FUZZY CLUSTERING BASED MODELS FOR SUPERVISION OF INDUSTRIAL PROCESSES

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Abstract: The application of fuzzy clustering techniques has recently become in a very useful alternative in the area of modeling and identification of complex industrial process. In particular, fuzzy clustering techniques such as fuzzy c-means and the Gustafson-Kessel (GK) algorithms will be analyzed and applied in details in this paper. These algorithms will be implemented in the construction of Takagi-Sugeno fuzzy models for the gas-liquid separation process and the water-oil separation process, which are important processes in the oil industry. This sort of modeling will be the base for the design of a supervisor control system. Validations of the obtained fuzzy models will be performed and some conclusions will be established.

Key Words: Fuzzy clustering, fuzzy c-means, Gustafson-Kessel (GK) algorithm, separator of production, artificial lift production methods.

1 Introduction

Modeling and identification are essentially the first steps previous for designing control, supervision and failure detection systems. The traditional approach for modeling is based on the precise knowledge of the system under consideration and its dynamic behaviour that along with an appropriate mathematical formulation, may lead to a representative model of the system [3]. This type of traditional first-principles models are usually known as white box models or mechanistic models which are based on deep knowledge of the nature of the system. However, there exists some complex physical process that are not amenable to conventional modeling approaches, due to the lack of precise, formal knowledge about the system, the presence of strong nonlinearities and high degree of uncertainty, or due to time varying characteristics[4].

A simple and intuitive technique for modeling complex processes can be achieved by dividing the global model of the system into a set of local models, where each local model has got its specific validity range. The global model of the system may be obtained by means of the integration of all the local models through the use of a fuzzy rules base that selects the appropriate local model according to the specific condition of the process [2]. The determination of the correct number of local models and the identification of these models are the major drawbacks of this sort of technique. There is an approach that contemplates the use of measured data of the process, the selection of the structure of the system based on fuzzy rules and the estimation of local parameters (parametric identification) [1]. Basically, fuzzy clustering algorithms are the most frequently used in the fuzzy identification of complex system [5][6], though there exists some other alternatives based for instance in neurofuzzy approaches [7].

In general, fuzzy identification is an effective tool for the approximation of dynamic nonlinear system on the basis of measured data [13]. Among the different fuzzy modeling techniques, the Takagi-Sugeno model has attracted most attention [14]. This model consists of IF-THEN rules with fuzzy antecedents and mathematical functions in the consequent part. The antecedent fuzzy sets partition the input space into a number of fuzzy regions, while the consequent functions describe the system's behaviour in these regions. The construction of a Takagi-Sugeno model is generally done in two steps. In the first step, the antecedent fuzzy sets are determined either by means of heuristic knowledge of the process under consideration or by using some data-driven techniques. In the second step, the consequent functions parameters are estimated. As these functions are usually chosen to be linear in their parameters, standard linear least-squares methods can be employed as estimation technique [15].

The rest of the paper is organized as follows. In section 2, the Takagi-Sugeno fuzzy model is analyzed taking into account the linear AutoRegressive with eXogenous input model (ARX). Section 3 describes in details the fuzzy c-means and Gustafson-Kessel algorithms. Section 4 will briefly describe some operations related to the oil industry and two cases of study will be presented. The cases of study deal with the determination of fuzzy models for the gas-liquid and water-oil separation process on the basis of measured data; some computer simulation result will be presented and a computer validation of the fuzzy models obtained will also be performed. Conclusions are given in section 6.

2 Takagi-Sugeno Fuzzy Model For Nonlinear System

Nonlinear dynamic systems are often represented in the Nonlinear AutoRegressive with eXogenous input (NARX) model form [16]. This kind of model establishes a nonlinear relation between the past inputs and outputs and the predicted output of the system, which can be represented as follows:

$$\begin{aligned}\mathbf{y}(\tau+1) = f([&y_1(\tau),\dots,y_1(\tau-n_y),\dots,y_p(\tau)\\ &,\dots,y_p(\tau-n_y),u_1(\tau),\dots,u_1(\tau-n_u),\dots u_q(\tau)\\ &,\dots,u_q(\tau-n_u)]^T)\end{aligned} \quad \textbf{(1)}$$

Here n_y and n_u denotes the maximum lags considered for the output and input terms, respectively. Equation (1) represents a NARX model with q inputs and p outputs and f represents the mapping of the NARX model. In every NARX model, there exists a term called state vector which is also known as the regressor matrix that can be written as follows:

$$\begin{aligned}\mathbf{x}(\tau) = [&y_1(\tau),\dots,y_1(\tau-n_y),\dots,y_p(\tau)\\ &,\dots,y_p(\tau-n_y),u_1(\tau),\dots,u_1(\tau-n_u),\dots u_q(\tau)\\ &,\dots,u_q(\tau-n_u)]^T\end{aligned} \quad \textbf{(2)}$$

If $\mathbf{x}(\tau)$ from (2) is substituted in (1), then the following compact equation is obtained:

$$\mathbf{y}(\tau+1) = f(\mathbf{x}(\tau)), \quad \textbf{(3)}$$

where $\mathbf{y}(\tau+1)$ is the regressand or outputs predicted vector(MIMO case).

