Particle Swarm Optimization for the continuous p-median problem

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Abstract: This work describes the application of the Particle Swarm Optimization (PSO) metaheuristic to the continuous version of the p-median problem. The problem consists in determining p points minimizing the weighted distance to a given set of points. This is an NP-hard problem and several evolutionary algorithms and other metaheuristics have been applied to solve it. We propose a new version of the PSO metaheuristic for this problem that takes into account the real meaning of the solution and that is combined with a Local Search. The proposed procedure is compared with the standard version of the PSO and its combination with the local search is compared with the Multi-Start method.

Key–Words: Particle Swarm Optimization, p-Median Problem, Facility Location, Metaheuristic, Evolutionary algorithm

1 Introduction

The Particle Swarm Optimization (PSO) is a promising relatively recent metaheuristic introduced by James Kennedy and Russel Eberhat [8] [11]. It is a evolutionary method inspired by the social behavior of individuals within clusters in nature, like flocks of birds or banks of fish. In order to do it a set of potential or alternative solutions of the problem are represented like members of the cluster that lie down to fly in the virtual space of the possible solutions. In the planning and logistic optimization it is necessary to adopt three types of decisions: strategic or long term ones (every several months or years), tactics or mid term ones (every few weeks or months) and operative or short time ones (several times in a day or a week). The metaheuristics are important in the support of the three types of decisions. The three types of problems more important in logistic, the location problems, route problems and load problems, correspond preponderantly to strategic, tactical and operative decisions, respectively. The intelligent evolutionary systems, as the PSO, are appropriated in decision environments where new elements often arise in the problem. For these circumstances, specific procedures that fit to the changing model are nonviable. In these environments, procedures that, like PSO, are not very depending on the characteristics of the problem are more and more necessary. Nevertheless, the initial tests of strategies for the application of PSO can become instances of the standard problems of location; the p-median problem. In this work we raised the application of the standard versions of the PSO and a specific approach that considers the real separation between two solutions. The procedure is combined with a local search. The improvement of the contributed parameters is analyzed experimentally.

The following sections introduce the standard version of the PSO. The continuous p-median problem is described. A new implementation of the PSO method combined with a local search sets out. Finally we offer a description of the computational experience made and brief conclusions.

2 The PSO

The PSO metaheuristic is inspired by the continuous movement of the particles that form a swarm taking into account the inertia and the attraction of the best members that lead the swarm. Two of the most important characteristics in the development of this metaheuristic are the easy implementation and the use of the evolution of social relations like computational model. In a PSO heuristic, particles or members of the swarm are interpreted like search agents who cross the virtual space of solutions. The PSO [6] is a procedure to solve optimization problems with continuous variables. In a problem with d continuous variables, each particle i of the swarm $S = \{1, 2, , s\}$ has associate position vector

$$\mathbf{x}_i = (x_{i1}, x_{i2}, , x_{ij}, ..., x_{id})$$

and its velocity or rate of change is

$$\mathbf{v}_i = (v_{i1}, ..., v_{ij}, , v_{id}).$$

Each particle i of the swarm communicates with a subgroup of the swarm or social neighborhood $N(i) \subseteq S$ that can vary dynamically in the evolution. Each particle keeps and uses information of its better position during the search process. It also can obtain the best position reached by the particles of his social neighborhood, that can be all the swarm or a part.

The information of the best positions influences in the behavior of particles. In all these cases the value of the objective, like fitness function, is also stored. The initial position and velocity of the particles are usually obtained randomly within their ranks. In each iteration, the particles update their position and velocity by means of recurrent formula. The position of each particle is modified using only its velocity. For updating velocity of a particle, in addition to the current value of the own velocity, the formula takes into account its own best position and the best position of the group of particles of the swarm to which it is related; its social neighborhood. These better positions, individual and collective, act with different weights, like attraction points for the particles. The vectorial equations to update the position $\mathbf{x}_i$ and velocity $\mathbf{v}_i$ of i-th particle of the swarm, in standard procedure PSO according to the proposal of Kennedy and Eberhart [12] are the following ones:

$$\begin{aligned} \mathbf{x}_i &\leftarrow \mathbf{x}_i + \mathbf{v}_i \\ \mathbf{v}_i &\leftarrow w \cdot \mathbf{v}_i + c_1 \cdot \xi \cdot (\mathbf{b}_i - \mathbf{x}_i) + c_2 \cdot \xi \cdot (\mathbf{g}_i - \mathbf{x}_i) \end{aligned}$$

where vectors $\mathbf{b}_i$ and $\mathbf{g}_i$ are the best position that has had the i-th particle since the procedure began and the best position between those that have had all particles of the group or social neighborhoods of this particle. The term ξ refers to a random number with uniform distribution in the unit interval $[0, 1]$ that is independently obtained each time. The parameter w represents the effect of the inertia whose mission is to control the magnitude of the velocity and to avoid that it grows indefinitely. The scalars c_1 and c_2 are the weights that represent the degree of confidence of the particle, in itself and in its social group, that in many versions agree. These quantities usually are positive and inferior to one and in several versions $w = 1$, or $c_1 = c_2$ or even $c_1 + c_2 = 1 - w$.

