Type-2 Fuzzy Set Representation of Stochastic Adding A/D Conversion

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Abstract: - Stochastic adding A/D conversion (SAADK) represents a method of measurement (or classification) which naturally can replace process of fuzzyfication of input quantities. In classical measurement in the whole measurement range absolute error is, more or less, the same, but in the case of control it is known that all measurement subintervals are not equally significant. Type-2 fuzzy set representation is able to model uncertainties of the stochastic error measurements using SAADK. The primary aim of the paper is to introduce this representation method.

Key-Words: - type-e fuzzy sets, A/D conversion

1 Introduction

Since the beginning of the fuzzy world, there are applications of type-1 fuzzy systems (FS) in which fuzzy system are used to approximate random data or to model an environment that is changing in an unknown way with time [2]. L.A. Zadeh introduced type-2 and higher-types FS in 1975 [6], to eliminate the paradox of type-1 fuzzy systems which can be formulized as the problem that the membership grades are themselves precise real numbers. It is not a serious problem for many applications, but in the cases when:

- the data generating system is known to be time-varying but the mathematical description of the time-variability is unknown,
- measurement noise is non-stationary and the mathematical description of the non-stationarity is unknown,
- features in a pattern recognition application have statistical attributes that are non-stationary and the mathematical description of the non-stationarity is unknown,
- knowledge is mined from a group of experts using questionnaires that involve uncertain words
- linguistic terms are used and have a non-measurable domain,

type-1 fuzziness results imprecise boundaries of FS-s [1]. The solution for this problem can be type-2 fuzziness, where fuzzy sets have grades of membership that are themselves fuzzy [2], [3]. At each value of the primary variable *x* on the universe *X*, the membership is a function, and not just a point value (characteristic value). It is the second level, or secondary membership function, whose domain is the primary membership value set. The secondary membership function is a function $MF2:[0,1]\rightarrow[0,1]$. It can be concluded that *MF2* gives a type-2 fuzzy set which is three-dimensional, and the third dimension design degree somehow the freedom for handling uncertainties.

In [2] Mendel defines and differentiates two types of uncertainties, random and linguistic. The first one is characteristic for example in statistical signal processing, and the linguistic uncertainties characteristic have in word-information based imprecision systems.

Operations on type-2 fuzzy sets are extended based on type-1 union, intersection, complementation and usually apply t-norms and conorms [7].

The developed measure method SAADK in [4], stochastic adding A/D conversion has several advantages compared to standard digital instrumentation. Its main advantages are extremely simple hardware and, consequently, simple implementation of parallel measurements, and on the other hand possibility to trade speed for accuracy. This instrument can be either fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership on the universe X is time-depending and with random uncertainties. In the paper the SAADK method is shortly represented. In further research the possible operator group, based on distance based operators, which are defined using min and max operators will be introduced [5]. It is an opportunity to use simply hardware realization of the operators and approximate reasoning process in the control problems based on the SAADK method.

2 Stochastic additive “analogue to digital” converter

In classical measurement in the whole measurement range absolute error is, more or less, is the same. This is characteristic of uniform quantizer, i.e. analogue to digital converter. On the other hand, in case of control we know that all measurement subintervals are not of equal signification. This means that equal precision is not necessary for control applications. In less significant subintervals it is sufficient to confirm that the system is in it. No more information is needed. Fuzzy systems serve as good mathematical models for this simple situation.

An interesting question arises: does the fuzzyfication belong to domain of measurements and metrology? The answer can be found in several definitions.

Metrology is the science of measurement. Metrology includes all aspects both theoretical and practical with reference to measurements, whatever their uncertainty, and in whatever fields of science or technology they occur. Measurement is a set of operations having the object of determining a value of a quantity. Measurable quantity is an attribute of a phenomenon, body or substance that may be distinguished qualitatively and determined quantitatively. Analog to digital conversion is the conversion of an analog quantity in its digital counterpart.

The possible conclusion from the above definitions is that fuzzyfication can be accepted as a kind of measurement, since by performing fuzzyfication we confirm membership functions, i.e. that fuzzyfied quantity has characteristics of defined fuzzy sets, depending on the moment - the time variable.