Let's consider the identification of the nonlinear system:

$$\mathbf{y}_k = f(\mathbf{x}_k) \quad \textbf{(4)}$$

based on some availabe input-output measured data $\mathbf{x}_k = [\mathbf{x}_{1,k}, \dots, \mathbf{x}_{n,k}]^T$ y $\mathbf{y}_k$, respectively, where k = 1,2,…,N denotes the individual samples of each variable. As it is difficult to get a model that describes the system as a whole, it is always possible to construct local linear models with representation capabilities around specific selected operating points. The modeling framework that is based on combining local models valid in predefined operating regions is called Operating Regime-Based Modeling [15]. In this framework, the model is generally given by:

$$\mathbf{y}_k = \sum_{i=1}^{c} \phi_i(\mathbf{x}_k)(\mathbf{a}_i^T \mathbf{x}_\mathbf{k} + b_i) \quad \textbf{(5)}$$

Where $\phi_i(\mathbf{x}_k)$ is the validity function for the i-th operating region and $\theta_i = [\mathbf{a}_i^T, b_i]^T$ is the parameter vector of the corresponding local linear model[16]. The operating regime can also be represented by fuzzy sets in which case the Takagi-Sugeno fuzzy model is obtained:

$$R_i:\ \textbf{If } \mathbf{x}_k \textbf{ is } \mathbf{A}_i(\mathbf{x}_k) \textbf{ Then } \mathbf{y}_k = \mathbf{a}_i^T \mathbf{x}_k + b_i, \quad [w_i] \quad i = 1,\dots c. \quad \textbf{(6)}$$

Here $\mathbf{A}_i(x)$ is a multivariable membership function, $\mathbf{a}_i$ y b_i are the parameters of the local linear model, and $w_i \in [0,1]$ is the weight of the rule. The value of w_i is usually chosen by the designer of the fuzzy system to represent the belief in the accuracy of the i-th rule. When such knowledge is not available w_i = 1, $\forall$i is used. The antecedent proposition "$\mathbf{x}_k$ *is* $\mathbf{A}_i(x)$" can be expressed as a logical combination of proposition with univariate fuzzy sets defined for the individual components of **x**, generally in the following conjunctive form:

$$\begin{aligned}R_i:\ &\textbf{If } \mathbf{x}_{1,k} \textbf{ is } \mathbf{A}_{i,1}(\mathbf{x}_{1,k}) \textbf{ and ... and } \mathbf{x}_{n,k} \textbf{ is } \mathbf{A}_{i,n}(\mathbf{x}_{n,k})\\ &\textbf{Then } \mathbf{y}_k = \mathbf{a}_i^T \mathbf{x} + b_i,\ [w_i] = 1,\dots,c\end{aligned} \quad \textbf{(7)}$$

The degree of fulfillment of the rule is then calculated as the product of the individual membership degrees and the rule's weight:

$$\beta_i(\mathbf{x}_k) = w_i \mathbf{A}_\mathbf{i}(\mathbf{x}_k) = w_i \prod_{j=1}^{n} A_{i,j}(\mathbf{x}_{j,k}) \quad \textbf{(8)}$$

The rules are aggregated by using the fuzzy-mean formula:

$$\mathbf{y}_k = \frac{\sum_{i=1}^{c} \beta_i(\mathbf{x}_k)(\mathbf{a}_i^T x + b_i)}{\sum_{i=1}^{c} \beta_i(\mathbf{x}_k)} \quad \textbf{(9)}$$

From (5) to (9) it can be observed that the Takagi-Sugeno fuzzy model is equivalent to the Operating Regime-Based Model when the validity function is chosen to be the normalized rule degree of fulfillment:

$$\phi_i(\mathbf{x}_k) = \frac{\beta_i(\mathbf{x}_k)}{\sum_{i=1}^{c} \beta_i(\mathbf{x}_k)} \qquad \textbf{(10)}$$

3 Fuzzy Clustering Algorithms

Fuzzy clustering techniques are mostly unsupervised algorithms that are used to decompose a given set of objects into subgroups or clusters based on similarity. The goal is to divide the data-set in such a way that objects (or example cases) belonging to the same clusters are as similar as possible, whereas objects belonging to different clusters are as dissimilar as possible. [8].

Fuzzy cluster analysis allows the formation of gradual membership of measured data points to clusters as degrees in the range [0, 1]. Furthermore, these membership degrees offer a much finer degree of detail of the data model. Aside from assigning a data point to clusters in shares, membership degrees can also express how ambiguously or definitely a data point should belong to a cluster. The concept of these membership degrees is substantiated by the definition and interpretation of fuzzy sets [9]. Clustering techniques can be applied to data that are quantitative (numerical), qualitative, or a mixture of both and in general, such data are related to observed data of some specific process [10].

