Mathematical Model for Diffusion- Sorption Processes in Layered Strata with Interlayers

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Abstract: - In this paper the three-dimensional system of the partial differential equations as a model of the diffusion with the sorption process for the porous layered medium with the semi-permeable interlayers is proposed. The generalized integral parabolic spline is used for the approximate transformation of the 2-D problem into the simpler 1-D system by the original method of the conservative averaging. This system of the 1-D partial differential equations with continuous coefficients fulfills in the averaged sense all the conservation laws of the original problem. The finite difference scheme for solving the averaged system is offered.

Key-Words: - Layered medium, Main layers, Interlayers, Diffusion, Sorption, Conservative averaging, Integral spline, Finite difference method.

1 Introduction

Many real processes take place in layered systems, especially in the underground systems, consisting of separate layers with different thickness and different physical properties [1]-[3]. Very often in the groundwater flows such systems are composed from the two types of layers which are located alternatively, e.g. aquifers and aquitards [1]. In these cases, in the mathematical model of such processes, we have a jump in the coefficients of differential equations on the surfaces between two layers. The usage of standard mathematical methods by discontinuity of the coefficients of the PDE (partial differential equations) brings out additional difficulties.

One of the authors has developed the conservative averaging method and he introduced a special new type of the spline for wide class of PDE problems with discontinuous coefficients [4]-[10].

In this paper we give the statement and propose the numerical method (finite difference scheme) for the layered system consisting of two different types of the layers: main and interlayers (semi-permeable layers). This type of the models automatically reduces (if the thickness of the interlayers trends to zero) to the models, which have only main layers - considered in previous works, e.g. [11]. Proposed method differs from methods traditionally used in various diffusion and transport processes in natural or artificial porous media. It allows analyzing broader spectrum of a physical phenomena and wider variety of the geometrical and physical parameters. In the paper [12] was considered a class of mathematical models for transport processes in layered strata with interlayers oriented for the applications to underground problems. The model for the transport process with sorption in the two-layer system with a one interlayer was considered in [13].

Here we bear in mind the diffusion and sorption processes in the traditional wood industry [14]-[16], as well as connected and oriented to a sustainable development and advanced zero emissions technologies.

2 The Mathematical Model of the Diffusion-Sorption in Layered System

For a homogeneous orthotropic medium the concentration fields' $U\left(x,y,z,t\right)\left(V\left(x,y,z,t\right)\right)$ equations for a dispersion-reaction process (without convection) can be written in the following form, e.g. [2]:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(D_u^{(x)}\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_u^{(y)}\frac{\partial U}{\partial y}\right)+$$
$$+\frac{\partial}{\partial z}\left(D_u^{(z)}\frac{\partial U}{\partial z}\right)+F_1\left(U,V\right), \qquad (1)$$
$$\frac{\partial V}{\partial t}=\frac{\partial}{\partial x}\left(D_v^{(x)}\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_v^{(y)}\frac{\partial V}{\partial y}\right)+$$

$$+\frac{\partial}{\partial z}\left(D_v^{(z)}\frac{\partial V}{\partial z}\right)+F_2\left(U,V\right). \quad (2)$$

The reaction terms in the equations (1), (2): the functions $F_i(U,V), i=1,2$ can be very different (linear, depending on one or several parameters, nonlinear etc.) for various processes.

We will restrict us by the narrow class in sense of the process (in comparison with the class of equations (1), (2)), but by the broader class in geometrical sense – the multilayer system with interlayers. The simplest typical example of such interest is a veneer, exacter plywood.

2.1 The description of the Geometry of the Layered Domain

Let it be given the domain $M \subset R^3$, where M is a multilayered domain with the base as semi-bounded (or bounded) in the plane $(x,y)\in D \subset R^2$ and with the height $H=b-a$:

$$M = D \otimes \left\{z \in \left[a=z_0, b=z_{N+1}\right]\right\}.$$

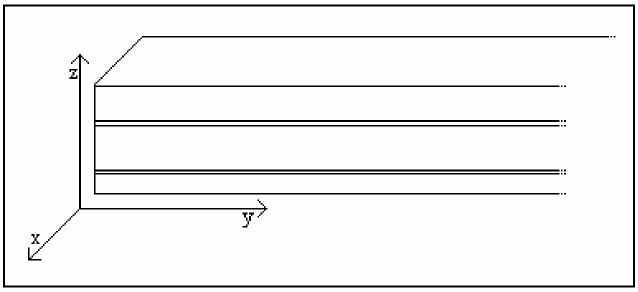

Fig.1
Multilayered domain with interlayers

So, in this paper we consider the layered system consisting of alternating two different types of layers: main and interlayers (semi-permeable layers).

