Decentralized Control of Platoons based on a Novel Adaptive Control of Lucid Geometric Interpretation

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Abstract: This paper is devoted to modeling and analyzing the behavior of a platoon that is formed of semi-automated vehicles and behaves like a virtual train in which the first vehicle is in the leading position while the other members of the train track just the preceding one. It is supposed that the dynamic models of the carts are only approximately known but their loads and the nature of the coupling (connection) between the carts and their lodas are completely unknown by the controller. It also is assumed that *various local tracking strategies* can be applied between the members as *"adaptive"* and *"non-adaptive" "distance and relative velocity tracking"*, and *"acceleration tracking"*. It is also assumed that the drives of the carts have limited driving forces that is modeled in the simulations by the use of smooth sigmoid functions. The carts are approximately modeled as rigid bodies while their loads are represented by masses connected to the appropriate carts by elastic springs having viscous friction. The aim is to guarantee the possible smoothest motion for the last element of the train that may be followed by various participants of the road communication whose driving comfort has to be kept in mind. This task corresponds to the control of an imprecisely known strongly coupled nonlinear system also having unmodeled coupled internal degrees of freedom. It is shown via computer simulation that the novel, geometrically interpreted adaptive control that in certain cases can be a simple alternative of Lyapunov's 2^{nd} Method can successfully solve this complicated task in decentralized manner, and can also reduce the jogging of the loads of the carts.

Key–Words: Coupled nonlinear dynamic systems, Adaptive control, Iterative learning, Banach spaces, Contractive mapping

1 Introduction

A particular application of more or less automated vehicles is the formation of platoons in which the main control task is solved by the leading vehicle, while the other members need not to be too intelligent: they simply have to follow the preceding vehicle in the convoy at a safe tracing/braking distance. They have to stop when the preceding engines stop, and have to come into motion when they come into motion. From control technical point of view this task has the specialty that the dynamic model of the members of the platoon are only approximately known. The carts may contain deformable loads as e.g. the liquid in an oil tank that is in dynamic interaction with the tank and the chassis carrying it. Besides the liquids, solid loads may be also considered as rigid bodies connected to the cart by some elastic, deformable junction generally having dissipative features, too. Normally the dynamical data of the loads are not known by the controller of the platoon and instead of the complicated task of developing/identifying their dynamic models/parameters application of some adaptive controller seems to be far more convenient and plausible practical solution.

Another important aspect in connection with incomplete modeling is the existence of two possible alternative approaches: application of a single, complex rough initial model containing each modeled degree of freedom, or tackling the problem in a "decentralized" manner in which certain subsystems are controlled by independent controllers modeling and controlling only certain degrees of freedom of the subsystem in their care. In this case, for the local, decentralized controllers, any dynamic coupling between the locally controlled subsystems appears as external perturbation influencing the behavior of the subsystem under their control. This problem was discussed in details e.g. in a plenary speech by D'Andrea in connection with the dynamic coupling of wings located in each other's vicinity in a wind channel [1]. On the inspiration by D'Andrea's paper similar effects were investigated in connection with the *centralized* and *de-*

centralized application of a previous version of the geometrically interpreted adaptive control developed at Budapest Tech [2]. The paradigm considered in these investigations consisted of two cart plus double pendulum systems coupled by an elastic spring. In each subsystem one of the pendulums represented the unmodeled internal dynamics. Besides that the coupling between the two carts was also unknown by the controllers. Heartened by the success of this approach in the present paper a novel, two parametric fixed point transformation based adaptive control is applied for the purpose of controlling the platoon. The idea of this new variant already was published in [3].