The available data samples are collected in matrix **Z**, formed by concatenating the regression data matrix **x** and the output vector **y**:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{x}_N^T \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_N \end{bmatrix}, \quad \mathbf{Z}^\mathbf{T} = [\mathbf{x}, \mathbf{y}] \quad \textbf{(11)}$$

Thus, each observation is a n+1- dimensional column vector expressed as: $\mathbf{z}_k = [\mathbf{x}_{1,k}, \ldots, \mathbf{x}_{n,k}, \mathbf{y}_k]^T = [\mathbf{x}_k, \mathbf{y}_k]^T$. Through clustering, the data set **Z** is partitioned into c clusters.

3.1 Fuzzy C-Means Objective Function

The most common objective function used in fuzzy clustering algorithms is given by:

$$J(Z;U,V) = \sum_{i=1}^{c}\sum_{k=1}^{N} (\mu_{ik})^m \left\| z_k - v_i \right\|_A^2 \qquad \textbf{(12)}$$

Where:

$$U = [\mu_{ik}] \in [0,1], \; 1 \le i \le c, \; 1 \le k \le N \qquad \textbf{(13)}$$

is a fuzzy partition matrix of Z,

$$V = [v_1, v_2, \ldots, v_c], \quad v_i \in \Re^n \qquad \textbf{(14)}$$

is a vector of cluster prototypes(centers) to be determined,

$$D_{ik\,A}^2 = \left\| z_k - v_i \right\|_A^2 = (z_k - v_i)^T A (z_k - v_i) \quad \textbf{(15)}$$

is a norm that is determined by the selection of the matrix A. In case matrix A is equal to the identity matrix, the norm becomes in the Euclidean distance. Finally,

$$m \in (1, \infty) \qquad \textbf{(16)}$$

is a parameter which determines the fuzziness of the resulting clusters. In general, the value of the cost function (12) can be seen as a measure of the total variance of z_k from vi [1].

3.2 Fuzzy C Means

The minimization of the c-means functional (12) represents a nonlinear optimization problem that can be solved by using a variety of methods, including iterative minimization. The most popular method is a simple Picard iteration through the first order conditions for stationary points of (12), known as the fuzzy c-means algorithms [10]. Let's define the next constraints:

$$\forall k \quad \sum_{i=1}^{c} \mu_{ik} > 0 \quad \textbf{(17)}$$

$$\forall i \quad \sum_{k=1}^{N} \mu_{ik} = 1 \quad \textbf{(18)}$$

Constraint (17) guarantees that no cluster is empty and constraint (18) ensures that the sum of the membership degrees for each data set is equal to 1, this means, that each data set receives the same weight in comparison to all other data and, therefore, all data set are equally included into the cluster partition.

The stationary points of the objective function (12) can be found by adjoining the constraint (18) to J by means of Lagrange Multipliers:

$$J(Z;U,V,\lambda) = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m D_{ik\,\mathbf{A}}^2 + \sum_{k=1}^{N}\lambda_k\left(\sum_{i=1}^{c}\mu_{ik} - 1\right) \quad \textbf{(18)}$$

and by setting the gradients of J with respect to U, V y λ to zero, it can be shown[18] that the stationary points correspond to:

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c}\left(\frac{D_{ik\,\mathbf{A}}^2}{D_{jk\,\mathbf{A}}^2}\right)^{\frac{2}{m-1}}};\ 1 \le i \le c \quad \mathbf{y}\ 1 \le k \le N \quad \textbf{(20)}$$

$$v_i = \frac{\sum_{k=1}^{N}(\mu_{ik})^2 z_k}{\sum_{k=1}^{N}(\mu_{ik})^2};\ 1 \le i \le c \quad \textbf{(21)}$$

Equation (21) gives a value for v_i that can be interpreted as the weighted average that belongs to a cluster and whose weights are given by the term μ_{ik}. The major problem that arises when using this kind of algorithm for the identification of fuzzy models is that the identified clusters have hiper-ellipsoids forms. For a future control application, it is not convenient to have this sort of cluster, instead, it will be adequate to get clusters with some kind of linear structure.[1]. There exist some other approaches applied to the fuzzy c means technique. These approaches consist in the use of an adaptive distance, where each norm is different for each cluster; thus it would be possible to get clusters of data with different structures.

3.3 Gustafson-Kessel Algorithm

Gustafson and Kessel proposed a powerful fuzzy clustering algorithm that is based on adaptive distance measures. It can be used to obtain a nonlinear model from a collection of local linear models. This technique has a model structure that is easy to understand and interpret, and can integrate various types of knowledge, such as empirical knowledge, derived from first-principles and measured data [11]

This algorithm has been frequently used because most of the time the clusters obtained have quasi-linear behaviour at different operating regimes that may exist in the data set and therefore, it is ideal for constructing Takagi-Sugeno Fuzzy Models [14].