2.2 Mathematical Statement of the Diffusion-Sorption Problem for the Layered Domain

The diffusion-sorption process in the $i-th$ main layer (type aquifer for underground processes, see, e.g., [1], [2], [11], [12]) (for $(x,y)\in D$ and $z_i < z < z_{i+1/2}$, $i=\overline{0,N}$) is described by a following system of the two partial differential equations together with the sorption kinetic function:

$$\frac{\partial U_i}{\partial t}+\frac{\partial V_i}{\partial t}=\frac{\partial}{\partial x}\left(D_i^{(x)}\frac{\partial U_i}{\partial x}\right)+$$

$$+\frac{\partial}{\partial y}\left(D_i^{(y)}\frac{\partial U_i}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_i^{(z)}\frac{\partial U_i}{\partial z}\right),$$

$$\frac{\partial V_i}{\partial t}=\beta_i\left(U_i-W_i\right), \quad (3)$$

$$V_i=F_i\left(W_i\right).$$

Evidently we can represent the first (most complicate equation) in the form with the source type term:

$$\frac{\partial U_i}{\partial t}=\frac{\partial}{\partial x}\left(D_i^{(x)}\frac{\partial U_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_i^{(y)}\frac{\partial U_i}{\partial y}\right)+$$

$$+\frac{\partial}{\partial z}\left(D_i^{(z)}\frac{\partial U_i}{\partial z}\right)+\beta_i\left(W_i-U_i\right), \quad (4)$$

$$\frac{\partial V_i}{\partial t}=\beta_i\left(U_i-W_i\right),$$

$$V_i=F_i\left(W_i\right).$$

In the equations for the interlayers (for $i=\overline{0,N-1}$) we neglect the possible diffusion parallel to the layers disposition (such assumption is typical for underground processes, see [1]-[3] and [17] and is natural for our class of processes too):

$$\frac{\partial U_{i+1/2}}{\partial t}=\frac{\partial}{\partial z}\left(D_{i+1/2}^{(z)}\frac{\partial U_i}{\partial z}\right)$$

$$+\beta_{i+1/2}\left(W_{i+1/2}-U_{i+1/2}\right), \quad (5)$$

$$\frac{\partial V_{i+1/2}}{\partial t}=\beta_{i+1/2}\left(U_{i+1/2}-W_{i+1/2}\right),$$

$$V_{i+1/2}=F_{i+1/2}\left(W_{i+1/2}\right).$$

In this paper we will restrict us by two types of source (sorption kinetics) functions $F(W)$:

a) linear: $F(W)=\dfrac{W}{\gamma}$;

b) with saturation: $F(W)=\dfrac{W}{\gamma+\delta W}$.

The initial conditions are in the traditional form:

$$\begin{aligned} U_i\big|_{t=0} &= U_i^0\left(x,y,z\right),\\ U_{i+1/2}\big|_{t=0} &= U_{i+1/2}^0\left(x,y,z\right),\\ V_i\big|_{t=0} &= V_i^0\left(x,y,z\right),\\ V_{i+1/2}\big|_{t=0} &= V_{i+1/2}^0\left(x,y,z\right). \end{aligned} \quad (6)$$

The boundary conditions on the outer planes (on the top and the bottom of the first and the last main layer) are in general form:

$$-\nu_0 D_0^{(z)}\frac{\partial U_0}{\partial z}+\lambda_0 U_0(a)=\Phi_0(y,z,t), \quad (7)$$

$$\nu_1 D_N^{(z)} \frac{\partial U_N}{\partial z} + \lambda_1 U_N(a) = \Phi_1(y,z,t). \quad (8)$$

Such form of the boundary conditions is typical for the partial differential equations. Indeed, if $\nu_0 = 0$ $(\nu_1 = 0)$ and $\lambda_0 = 1 (\lambda_1 = 1)$, we obtain Dirichlet BC, if $\nu_0 = 1(\nu_1 = 1)$ it gives Neumann BC (for $\lambda_0 = 0$ $(\lambda_1 = 0)$) and Robin BC (for $\nu_0\lambda_0 > 0$ $(\nu_1\lambda_1 > 0)$).