In the standard version PSO-2007 proposed by Maurice Clerc[1], the structure of neighborhoods is obtained at random from a fixed number K as follows. Each possible link is activated with probability $p = 1 - (1 - 1/s)^K$. The neighborhoods are changed whenever best global position $\mathbf{g}^*$ does not improve. The values of the parameters assumed in this standard version are: $w = 1(2 + \; ln2) = 0.721$ and $c_1 = c_2 = 0.5 + \ln 2 = 1.193$; in addition $K = 3$ and the size to the swarm is fixed to $|S| = 10 + 2\sqrt{d}$ where d is the dimension of the space of solutions.

Aside from the random selection of the neighborhoods, the two more current topologies for the structure neighborhoods are the ring N_r and stars N_s neighborhood. With the ring structure each particle interacts with the previous one and later (in a arrangement considered cyclical) and in the star structure each particle interacts with all particles. Formally, these neighborhoods come defined by:

$$N_r(i) = \{i - 1, i, i + 1\}, 1 < i < s$$

$$N_r(1) = \{s, 1, 2\}, N_r(s) = \{s - 1, s, 1\}.$$

and

$$N_s(i) = S, 1 \leq i \leq s.$$

3 The continuous p-median problem

The p-median problem is one of the most important problems in location of services and constitutes one of the centers of interest in the logistic planning [4]. This problem corresponds to an ideal situation in the analysis of important strategic decisions in logistic, as they are those of location. Nevertheless, the p-median problem usually does not appear in pure state in real situations but with restrictions or additional costs, and mixed with other problems. One of the questions that confer that importance to the p-median problem is that it serves as model and prototype for other problems on which the alternative solutions are based on choosing a fixed number of elements.

In order to formalize the p-median problem as the problem of locating a service in a space E, let $Z = \{z_1, z_2, \cdots, z_n\}$ be a set of n demand points in E where are the users of the service, and let $L \subseteq E$ be a set of possible location points for the service. Each demand point z_i usually has associate a weight w_i, that represents the amount that it demands or the number of users in this point. The p-median problem consists of determining p simultaneous positions in L (the medians) in which to establish the service, so that the total cost of transports necessary to satisfy all the demands of the users is minimized, supposing that this cost is proportional to amount of demand and to the distance traveled. Therefore, each user will be served by its closest solution point (median) than has a unbounded capacity. In the standard continuous problem

[1] http://www.particleswarm.info/standard_pso_2007.c

the space E in which are the demand points and the possible locations for the services is the whole plane with euclidean distance, and L is the whole space E. In other continuous problems, the solution space is of greater dimension or is a limited convex region that includes Z.

The continuous p-median problem in the euclidean plane is an NP-hard optimization problem ([14] and [10]). The continuous p-median problem is also named the multiple facility Weber problem [18] and it is an important problem in location since the initial work of Cooper [7] and it is also an interesting problem from the point of view of computational geometry. Formally, the problem consists of, given n points $Z = \{z_1, z_2, , z_n\}$ with their corresponding weights w_i, to find the set X of p points that minimizes the function:

$$F(X) = \sum_{i=1}^{n} w_i \min_{x \in X} d(x, z_i).$$

The distance between a set of points X and a demand point z is

$$d(X, z) = \min_{x \in X} d(x, z).$$

So the problem is to minimize the weighted sum of the distance between X and the demand points. Note, that each demand point is served by its nearest median $x \in X$. Therefore if $X = \{x_1, ..., x_j, ..., x_p\}$ is the optimal solution of the p-median problem and

$$Z(x_j) = \{z \in Z : d(X, z) = d(x_j, z)\}$$

then x_j must be the optimal solution of the 1-median problem with demand point set $Z(x_j)$. The 1-median problem is also known as the simple Weber problem or continuous median problem [16] and consists of finding the point of the plane that minimizes the weighed sum of the distances to a finite set of points Z'; the optimal solution is named the (weighted) median of the demand set.