Then, if a different norm B_i is selected for each cluster and if equation (15) is taken into account then:

$$D_{ik\,B_i}^2 = \|z_k - v_i\|_{B_i}^2 = (z_k - v_i)^T B_i (z_k - v_i) \quad \textbf{(22)}$$

Now, these matrices are used as the variables for the optimization of function (12) adapting the specific norm for each cluster according to its characteristics. Let's define $B_i = \{B_1, B_2, \ldots, B_C\}$ as the vector that contains the v norms. The new function to be minimized taking into account (13), (14) and (16) is:

$$J(Z;U,V,B) = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m \|z_k - v_i\|_{B_i}^2 \quad \textbf{(23)}$$

This objective function can not be directly minimized with respect to B_i , since it is linear with respect to B_i .To obtain a feasible solution, B_i must be constrained in some way. The usual way of accomplishing this is by constraining the determinant of B_i :

$$|B_i| = \rho_i, \quad \rho > 0 \quad \textbf{(24)}$$

With ρ_i constant for each cluster. Once the Lagrange multiplier is used, the next solution is obtained:

$$|B_i| = [\rho_i \det(F_i)]^{\frac{1}{n}} F_i^{-1} \quad \textbf{(25)}$$

Where F_i is the fuzzy covariance matrix of the ith cluster given by:

$$F_i = \frac{\sum_{k=1}^{N}(\mu_{ik})^m (z_k - v_i)(z_k - v_i)^T}{\sum_{k=1}^{N}(\mu_{ik})^m} \quad \textbf{(26)}$$

Once all the clusters are obtained, the determination of a Takagi-Sugeno model can be achieved. The antecedent membership functions A^k can be computed, and hence the consequent parameters $\mathbf{a}_i^T$ and b_i can be calculated using the least-squares method. [17]

For the determination of the antecedent membership functions, it is important to consider that each cluster obtained is a Takagi-Sugeno fuzzy rule. The multidimensional membership function A^k are given analytically by computing the distance of $\mathbf{x}(\tau)$ (Regressor Matrix) from the projection of the cluster prototypes center v_i onto X, and then computing the membership degree in an inverse proportion to the distance. Denote with F_x , a submatrix of F_i, which, describes the form of the cluster in the antecedent space X. Let $V_x = [v_1, v_2,\ldots,v_k]^T$ denote the projection of the cluster center onto the antecedent space X. From the Gustafson-Kessel algorithm, the inner product distance norm, given by:

$$d_{kl} = (x(\tau) - V_x)^T |F_x|^{1/n} (F_x)^{-1} (x(\tau) - V_x) \quad \textbf{(27)}$$

is converted into the membership degree by:

$$\mu_{A^k}(\mathbf{x}(\tau)) = \frac{1}{\sum_{j=1}^{K} \left(d_{kl} / d_{jl}\right)^{2/(m-1)}} \quad \textbf{(28)}$$

For the determination of the consequent parameters, as it was pointed out before, least-squares methods can be used. Let $\Gamma_k \in \Re^{NxN}$ denote the diagonal matrix having the membership degree $\mu_{Ak}x(\tau)$ as its k-th diagonal element. By appending a unitary column to **X**, the extended matrix $X_e = [\mathbf{X},1]$ is created. Furthermore, denote X', the matrix composed of the products of matrices Γ_i y X_e:

$$X' = [\Gamma_1 X_e, \Gamma_2 X_e, \ldots, \Gamma_k X_e] \quad \textbf{(29)}$$

The consequent parameters $\mathbf{a}_i^T$ y b_i are lumped into a single parameter vector:

$$\theta = [\mathbf{a}_1^T, b_1, \mathbf{a}_2^T, b_2, \ldots, \mathbf{a}_k^T, b_k] \quad \textbf{(30)}$$

Given the data **X** and **y,** it is well known that the solution of $\mathbf{y} = X'\theta + \varepsilon$, by means of least-squares techniques has the solution:

$$\theta = [(X')^{-1} X']^{\mathbf{T}} (X')^T \mathbf{y} \quad \textbf{(31)}$$

This is an optimal least-squares solution which gives the minimal prediction error. Nevertheless, a weighted least-squares approach can be applied and then the next solution is obtained:

$$[\mathbf{a}_k^T, b_k]^T = [X_e^T \Gamma_k X_e]^{\mathbf{-1}} X_e^T \Gamma_k \mathbf{y} \quad \textbf{(32)}$$

4 Major Operations in a Petroleum Company

A petroleum company has a set of fundamental operations, which in general can be divided in three phases: extraction, processing and transport. In the process of extraction, since some petroleum's volume has low gravity and due to some specific characteristic of some wells (heavy oil), it is necessary the installation of some artificial lift methods in order to extract the petroleum from the oilfields. Among these artificial lift methods, the most common are: mechanical pumping, progressive cavity pumping and electrosubmersible pumping. In the extraction phase, the petroleum (crude) in its original state (gas, water and petroleum) is pumped to a specific place called discharge station. In the discharge station, the sum of the petroleum's flow of a determined group of wells is received by a production multiple. In the production multiple, the petroleum's flow is distributed to a group of production separator.