Besides reducing jogging or rumbling the loads carried the really important factor is the nice, smooth, conveniently followable behavior of the last vehicle that actually is "visible" for the other participants of public communication. In order to realize the above outlined program the following factors have to be taken into consideration: a) the controller to be designed will work as that of a *Single Input – Single Output (SISO)* system, since the acceleration of the last vehicle can be regarded as the *"output"* of the controller, while the acceleration of the first one can be manipulated as an *"input"*. However, in the reality the convoy with an internal control is a "*Multiple Input – Multiple Output (MIMO)*" system in which certain internal degrees of freedom are not directly controlled though their equations of motion are coupled with that of the directly controlled ones. In the simplest model in which the tracking rule takes into account only the distance and the 1^{st} time–derivative of the distance between the neighboring carts this system has high order: the 2^{nd} time–derivative of the trajectory of the preceding cart is related to higher derivative of the trajectory of the tracing one. If the 2^{nd} time derivative of the preceding cart is taken into account in the tracing policy of the tracing one the whole system remains a 2^{nd} order one.

Adaptivity can be built into this system in various manners. The simplest possibility is to prescribe a smooth nominal trajectory for the last cart, on which basis a trajectory "shifted" by the sums of the safe tracking distances can be prescribed for the 1^{st} cart. Since the motion of the last cart is realized through the chain of coupling between the platoon members the motion of the leading vehicle can adaptively controlled according to the observed motion of the last one. This solution corresponds to the application of a big adaptive loop for a system considered to be of "SISO" nature in which considerable perturbation of the trajectory of the leading cart can be expected due to inappropriate modeling of the chain as a whole. The operation of this construction expectedly can be improved if similar adaptive loops are introduced in decentralized manner to each cart within the train in order to more precisely realize the prescribed tracking algorithm. In the sequel at first the system's model used in the simulations is described. Following that simulation results are analyzed and conclusions are drawn.

2 The Model of the Coupled Vehicles

Due to the communication possibility between the vehicles and their local controllers the required *nominal acceleration* of the n^{th} member $[n > 1]$ is coupled to the motion of the $(n-1)^{th}$ one as follows:

$$\begin{gathered}\delta x_n := x_n - x_{n-1} + L_n, \delta\dot{x}_n := \dot{x}_n - \dot{x}_{n-1}\\ F_n^{Calc} = \widetilde{M}_n(\mu_n\ddot{x}_{n-1} - P_n\delta x_n - D_n\delta\dot{x}_n)\\ F_n^{Drive} = h_n\left(F_n^{Calc}\right)\\ \ddot{x}_n^{Nom} = (F_n^{Drive} + F_n^{Cont})/M_n\end{gathered} \tag{1}$$

in which L_n (m) denotes the required constant nominal distance between the center points of the n^{th} and the $(n-1)^{th}$ vehicles, P_n (s^{-2}), and D_n (s^{-1}) are the coefficients of the proportional and the derivative coupling between these vehicles. Function h_n is a sigmoid (i.e. strictly increasing function with upper and lower bounds yielding zero output to zero input) that represents the saturation in the available driving force at the appropriate local controller using the modeled mass $\widetilde{M}_n$ instead of the actual one M_n [both in (kg)]. The dimensionless parameter μ_n can take the value 1 if the acceleration of the preceding vehicle with respect to an inertial frame (i.e. the Earth) is taken into account, otherwise it takes 0. Phenomenologically (1) can be realized: the actual distance $x_n - x_{n-1}$ (m) can be measured by local sensors, while $\ddot{x}_{n-1}$ $(m \cdot s^{-2})$ is measurable by local accelerometers since the road is fixed with respect to the Earth that approximately corresponds to an inertial system. The contact force arising due to the internal dynamic interaction between the cart and its load also gives a contribution to it

$$F_n^{Cont} = k_n(x_n^{Load} - x_n) + \nu_n(\dot{x}_n^{Load} - \dot{x}_n) \tag{2}$$

in which k_n (N/m) and ν_n (Ns/m) denote the appropriate spring and damping constants, respectively. The acceleration of the n^{th} load is $\ddot{x}_n^{Load} = -\frac{F_n^{Cont}}{M_n^{Load}}$. The existence of these terms is unknown by the controller. In the case of some adaptive feedback $\ddot{x}_n^{Nom}$ can be replaced by a *desired value* $\ddot{x}_n^{Des}$ also containing the feedback corrections. It is worth noting that in (1) and (2) the various carts may have different parameters not exactly known by the controller in the leading vehicle determining $\ddot{x}_1$. Application of an adaptive controller operating on the basis of an approximate uniform model that need not to be "programmed" in the stage of collecting the convoy would also be a great practical advantage.