There is not an algorithm that provides the optimal solution of this problem in a finite number of steps and the most practical algorithm consists of applying to the iterative Weiszfeld method [17]. The Weiszfeld method to get the median of a set $Z' = \{z'_1, z'_2, , z'_m\}$ with weights w'_i consists of iteratively applying the following transformation to an arbitrary point of the plane:

$$T(x) = \frac{\displaystyle\sum_{i=1}^{m} \frac{w'_i}{d(x, z'_i)} z'_i}{\displaystyle\sum_{i=1}^{m} \frac{w'_i}{d(x, z'_i)}}.$$

The sequence of points of the plane given by the equation

$$x_{i+1} = T(x_i)$$

converges to the median of Z', unless in some iteration this points belongs to Z'. This situation, although possible is very improbable; the set of starting points, from which the succession reaches a point of Z' that is not solution of the problem, is a numerable set. A later modification of the algorithm allows to solve this case; if the succession stagnates in a point it is simple to verify if this point is the solution of the problem or not. In case that it is not the solution, the algorithm restarts the sequence from a different point, in the hope of which it does not return to happen the same; what is almost sure.

The continuous p-median problem has been extended in multiple directions: without fixing the number of locations and including capacity constraints, location costs, barriers, different type of norms or pseudo-norms instead of the Euclidean norm, etc. A revision of the heuristic methods applied to the continuous p-median problem is in [5]; with posteriority other heuristics have appeared in [9], [1] and [15].

4 New PSO for the p-median

In order to apply the standard version of the PSO heuristic to the continuous p-median problem in the plane we represented each possible solution consisting of p points by the $2p$ paired coordinates. Therefore, the position and velocity vectors are vectors in the $2d$-dimensional space without constraints. Nevertheless, simple tests show that the election by defect of the parameters for the standard version can be improved.

Our new version of the PSO considers the real separation between the solutions when they are understood like sets of points. In addition the Weiszfeld algorithm is used to improve the position of particles derived from its velocity. The proposed version tries to physically reflect the attraction of the best position of the particle and its group of informants. Instead of using the vectorial difference of the leader position and the current one, component to component, without considering its arrangement, in the update of the velocity, we use the proximity of medians of the leader position to direct the attraction on each point of the position of the particle.

The second adding of the equation of update of the velocity is calculated as follows. For each one of the p points of the set represented by the position of the particle, the closest point of the solution represented by the leader particle is determined. The vectorial difference between both points of the plane is multiplied by the random factor. This is done with the best position of the particle as a leader and with the best position of the group in the third adding of the update.

Formally, the position of a particle is given by the vector from dimension $2p$. The position of the i-th particle is given by:

$$\mathbf{x}_i = ((x_{i1}^1, x_{i1}^2), ..., (x_{ij}^1, x_{ij}^2), ..., (x_{ip}^1, x_{ip}^2))$$

o equivalently by

$$\mathbf{x}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{ij}, ..., \mathbf{x}_{id})$$

where for each $j = 1, 2, ..., p$, the component $\mathbf{x}_{ij}$ of the position is the point of the plane given through its coordinates by $\mathbf{x}_{ij} = (x_{ij}^1, x_{ij}^2)$. If the equation of update of velocity of a particle is expressed by

$$\mathbf{v}_i = c_1 \cdot \mathbf{v}_i + c_2 \cdot \xi \cdot (\mathbf{b}_i \ominus \mathbf{x}_i) + c_3 \cdot \xi \cdot (\mathbf{g} \ominus \mathbf{x}_i),$$

then the operation $\ominus$ acts as follows. If **x** and **y** represent two sets of p points of the plane, the vector $z = x \ominus y$ is obtained in the following way. For each point x_j of x the closest point y_j^* of y to x_j is determined; that is the point that minimizes $d(x_j, y_k)$ and the differences constitute the components of z that correspond to the point x_j. Formally:

$$z_j = x_j - y_j^* \text{ where } d(x_j, y_j^*) = \min_k d(x_i, y_k).$$

Another aspect in which the original version of the PSO is modified is the update of the position. After obtaining the new position x_j by algebraically adding the velocity v_j, the demand points in Z are assigned to the closest point of the solution represented by particle. With each one of these points, x_j, as starting point, the procedure applies the Weiszfeld algorithm with the set the demand points that have been assigned to it, $Z(x_j)$. Formally, these sets are

$$Z_j = \{z\, inZ : d(z, x_j) = \min_k d(z, x_k)\}.$$

The proposed PSO replaces each point x_j of the particle by the result x_j^* of applying the Weiszfeld algorithm to the set Z_j with x_j as starting point.

5 Experiments

The objective of the experiments is to show the behavior of the new version of proposed PSO (without the local search) in comparison with the standard version of the method. The combination of the new and standard versions of the PSO with the local search is also compared with a Multi-Start method [13] the perform the same number of local searches from random initial solutions. For the computational experience several problems of the OR-Library [2], that are often used to test heuristics for the continuous of p-median problem, were selected. Two sets of 287 and 564 points were used, with its original corresponding weights of [3] that have also been used in [1], [9] and [15].