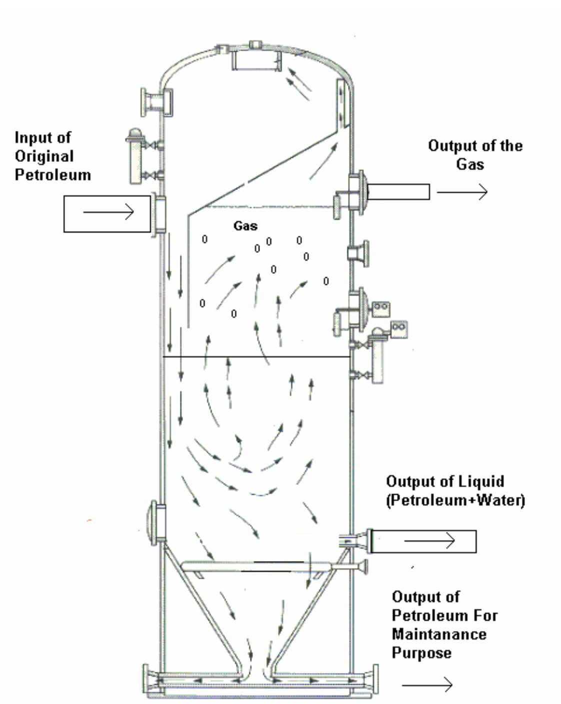

Figure 1: Drawing of a production separator

Figure 1 shows the drawing of a standard production separator. A separator for petroleum production is a large drum designed to separate production fluids into their constitute components of oil, gas and water. It operates on the principle that the three components have different densities, which allows them to stratify when moving slowly with gas on top, water on the bottom and oil in the middle. A standard separator has two liquid-output. In the first output, there is a control valve which is used to regulate the liquid of the separator; the second output is used only for maintenance purpose. There is also a gas-output, where the gas flows toward a system that keeps some other instruments operating, for instance, all the actuators of the stations. Also, the gas is used as a combustible material for keeping the furnaces on.

The liquid component at the output of the separator flows toward the boilers, where, the temperature of the liquid is increased from 95 Fahrenheit degrees to 190 Fahrenheit degrees, thus reducing the viscosity of the liquid. Once the temperature of the liquid is increased, the liquid flows toward some special tanks (Washing-Tanks) where the oil and water are completely separated. The oil is pumped to a major station either for being refined or for marketing purposes.

4.1 First Case of Study: Gas-Liquid Separation Process

As it was mentioned before, the basic function of a separator is to separate the gas and liquid phases. Because of the significant variations in the productions of some wells, the flow of petroleum at the input of the separator has a high nonlinear behaviour. Currently, the control system is based on PID algorithms and its behaviour has a poor performance.

In order to implement a more reliable control system, it is necessary to determine a fuzzy model of the separator. Based on the fact, that a fuzzy model-based control will be implemented in the future, a fuzzy model will be constructed based on historical data.

A separator has a series instruments installed for measuring certain variables. These variables are: the pressure of the separator, the temperature of the liquid, the level of the separator and a valve control with a pneumatic actuator. Also, there are switches for detecting low level and high level of petroleum and high pressure in the separator.

A set of 1000 sampled data per variable was recorded with a time sample of one second. It was taken into account three operation regimes: level of liquid greater than the level of reference, level of liquid near the level of reference and level of liquid less than the level of reference.

A MISO model was proposed where the input variables were: Current Level of the Separator $y(\tau)$ and Valve Position $u(\tau)$. The output variable was the level at the next sampling instant $y(\tau+1)$. The model consisted of three rules with linear consequents, including the bias term. The rule base represents a nonlinear first-order regression model that can be written as follows:

$$y(\tau+1) = f(y(\tau), u(\tau)) \qquad \textbf{(33)}$$

The Fuzzy C-Means algorithm was applied to the set of 1000 data sampled. Figure 2 shows the clusters obtained. The same procedure was applied for the Gustafson-Kessel algorithm. Figure 3 shows the clusters obtained. The MATLAB Software was used to do the simulations.

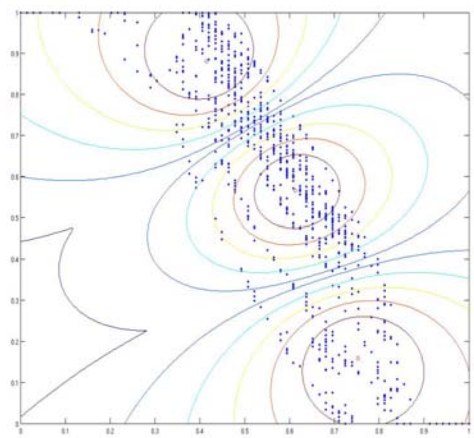

Figure 2: Application of Fuzzy C-Means algorithm based on historical data of the Separator of Production

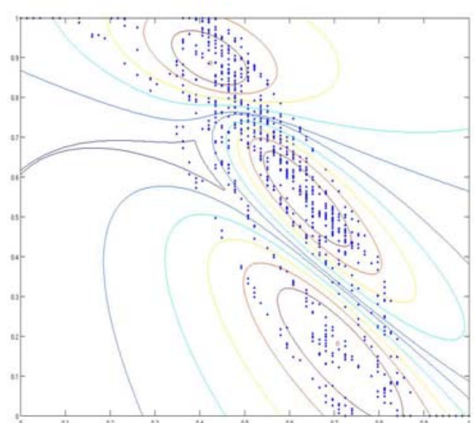

Figure 3: Application of Gustafson-Kessel algorithm based on historical data of the Separator of Production

For the construction of the Takagi-Sugeno fuzzy model, first the antecedent membership function were determined by projecting the fuzzy partition matrix onto the two antecedent variable as it is shown generically in figure 4. Basically, equations (27) and (28) are used for obtaining the membership functions. Figure 5 shows the resultant membership functions obtained from the simulation.