Human beings are not capable of processing too many pieces of information, so they process one information at a moment, possiblly, the most significant one in that moment. The process of choosing this most significant information is a kind of control process. Humans have the capability to do this. Adaptive measurement system puts this human characteristic in the area of automatic control systems. Let us imagine that we have system with many inputs, one multipexer and one system for processing inputs. On input we have information with low resolution, defined with some synchronizing pulse, but using longer processing time we get information with higher resolution. If processing system works longer on a specific input the information is more detailed and more reliable, in the opposite case the information is rough and less reliable.

The processing cycle for all inputs lasts the same time, in every case, but work on specific inputs can be variable, depending on the state of the system. So there is a need for an instrument capable of giving rough and fast measurement information in the shorter time or precise and reliable measurement information in longer time interval. One of such instruments is stochastic additive A/D converter.

The developed measure method, stochastic adding A/D conversion has several advantages compared to standard digital instrumentation [4]. Its main advantages are:

1. extremely simple hardware and, consequently, simple implementation of parallel measurements, and
2. possibility to trade speed for accuracy.

This instrument can be either fast and less accurate, or slow and very accurate. The choice can be made by specifying the frequency of reading its output. This is a kind of adaptation which the master processor performs. In the case of multi-channel measurements, adaptation can be performed on each channel independently.

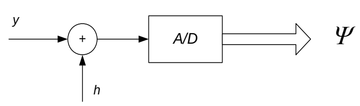

Fig. 1 *The most simple outline of the instrument*

In Fig.1. a schematic picture of the instrument is shown. The dithering signal *h* is random, uniform and satisfies Widrow's conditions

$$0 \le |h| \le \frac{a}{2} \tag{1}$$

$$p(h) = \frac{1}{a}, \tag{2}$$

where a is a quantum of the uniform quantiser, and $p(h)$ is the corresponding probability density function of h.

2.1 Theory of operation - DC inputs

Let us observe the output of *AD* converter Ψ. Let $y = const = na + |\Delta a|$ be the corresponding input voltage located between quantum level na and $(n+1)a$, at the distance $|\Delta a| \le a/2$ from the closest quantum level na shown in Fig. 2.

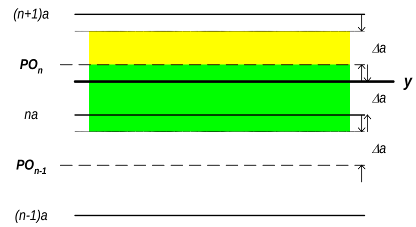

Fig.2: *The situation for y=const*

For the situation depicted in Fig.2 the quantized level of $y+h$ is $\Psi \in \{na, (n+1)a\}$. The expectation $\overline{\Psi}$ is given by

$$\overline{\Psi} = \Psi_1 \cdot p_1 + \Psi_2 \cdot p_2 = (n+1) \cdot a \cdot \frac{|\Delta a|}{a} + na\frac{(a - |\Delta a|)}{a} =$$
$$= n \cdot |\Delta a| + |\Delta a| + na - n \cdot |\Delta a| = na + |\Delta a| = y$$

$$\overline{\Psi} = y \tag{3}$$

The corresponding variance is:

$$\overline{e^2} = \sigma_\Psi^2 = \left(\Psi_1 - \overline{\Psi}\right)^2 \cdot p_1 + \left(\Psi_2 - \overline{\Psi}\right)^2 \cdot p_2 =$$
$$(a - |\Delta a|)^2 \cdot \frac{|\Delta a|}{a} + |\Delta a|^2 \cdot \frac{(a - |\Delta a|)}{a} =$$
$$\sigma_\Psi^2 = (a - |\Delta a|) \cdot \left((a - |\Delta a|) \cdot \frac{|\Delta a|}{a} + \frac{|\Delta a|^2}{a} \right) =$$
$$(a - |\Delta a|) \cdot (|\Delta a|)$$
$$\sigma_\Psi^2 = (a - |\Delta a|) \cdot (|\Delta a|) \tag{4}$$

An interesting question is: what is the error if we have finite number N of dithered samples? The answer gives theory of samples and central limit theorem. Suppose that we have next set of samples: $\Psi_1, \Psi_2, ..., \Psi_n$.

Then measurement quantity is

$$\overline{\Psi} = \frac{1}{N}\sum_{i=1}^{N} \Psi_i \approx const. \tag{5}$$

Central limit theorem gives next result

$$\sigma_{\overline{\Psi}}^2 = \frac{\sigma_\Psi^2}{N}. \tag{6}$$

The equations (4), (5) and (6) completely define the situation if we have a finite number N of dithered samples.

The shape of σ_Ψ^2 as function of Δa, for the case of $y=const$, is given in Fig. 3.

