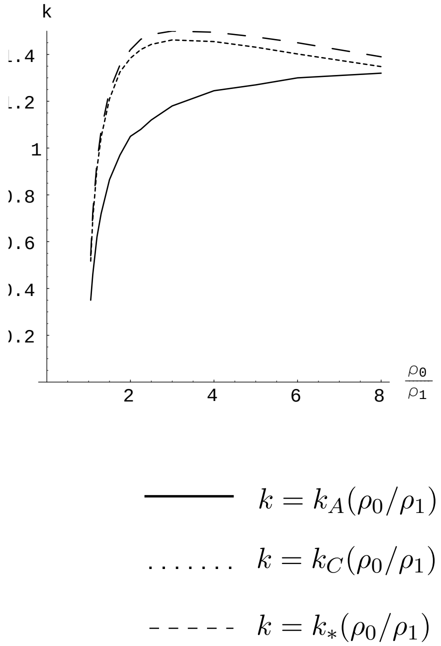

Figure 7: The curves k_A, k_C and k_*.

by solving the quasi-one-dimensional Riemann problem at Ξ_s, we can reduce the original Riemann problem to a problem for the nonuniform subsonic flow in the domain bounded by the reflected shocks and the point Ξ_s. Moreover, this is a free boundary problem since the position of the reflected shocks is not known apriori and it depends on the unknown subsonic flow. We claim that the structure of the solution for ρ is as in Figure 8.

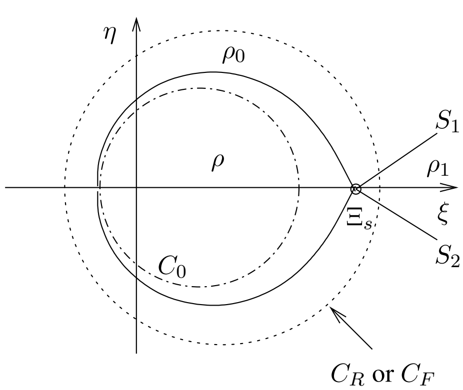

Figure 8: Strong regular reflection.

- Given densities $\rho_0 > \rho_1$ and the parameter $k > k_*$, we have two possibilities. If the solution at the point Ξ_s is assumed to be $\overline{U}_F$, then the solution again models strong regular reflection. However, if we assume that the solution at the point Ξ_s is $\overline{U}_R$, then since Ξ_s is supersonic with respect to $\overline{U}_R$, we have that the two reflected shocks are hyperbolic and rectilinear until they intersect the sonic circle C_R. After that they become transonic and curved, and, again, they do not exit the sonic circle C_R, by causality, nor they enter the sonic circle C_0, by the Lax entropy condition. Hence, the solution is equal to the constant state $\overline{U}_R$ in the region bounded by the rectilinear part of the reflected shocks and the sonic circle C_R. The flow inside the region bounded by C_R and the curved part of the reflected shocks is unknown. Since the position of transonic reflected shocks is also unknown, we again obtain a free boundary problem. The structure of the solution for ρ is depicted in Figure 9. This solution models *weak (or supersonic) regular reflection*.

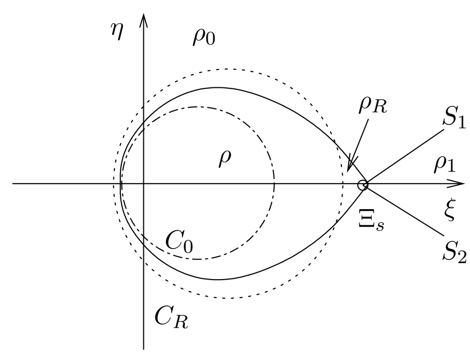

Figure 9: Weak regular reflection.

5 Conclusion

In this paper we consider a sectorial Riemann problem for the two-dimensional isentropic gas dynamics equations. By writing the problem in self-similar coordinates, by resolving the interactions in the hyperbolic region and by solving a quasi-one-dimensional Riemann problem at the reflection point, we derive conditions on the initial data for which the solution will give rise to strong and weak regular reflection. Moreover, the original Riemann problem can be reduced to a free boundary problem for the reflected shock and the subsonic flow in (ξ, η)-plane.

The author, in collaboration with Keyfitz and Čanić, has started local analysis (near the reflection point Ξ_s) of the free boundary problem arising in the case of strong regular reflection. Our study follows the approach outlined in [20]; however due to complicated nonlinear coupling of the isentropic gas dynamics equations, novel techniques are needed to establish the existence of the solution. We leave this analysis for a future paper.

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