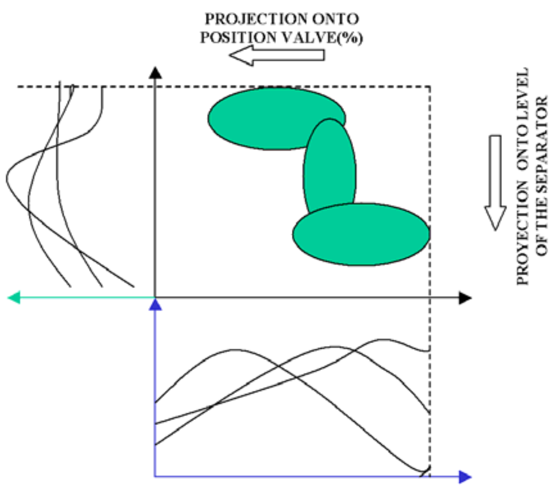

Figure 4: Projection of fuzzy clusters onto the antecedent variables

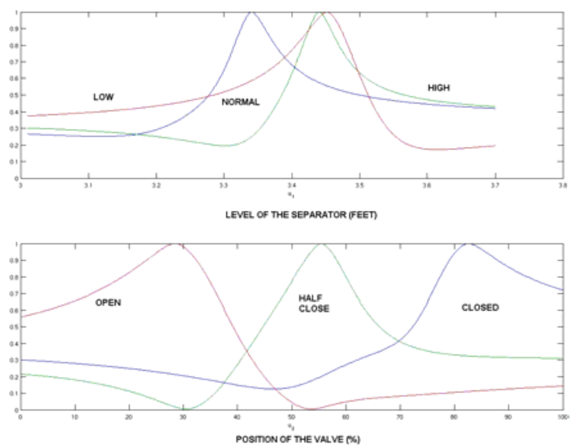

Figure 5: Membership functions obtained from the simulations

Consequent parameters are estimated by least-squares method such as the solution proposed in (31). The resultant rule base obtained from the simulation was:

1. **If** $y(\tau)$ is Low **AND** $u(\tau)$ is Open **Then** $y(\tau+1) = 1.0697y(\tau) + 0.0002u(\tau) - 0.2491$
2. **If** $y(\tau)$ is Normal **AND** $u(\tau)$ is Half-Closed **Then** $y(\tau+1) = 0.9375y(\tau) + 0.2170$
3. **If** $y(\tau)$ is High **AND** $u(\tau)$ is Closed **Then** $y(\tau+1) = 1.0540y(\tau) + 0.003u(\tau) - 0.1978$

This fuzzy model was validated with 775 data sampled different from the set of 1000 data sample. Figure 6 shows the validation of the model.

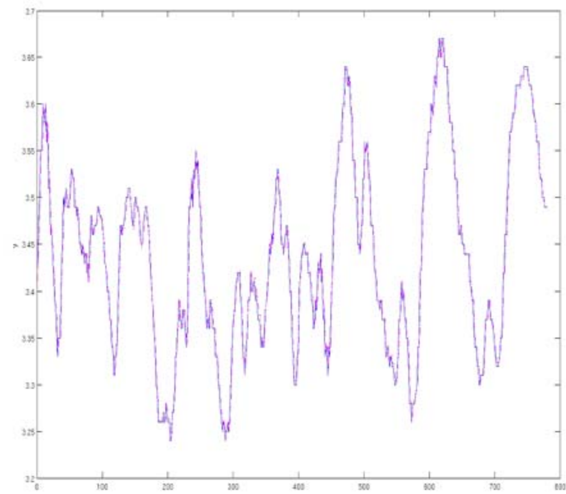

Figure 6: Validation of the fuzzy model obtained

4.2 Second Case of Study: Water-Oil Separation Process

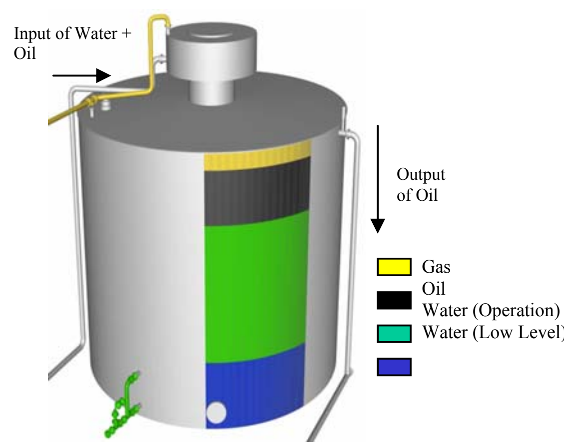

Figure 7: Drawing of a washing tank

Separating the oil from the water is one of the most complex processes in the oil industry. Standard washing tanks are used for this purpose. Washing tanks are passive, physical separation systems designed for removing oil from water, where, the oil-water mixture enters the tank and is spread out horizontally, distributed through an energy and turbulence diffusing device.