We assume fulfilling one of all traditional BC on the lateral boundary $\partial D \otimes \{z \in [a,b]\}$. Its specific properties don't play an important role for the proposed method of the conservative averaging, but, of course, they are important by solving concrete problems.

3 Problem Solution

Further, to explain the main idea, we concentrate our attention to the 2-D problem (without y − argument). We show how the generalized integral parabolic spline [7], [10], [12] allows diminishing by one the dimension of the original problem: instead of the 2-D problem for the partial differential equation we will obtain a system of the 1-D partial differential equations with continuous coefficients. The order of this system is equal to the number of main layers. Then for the numerical solving of this system of the 1-D partial differential equations will be proposed a special difference scheme of the type of the classical Peaceman-Rackford scheme (the role of second space coordinate in our case plays the index in main layer).

In case of full the 3-D problem we could use so called additive or locally one-dimensional finite difference schemes [18].

3.1 Generalized Integral Parabolic Spline

Let it be given a continuous, piecewise-smooth function $U(z)$, $z \in [a,b]$, for which the first derivative $U'(z)$ has first kind discontinuities in the $2N$ different inner points $z_i, z_{i+1/2}$, $i = \overline{1,N}$:

$$\begin{aligned} &D_{i-1}U'(z_{i-1/2} - 0) = D_{i-1/2}U'(z_{i-1/2} + 0), \\ &D_{i-1/2}U'(z_i - 0) = D_i U'(z_i + 0). \end{aligned} \quad (9)$$

The continuity property of the function $U(z)$ gives following $2N$ equalities:

$$\begin{aligned} &U(z_{i-1/2} - 0) = U(z_{i-1/2} + 0), \\ &U(z_i - 0) = U(z_i + 0). \end{aligned} \quad (10)$$

Here the diffusion function (coefficients) $D(z)$ is a piecewise-constant function (practically $D_i = D_i^{(z)}$ $(D_{i-1/2} = D_{i-1/2}^{(z)})$ from previous sub-section):

$$D(z) = \begin{cases} D_{i-1/2}, & z \in (z_{i-1/2}, z_i), \\ D_i, & z \in (z_i, z_{i+1/2}). \end{cases}$$

We assume fulfilling the properties $D_{i-1/2} << D_{i-1}$, $D_{i-1/2} << D_i$. Here must be mentioned that the $z_{N+1/2} = z_{N+1} = b$, it means the function $U(z)$ can neither end nor begin with a linear part (in the physical sense: we start and finish with the main layer, not the interlayer) of the segment $[a,b]$.

Additionally there are given the $N+1$ values u_i of the function $U(z)$ over the sub-segments $[z_i, z_{i+1/2}]$:

$$\tilde{u}_i = \frac{1}{\tilde{H}_i} \int_{z_i}^{z_{i+1/2}} U(z)dz, \qquad \tilde{H}_i = z_{i+1/2} - z_i, \; i = \overline{0,N}. \quad (11)$$

Further, it is known, that on the sub-segments $[z_i, z_{i+1/2}]$, $i = \overline{0,N}$ the function can be approximated by the linear function. Finally, the BC of type (7), (8) must be fulfilled.

In the paper [10] it was proved that here exists exactly one spline fulfilling all mentioned conditions. We will seek this spline in the form:

$$\tilde{S}(z) = \begin{cases} \tilde{u}_i + \tilde{m}_i(z - \tilde{z}_i) + \tilde{e}_i\left[\dfrac{(z-\tilde{z}_i)^2}{D_i\tilde{H}_i} - \dfrac{\tilde{G}_i}{12}\right], \\ z \in [z_i, z_{i+1/2}], \tilde{z}_i = \dfrac{z_i + z_{i+1/2}}{2}, i = \overline{0,N}; \\ u_{i-1/2} + m_{i-1/2}(z - \tilde{\tilde{z}}_{i-1/2}), z \in [z_{i-1/2}, z_i], \\ \tilde{\tilde{z}}_{i-1/2} = \dfrac{z_{i-1/2} + z_i}{2}, \quad i = \overline{1,N}. \end{cases} \quad (12)$$