3 On the Possible Control Approaches

The complexity of the above task justifies seeking appropriate nonlinear control approach. Due to space limitation only very brief survey of the possible traditional approaches can be given.

3.1 Possible Traditional Approaches

In the possession of some analytical system model *Lyapunov's* 2^{nd} *Method* published in 1892 [4] is a widely used technique in the analysis of the stability of the motion of the *non-autonomous dynamic systems* of equation of motion as $\dot{x} = f(x,t)$. It has the great advantage that it does not require to analytically solve the equations of motion. Instead of that uses relatively simple estimations that can be done in the case of strongly nonlinear systems, too. In spite of its lucid and simple geometric interpretation this method in the practice is rather an "art" than some "mechanically applicable" tool. It requires not only great practice but also needs good intuition. Furthermore, the computational need of realizing the method is not always negligible. The "traditional" *Soft Computing* approaches are based on *Kolmogorov's Approximation Theorem* elaborated for multiple variable *continuous functions* [5]. In general these approaches suffer from the *"curse of dimensionality"* that is related to the fact that very extreme continuous functions exist. (The first example of a function that everywhere is continuous but nowhere is differentiable was given by Weierstraß in 1872.) On these reasons in this paper alternative solutions are sought for.

3.2 The Excitation and Response Scheme Combined with Fixed Point Transformation

It is worth noting that in this approach the desired response can freely be defined on the basis of purely kinematic considerations quite independently of the dynamic behavior of the system. *The task of realizing this desired response remains exclusively that of the adaptive part of the controller. This feature of this novel approach considerably differs from the characteristics of the Lyapunov functions based methods in which the dynamics of the error relaxation is an "inseparable, organic part" of the control. Furthermore, since this approach is based on the application of simple iterations, it cannot be formulated in single closed analytical formula as some "control statement" that normally occurs at the application of Lyapunov's method in which $\dot{V}$ can simply be prescribed. Instead of that an infinite series of cascade expressions $h(h(h(...)))$ appears for which appropriate convergence has to be guaranteed.*

Each control task can be formulated by using the concepts of the appropriate *"excitation"* Q of the controlled system to which it is expected to respond by some prescribed or *"desired response"* r^d. The appropriate excitation can be computed by the use of some *inverse dynamic model* $Q = \varphi(r^d)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a *realized response* r^r that differs from the desired one: $r^r \equiv \psi(\varphi(r^d)) \equiv f(r^d) \neq r^d$. It is worth noting that the functions $\varphi()$ and $\psi()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or "deform" the input value from r^d so that $r^d \equiv \psi(r^d_*)$. Other possibility is the manipulation of the output of the rough model as $r^d \equiv \psi(\varphi^*(r^d))$. In the sequel it will be shown that for *SISO* systems the appropriate deformation can be defined as some *Parametric Fixed Point Transformation*.

For this purpose consider the simple iteration described by (3) that is suggested by the similar triangles of Fig. 1.

$$
\begin{array}{c}
g(x|x^d, D_-, \Delta_+) := \\
= \frac{(f(x)-\Delta_+)(x-D_-)}{x^d-\Delta_+} + D_-, \\
if \quad f(x_\star) = x^d \quad then \quad g(x_\star) = x_\star, \\
g' = f'(x)\frac{x-D_-}{x^d-\Delta_+} + \frac{f(x)-\Delta_+}{x^d-\Delta_+}, \\
g'(x_\star|x^d, D_-, \Delta_+) = 1 + f'(x_\star)\frac{x_\star - D_-}{x^d-\Delta_+}
\end{array}
\qquad (3)
$$