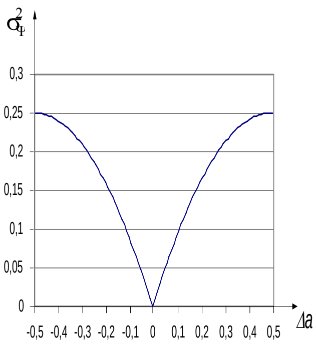

Fig. 3. *The diagram of* σ_Ψ^2 *(Δa) for y=const and a=1.*

From Fig.3. it is obvious that the maximum value of error is in the case of $y = c = \frac{a}{2} + na$, i.e. when measurement voltage has the value of threshold voltage, but the minimum value of error is when $y=c=na$, i.e. when measurement voltage has the value of quantum level.

2.2. **Function-based measurements and the numeric processing of the measurements**

The stochastic additive A/D converter basically works with equidistant comparators, but this is not obligatory. If we make A/D converter with non-equidistant threshold levels, then we have stochastic additive analog to fuzzy converter (SAAFC).

Defining the membership functions on the output of the stochastic additive analog to fuzzy converter the measurement range is divided in to n intervals. There is one deciding threshold in every interval. (n +1) sets are defined by n deciding thresholds. The first set is approximately the minimum value of the measurement range, $(n+1)^{st}$ set is approximately the maximum value of the measurement range. Between these two sets there are (n-1) sets „approximately A_j" where x_{A_j} represents the middle of the two deciding thresholds: PO_j and PO_{j-1} where $j = 2,...,n$. The deciding thresholds are marked from PO_1 to PO_n. There exists the following limitation: $PO_1 > PO_{j-1}$ where $j = 2,...,n$.

In general case, the membership of the defined sets can be described in the following way:

$$A_j(x) = \{x_i | x_i \geq PO_{j-1} \wedge x_i < PO_j\}$$

where $j = 2,...,n$ (7)

For the first and the last set:

$$A_1(x) = \{x_i | x_i < PO_1\} \quad (8)$$

$$A_{n+1}(x) = \{x_i | x_i \geq PO_n\} \quad (9)$$

The fuzzy intervals or fuzzy numbers are defined on these sets.

SAAFC makes the quantification on this sum:

$$x_i = x(t) + h(t) \quad (10)$$

where:

x_i is the ith value of input sum into the flash A/D converter,

i is the serial number of quantification in the measurement cycle,

$x(t)$ is the value of the variable which is fuzzyficated in the moment of sampling,

$h(t)$ is the value of random variable of uniform distribution in the moment of sampling.

Membership function of fuzzy sets (A_j) is defined depending on the relative frequency of appearing of the measuring result in (A_j) during the measuring cycle.

$$\mu_{A_j}(x) = \frac{a_j}{N} \quad (11)$$

where:

j is the serial number of the fuzzy set, j = 1, 2,...,(n+ 1),

a_j is the number of appearing value on the output of the fuzzy set A_j,

N is the total quantification number during the measuring cycle.

The following limitation is in force for h:

$h \leq \min(PO_j - PO_{j-1})$

where $j = 2,...,n$. (12)

where h must be smaller or equal to the minimum difference between the two neighboring deciding thresholds.

The elements of continuous set are ordered to the membership function values as a secondary fuzzy set, constructed a type-2 fuzzy environment.

If we deal with system with great number of inputs and with limited capability, not only for measurements, but for processing as well, we can use this system more efficiently if we apply the above mentioned idea: if processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable. Similarly to human reasoning, the system can concentrate on the most significant input and the most significant piece of information is processed. For other inputs, the system only confirms that they are under control.

The appearance of the signal in the first instant of quantization indicates the rough estimation of specific input and it can define the processing time interval. If the estimation says that the input is under control, we can process the next input immediately. But if the estimation idicates that the input is not under control, the system pays more attention (longer processing time) to go back under control.

3 Conclusion

The adaptivity and functional time-depending representation, as type-2 fuzziness representation of the accuracy of measurement and the possibility of direct fuzzyfication decreases hardware complexity and increases the speed and reliability of the system work using SAADK.

The main goal of further research is to confirm that the SAADK can be represented in the type-2 fuzzy environment, and to suggest that the method of combined SAADK and type-2 applications is realizable in control problems, fuzzy rule bases, fuzzy approximate reasoning using minimum maximum based operators – the distance based operator group.

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