A standard washing tank has a series instruments installed for measuring certain variables. These variables are: the level of water inside the tank, the temperature in the input of the tank, the water-cut in the output of the tank and a valve control with a pneumatic actuator for the regulation of water level. Also, there are switches for detecting low level of water and high level of oil.

Currently, the control system is based on PID algorithm. The performance can be considered poor because of some nonlinearities associated with the time of separation between the molecules of oil and gas. Based on the fact, that a fuzzy model-based control and a supervisor control system will be implemented, a fuzzy model of the washing tank will be constructed base on historical data.

Based on the same procedure used in the first case of study, a MISO model was proposed based on the same variables: the level of water inside the tank $y(\tau)$ and the valve position $u(\tau)$ (Input variables). The output variable was the level of water at the next sampling instant $y(\tau+1)$. Given the fact, that this sort of process is quite slow, some tests were done in order to determine the appropriate time sample. A value of 20 second was considered as an appropriate time sample.

The resultant fuzzy model was similar to the one obtained for the production separator: three rules with linear consequents including the bias term. The FCM and the GK algorithms applied to this particular process are showed in figure 8 and 9.

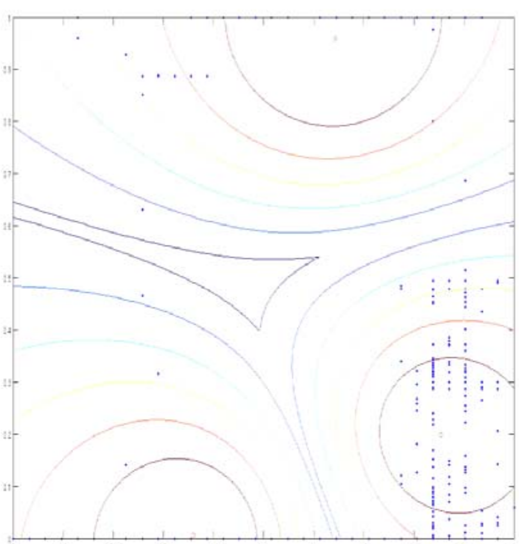

Figure 8: Application of Fuzzy C-Means algorithm based on historical data of the Washing Tank

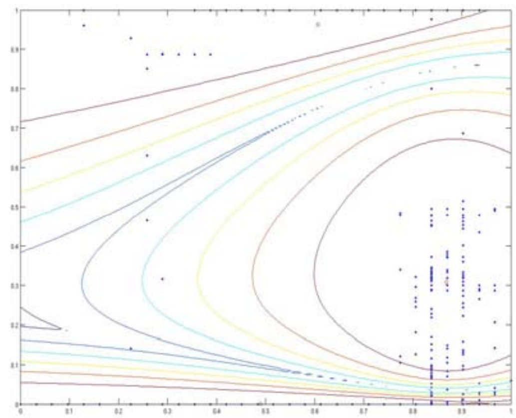

Figure 9: Application of Gustafson-Kessel algorithm based on historical data of the Washing Tank

The determination of the Takagi-Sugeno fuzzy model was done through the simulation. The resultant rule base obtained from the simulation was:

1. **If** $y(\tau)$ is Low **AND** $u(\tau)$ is Closed **Then** $y(\tau+1) = y(\tau) + 7{,}831u(\tau) + 0.0005$
2. **If** $y(\tau)$ is Normal **AND** $u(\tau)$ is Half-Open **Then** $y(\tau+1) = 0.9375y(\tau) + 0.0005$
3. **If** $y(\tau)$ is High **AND** $u(\tau)$ is Open **Then** $y(\tau+1) = 0{,}7y(\tau) + 4{,}7$

This fuzzy model was validated with 229 data sampled different from the set of 628 data sample. Figure 10 shows the validation of the model.

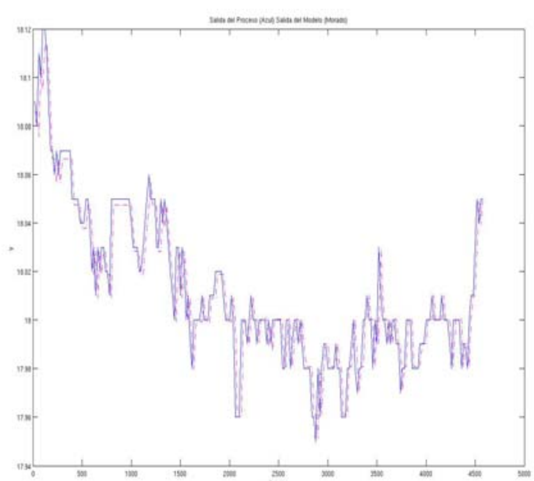

Figure 10: Validation of the fuzzy model obtained

The resultant membership functions are showed in figure 11:

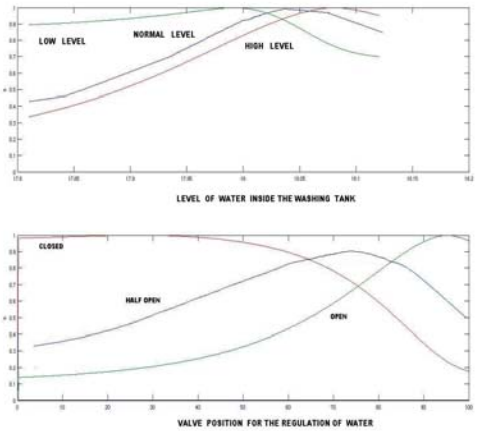

Figure 11: Membership functions obtained from the simulations

5. Conclusions

The Fuzzy C-Means and the Gustafson-Kessel algorithms were analyzed in details in this paper. Two cases of study related with the gas-liquid separation process and the water-oil separation process were presented for the implementation of these techniques.

Both models were constructed based on a MISO framework. Only two inputs and one output parameters were defined for each model as elements of the fuzzy model with a set of 1000 sampled data for the production separator and 628 sampled data for the washing tank.

The fuzzy models were computationally validated with a set of different data samples (775 different data sample for the production separator and 229 different data sample for the washing tank). Both models showed an excellent generalization. This kind of modeling technique applied to complex processes gives good results in a relatively short period of time, so it will be used for the future design of supervisor control system.

References:

[1] Diez J.L., Navarro J.L, Salas A. (2004). "Algoritmos de Agrupamiento en la Identificación de Modelos Borrosos". Departamento de Ingeniería de Sistemas y Automática, Universidad Politécnica de Valencia, España, pp-1-6..

[2] Johansen T.A, Murray-Smith,R. (1997). "The Operating Regime Approach to Nonlinear Modeling and Control". Multiple Model Approaches to Modeling and Control. Taylor & Francis, London.

[3] Carbonell P., Diez J.L., Navarro J.L. (2004). "Aplicaciones de Técnicas de Modelos Locales en Sistemas Complejos". Departamento de Ingeniería de Sistemas y Automática, Universidad Politécnica de Valencia, España, pp.1-9.

[4] Babuska, R. (1998). "Fuzzy Modeling and Identification Toolbox", For use with MATLAB, pp3-5.

[6] Sugeno M., Yasukawa T. (1993). "A Fuzzy-Logic Based Approach to Qualitative Modeling". Transaction on Fuzzy Systems, Vol 1, número 1, pp 7-10.

[7] Babuska, R. (2004). "Modeling and Identification", PhD dissertation. Delft University of Technology, Delft, the Netherlands

[8] Nelles, O. (1999). "Nonlinear System Identification with Local Linear Neuro-Fuzzy Models". PhD dissertation. Darmstadt University of Technology, Darmstadt, Germany,pp 1-8.

[9] Kruse, R., Doring C., Lesot M. (2007). "Fundamental of Fuzzy Clustering", Department of Knowledge Processing and Language Engineering, University of Magdeburg, Germany, pp. 1-10.

[10] Babuska, R. (2004). "Fuzzy and Neural Control DISC Course Lecture Notes". Delft Center For Systems and Control, The Netherlands.

[11] Sousa J., Kaymak U. (2002). "Fuzzy Decision Making in Modeling and Control". World Scientific Series in Robotics and Intelligent Systems, pp. 93-108.

[12] Gustafson, D. E., Kessel W.C. (1979). "Fuzzy Clustering with a Fuzzy Covariance Matrix". Proc. IEEE CDC, San Diego, USA, pp. 761-766.

[13] Hellendoorn H., Driankov D. (1997) "Fuzzy Model Identification". Selected Approaches. Springer, Berlin, Germany, pp. 5-8.

[14] Takagi T., Sugeno M. (1985). "Fuzzy identification of system and its application to modeling and control". IEEE Transactions on Systems, Man and Cybernetics, pp. 116-120.

[15] Abonyi J., Babuska R., Szeifert F. (2004). "Modified Gath-Geva Fuzzy Clustering for Identification of Takagi-Sugeno Fuzzy Models". Delft University of Technology, Department of Information Technology and Systems, The

Netherlands pp. 1-6.

[16] Abonyi J., Choban T., Szeifert F.(2000) "Identification of Nonlinear Systems using Gaussian Mixture of Local Models". University of Vezprem, Department of Process Engineering, Hungary, pp3-8.

[17] Diez J.L., Navarro J.L. (1999). "Fuzzy Models of Complex Systems by Means of Clustering Techniques". Proc. 2nd Intelligent Systems in Control and Measurement, pp. 147-153.

[18] Besdek J.C.(1980). "A convergence theorem for the fuzzy ISODATA clustering algorithms". IEEE transactions, Machine Intelligent. PAMI-2(1), pp. 1-8.