According to (3) and the caption of Fig. 1 it is evident that the requested solution $x_\star$ just is the fixed–point of the function g, and that by appropriately choosing the initial model and the control parameters D_-, and Δ_+ the condition of $|dg/dx| \leq K < 1$ can be achieved in a vicinity of $x_\star$. This region around $x_\star$ serves as a basin of attraction for the iteration $x_{n+1} = g(x_n)$ since if a and b are within this region then $|g(a) - g(b)| = |\int_a^b g' dx| \leq \int_a^b |g'| dx \leq K|a-b|$, i.e. $g()$ realizes a *contractive mapping* resulting in a *Cauchy Sequence* in the complete normed linear space of the real numbers with the norm defined by the absolute value. Really, for arbitrary natural number L it

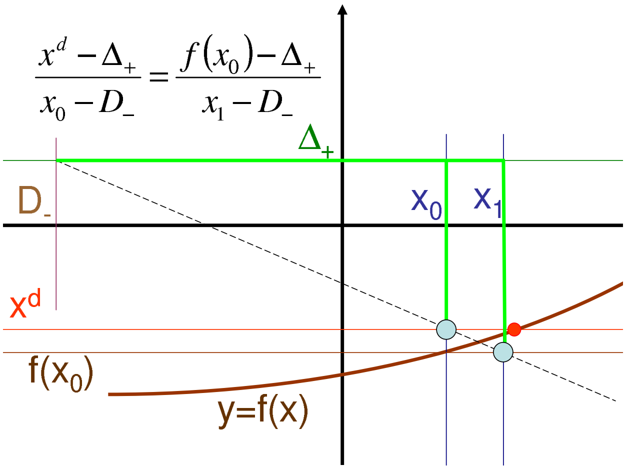

Figure 1: Fixed point transformation for $f'(x) > 0$ with parameters D_- and Δ_+ belonging to Eq.(3); if $x_\star > D_-$, $x^d < \Delta_+$, $f'(x_\star) > 0$, and $|f'(x_\star)|$ is small enough, the iteration generated by the function $g(x|x^d, D_-, \Delta_+)$ converges to $x_\star$

can be stated that

$$\begin{array}{c} |x_{n+L} - x_n| = |g(x_{n+L-1}) - g(x_{n-1})| \leq \dots \\ \leq K^n |x_L - x_0| \to 0 \;\; as \quad n \to \infty \end{array}, \tag{4}$$

therefore the sequence $\{x_n\}$ must converge to a limit value c. This limit value evidently must be equal to the fixed point $x_\star$ since $|g(c) - c| \equiv |(g(c) - x_n) + (x_n - c)| \leq |g(c) - x_n| + |x_n - c| = |g(c) - g(x_{n-1})| + |x_n - c| \leq K|c - x_{n-1}| + |x_n - c| \to 0$ as $n \to \infty$. For utilizing this convergence in the adaptive control it is just enough to prescribe some appropriate *desired response* on the basis of simple kinematical considerations. If the variation of x^d in time is far slower than the speed of convergence of the above iteration the idea of *Complete Stability* can similarly be applied as e.g. in the case of fast real-time image processing [7], and a good adaptive tracking control can be achieved. (More precisely, within one control cycle only one step of iteration can be executed.) This expectation is also supported by the results of previous investigations made in connection with less lucid fixed–point transformations [8]. In the sequel this idea will be used in the proposed adaptive control of the platoon.