Table I and II show the results for the comparisons. The first column are the number of medians considered in the experiments and the rest of the column show the values of the objective function with the solutions found. Second and third columns show the objective values of the solutions found with the standard version of the PSO proposed by Maurice Clerc (StPSO) and with the new version proposed here (NPSO). The swarm was evolving for one hundred thousand generations in each case. Columns four and five of these two tables show the the objective values of the solutions found with the standard version of the PSO and with the new version combined with the Weiszfeld local learch (StPSO+LS and NPSO+LS). The sixth column includes, for comparison purpose, the best objective valued reached with a multi-start method (MS+LS) that used the same number of local searches form random initial solutions.

In order to make comparisons on the continuous p-median problem, standard version PSO-2007 for $2p$ variable was executed. The rest of the parameter for the new PSO were the same proposed for the standard version. The size of the swarm that in the standard version PSO-2007 depends on the number of variables $d = 2p$ by the equation: $s = 10 + 2\lfloor\sqrt{d}\rfloor$ where $\lfloor.\rfloor$ is the integer part. This value stays for the four compared versions. The neighborhood structure in PSO-2007 is random with the number of communicants of each particle $k = 3$. Finally, the coefficients of inertia and confidences proposed in the standard version according to the Clerc's Stagnation Analysis were $w = 1/(2 + \log 2) = 0,721$ and $c_1 = c_1 = 1/2 + log2 = 1.193$.

6 Conclusions

In this work the application of the PSO metaheuristic to the continuous p-median problem has been analyzed. The standard version of the PSO, PSO-2007, proposed by Maurice Clerc has been adapted to the continuous p-median problem. A New version of PSO is proposed considering the attraction between the solutions represented by particles following a faithful interpretation the problem. In addition it has been proposed the incorporation of a local search to improve the positions of particles after each update. The obtained experimental results show that our proposal

Table 1: Comparing the results for the instance with 287 points

p	StPSO	NPSO	StPSO+LS	NPSO+LS	MS+LS	Optima
2	14488.70	14448.88	14434.48	14431.92	14434.41	14427.59
3	13687.60	12563.79	12143.47	12161.42	12137.85	12095.44
4	12784.38	11734.25	10895.05	10735.74	10969.41	10661.48
5	12205.65	10588.56	10129.17	9836.13	10128.65	9715.63
6	11790.50	10243.52	9581.41	9068.72	9800.56	8787.56
7	11245.15	9732.47	8972.99	8354.21	9403.97	8160.32
8	11435.88	9428.73	8943.19	8047.47	9055.68	7564.29
9	10948.41	8877.79	8717.08	7844.99	8791.66	7088.13
10	10901.40	8869.72	8574.84	7492.21	8456.48	6705.04
11	10177.23	7938.26	8608.71	7016.39	8313.94	6351.59
12	10195.75	7535.52	8344.82	6609.31	7957.18	6033.05
13	9970.47	7509.22	8128.71	6705.37	7844.16	5725.19
14	9741.95	7345.54	7766.41	6279.56	7567.33	5469.65
15	9824.27	7540.35	7829.33	6195.82	7767.35	5224.70
16	9760.08	6984.96	7691.48	6472.31	7683.83	4981.96
17	9574.54	7270.75	7393.86	6132.09	7450.52	4755.19
18	9549.47	6934.07	7130.35	5677.44	7232.64	4547.37
19	8742.14	6642.70	7064.38	5683.48	6888.95	4342.06
20	9148.22	6634.67	7132.89	5436.79	7244.67	4148.84

improves the quality of the resulting solutions. Also the introduction of the local search is able to improve still more the quality of the contributed solutions and also the solutions obtained with the standard PSO. These combinations of PSO with the Weiszfeld local search provided better results that a multi-start procedure that applies the same number of local searches. As future investigations will be deepened in the experimental study of the aspects considered here. In addition we will study possibility of using other neighborhood structures and to parallelize the procedure. Also the application to other standard problems in logistic planning and to mixed and more realistic problems will be considered.

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Table 2: Comparing the results for the instance 654 points

p	StPSO-2007	NPSO	StPSO+LS	NPSO+LS	MS+LS	Optima
2	816039.01	815637.89	815313.30	815313.30	815313.30	815313.30
3	571972.87	557574.70	551062.91	551062.91	551062.91	551062.88
4	380162.03	346377.24	288203.57	288200.68	288195.18	288190.99
5	314705.56	285146.13	209080.62	209076.34	209079.72	209068.79
6	305368.74	284262.86	180610.41	180551.82	180547.79	180488.21
7	269924.58	234270.74	164105.13	163951.16	163991.10	163704.17
8	239756.84	218274.21	148306.09	147634.04	148132.56	147050.79
9	235506.89	237490.09	131724.85	132028.37	132655.15	130936.12
10	235394.36	229802.04	117747.76	117041.09	116775.22	115339.03

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