4 Computational Results

The desired distance between the platoon members (totally 4 carts) was set to 6 (m). The control had the PD-type feedback for the last cart as $\ddot{x}_4^d = \ddot{x}_4^{Nom}(t) + P_{contrl}(x_4^{Nom}(t) - x_4(t)) + D_{contrl}(\dot{x}_4^{Nom}(t) - \dot{x}_4(t))$ with $P_{contrl} = 0.3\ (1/s^2)$ and $D_{contrl} \approx 5.477\ (1/s)$ that guarantees oscillation–free tracking. The carts had the mass of $M_n = 1200\ (kg)$, the loads had

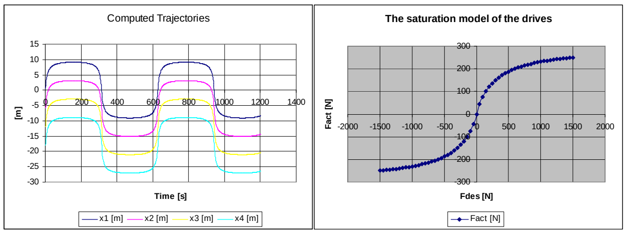

Figure 2: The trajectories to be traced by the members of the platoon vs. time (LHS), and the sigmoid function used for modeling saturation of the drives (RHS)

$M_n^{Load} = 200\ (kg)$ masses, each. The springs between the loads and the carts had $k_n = 300\ (N/m)$ spring constants and $\nu_n \approx 346\ Ns/m$ damping coefficients. The model values were $\widetilde{M_n} = 500\ (kg)$, and the control rule for tracking the preceding carts had $P_n = 0.4\ (s^{-2})$ and $D_n \approx 6.325\ (s^{-1})$, each. It is worth noting that keeping the cart–load displacements within realistic limits very stiff springs with very strong viscous damping had to be supposed. The adaptive fixed–point transformation in each run had the parameters as $D_- = -20$ and $\Delta_+ = 80$ [both in (m/s^2)], both for the "big" adaptive loop directly connecting to each other the motion of the first and the last carts, and for the internal loops of the decentralized controllers. The internal adaptive loops were used in each result presented, the qualifiers as *"adaptive"* and *"non-adaptive"* concern only the big external loop. The trajectories to be traced as well as the nature of the sigmoid function modeling saturation issues are described in Fig. 2. They contain flat parts, and sharply accelerating/decelerating stages, too. In each case investigated within the scaling of this chart only minimal differences can be observed, therefore in the sequel the tracking errors will be used instead of the counterparts of this chart.

As it can well be seen in Fig. 3 the worst result is obtained in the case of the *"non-adaptive distance and relative velocity control"* as it was evidently expected. Due to the exisence of the decentralized internal adaptive loops of the individual carts, there is no significant difference between the 2^{nd} (*"adaptive distance and relative velocity control"*) and the 3^{rd} (*"non-adaptive acceleration control"*) rows results that can be simply understood: when the outer adaptive loop is switched off just the acceleration of the last cart is prescribed for the leading one. Since the internal adaptive loops guarantee precise acceleration tracking the proper acceleration of the last cart is guaranteed, too. The situation was quite different when no internal decentralized adaptive loops were in use. Regarding the 2^{nd} row, when the precise distance and relative velocity tracking also is guaranteed by the internal adaptive loops,

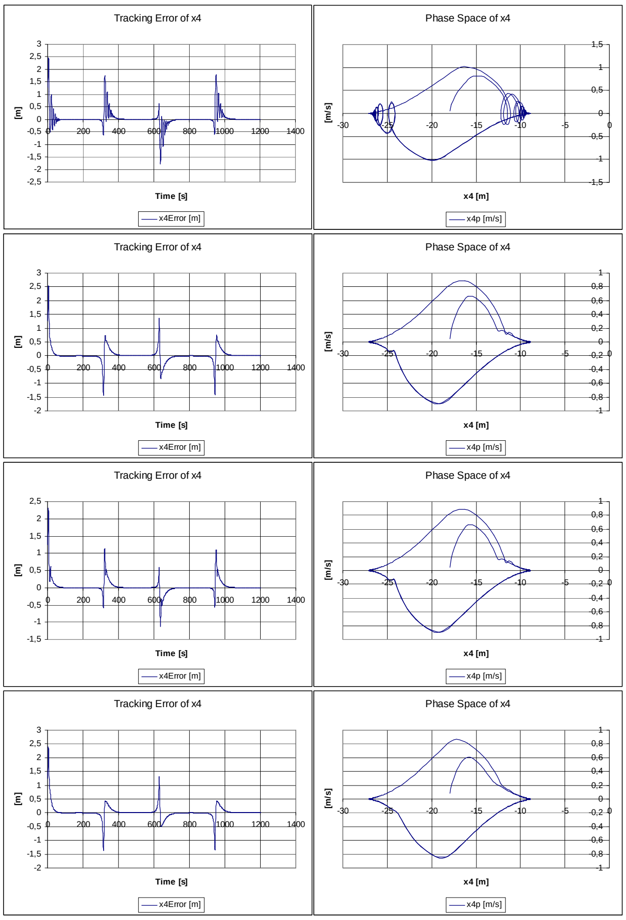

Figure 3: The tracking error vs. time (LHS), and the phase space (RHS) of the motion of the last cart (No. 4) for *"non-adaptive distance and relative velocity control"* (1^{st} row), *"adaptive distance and relative velocity control"* (2^{nd} row), *"non-adaptive acceleration control"* (3^{rd} row), and *"adaptive acceleration control"* (4^{th} row)

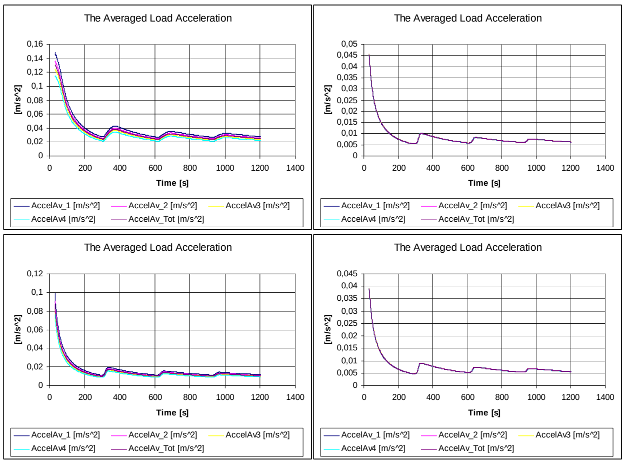

Figure 4: Time-averaged absolute values of the load accelerations vs. time: *"non-adaptive distance and relative velocity control"* (upper left), *"adaptive distance and relative velocity control"* (upper right), *"non-adaptive acceleration control"* (lower left), and *"adaptive acceleration control"* (lower right)

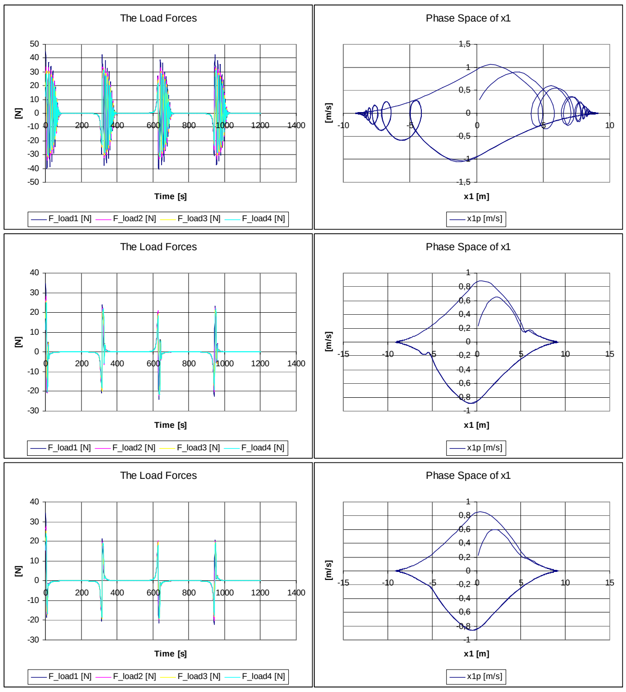

Figure 5: The load–vehicle contact forces vs. time (LHS), and the phase space (RHS) of the motion of the first cart for *"non-adaptive distance and relative velocity control"* (1^{st} row), *"adaptive distance and relative velocity control"* (2^{nd} row), and *"adaptive acceleration control"* (3^{rd} row)

this simple strategy yields good results, too. However, the best result is obtained for the *"adaptive acceleration control"* (4^{th} row).

The time-averaged absolute values of the accelerations of the loads with respect to an inertial frame are depicted in Fig. 4 according to which the *"adaptive acceleration tracking"* seems to be the superior solution. The details described on Fig. 5 substantiate the same observation by displaying the contact forces and the phase space of the motion of the leading cart.

Alternative way for describing jogging of the loads is chsoen in Fig. 6 in which the phase spaces of the cart–load relative motions became better by the fully adaptive control. It reveals that fine adaptivity reduces the unnecessary push–pull sections that can excite the coupled, directly not controlled internal degrees of freedom.

5 Conclusions

In this paper approximate modeling and adaptive control of platoons equipped with various internal tracking strategies were presented. It was shown that by introducing nonlinear saturation of the forces of the drives to represent their limited capacities this system becomes a strongly coupled, partially and inaccurately modeled nonlinear system within which cou-

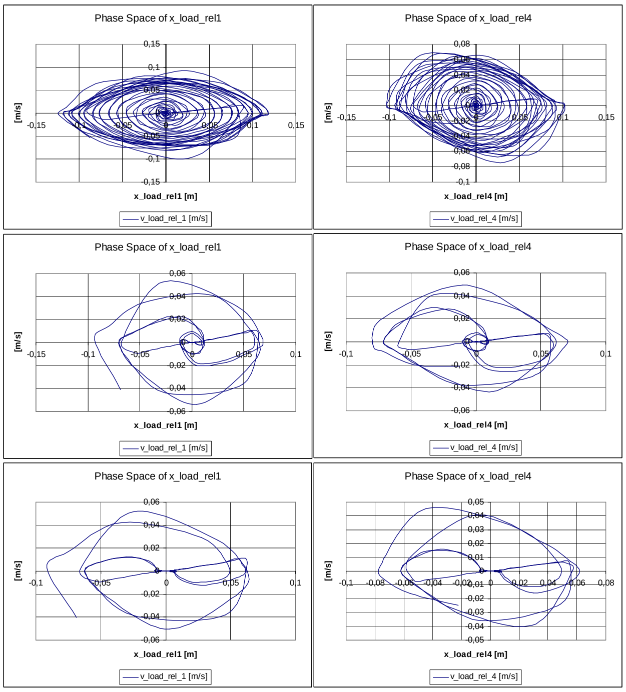

Figure 6: Phase spaces of certain load-vehicle relative motions for *"non-adaptive distance and relative velocity control"* (1^{st} row), *"adaptive distance and relative velocity control"* (2^{nd} row), and *"adaptive acceleration control"* (3^{rd} row)

pled internal degrees of freedom unknown by the controller can be excited. To develop an adaptive control for such a system various traditional theoretical possibilities were pondered or considered.

Due to its simplicity a novel adaptive controller developed at Budapest Tech in the recent years was proposed. As an alternative of Lyapunov's 2^{nd} Method this controller transforms the control task into a fixed point problem of a contractive map. It is well known that in complete linear metric spaces the iterations generated by a contractive map converge to its fixed point. The control algorithm corresponds to executing exactly one iterative step within each control cycle. It was found that the use of decentralized local controllers and a single big adaptive loop resulst in nice and precise control of this system.

In the further research investigation of the dynamic asymmetries present in the platoon would be expedient.

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