Here $\tilde{G}_i = \dfrac{\tilde{H}_i}{D_i}$ are the lengths parameters reduced by the diffusions coefficient, which can be called as "characteristic diffusion lengths". Similarly we introduce the N additional lengths parameters for second type of layers (interlayers) $G_{i-1/2} = \dfrac{H_{i-1/2}}{D_{i-1/2}}$, $i = \overline{1,N}$, which will be used immediately. The continuity property (10) of the generalized integral parabolic spline (GIPS) at the points $z_{i+1/2}$, $i = \overline{0,N-1}$ gives following equalities:

$$\tilde{u}_i + D_i \tilde{m}_i \frac{\tilde{G}_i}{2} + \tilde{e}_i \frac{\tilde{G}_i}{6} = \\ = u_{i+1/2} - D_{i+1/2} m_{i+1/2} \frac{G_{i+1/2}}{2} \quad (13)$$

The same property at the points z_{i+1}, $i = \overline{0,N-1}$ gives similar equalities:

$$\tilde{u}_{i+1} - D_{i+1} \tilde{m}_{i+1} \frac{\tilde{G}_{i+1}}{2} + \tilde{e}_{i+1} \frac{\tilde{G}_{i+1}}{6} = \\ = u_{i+1/2} + D_{i+1/2} m_{i+1/2} \frac{G_{i+1/2}}{2}. \quad (14)$$

The conjugation conditions (9) lead to the equalities for $i = \overline{0,N-1}$:

$$D_i \tilde{m}_i + \tilde{e}_i = D_{i+1/2} m_{i+1/2} = D_{i+1} \tilde{m}_{i+1} - \tilde{e}_{i+1} \quad (15)$$

The equalities (15) allow us to exclude $D_{i+1/2} m_{i+1/2}$ and $D_{i+1} \tilde{m}_{i+1}$ from the (13), (14). We obtain the following generalized chain of the equalities for $i = \overline{0,N-1}$:

$$D_i \tilde{m}_i \left(\tilde{G}_i + 2G_{i+1/2} + \tilde{G}_{i+1} \right) + \\ + \tilde{e}_i \left(\tilde{G}_i / 3 + 2G_{i+1/2} + \tilde{G}_{i+1} \right) + \frac{2}{3} \tilde{e}_{i+1} \tilde{G}_{i+1} = \\ = 2\left(\tilde{u}_{i+1} - \tilde{u}_i \right) \quad (16)$$

And for the $i = \overline{1,N}$ we obtain:

$$D_i \tilde{m}_i \left(\tilde{G}_i + 2G_{i-1/2} + \tilde{G}_{i-1} \right) + \\ + \tilde{e}_i \left(\tilde{G}_i / 3 + 2G_{i-1/2} + \tilde{G}_{i+1} \right) - 2/3 \tilde{e}_{i-1} \tilde{G}_{i-1} = \\ = 2\left(\tilde{u}_i - \tilde{u}_{i-1} \right) \quad (17)$$

The final step consists of excluding the term $D_i \tilde{m}_i$ from last two chains of equalities. We obtain:

$$\left(1 + \tilde{a}_0 + \tilde{b}_0\right) \tilde{e}_0 + \tilde{b}_0 \tilde{e}_1 = \\ = \tilde{f}_0^- \tilde{u}_{-1} - \tilde{f}_0 \tilde{u}_0 + \tilde{f}_0^+ \tilde{u}_1,$$

$$\tilde{a}_i \tilde{e}_{i-1} + \left(1 + \tilde{a}_i + \tilde{b}_i\right) \tilde{e}_i + \tilde{b}_i \tilde{e}_{i+1} = \\ = \tilde{f}_i^- \tilde{u}_{i-1} - \tilde{f}_i \tilde{u}_i + \tilde{f}_i^+ \tilde{u}_{i+1}, \quad i = \overline{1,N-1}, \\ \tilde{a}_N \tilde{e}_{N-1} + \left(1 + \tilde{a}_N + \tilde{b}_N\right) \tilde{e}_N = \\ = \tilde{f}_N^- \tilde{u}_{N-1} - \tilde{f}_N \tilde{u}_N + \tilde{f}_N^+ \tilde{u}_{N+1}. \quad (18)$$

Here the coefficients of the linear algebraic system have following expressions:

$$\tilde{a}_i = \tilde{G}_{i-1} / \left(\tilde{G}_i + \tilde{G}_{i-1/2} + \tilde{G}_{i-1} \right), \\ \tilde{b}_i = \tilde{G}_{i+1} / (\tilde{G}_i + \tilde{G}_{i+1/2} + \tilde{G}_{i+1}), \\ \tilde{f}_i^- = 3 / \left(\tilde{G}_i + \tilde{G}_{i-1/2} + \tilde{G}_{i-1} \right), \\ \tilde{f}_i^+ = 3 / (\tilde{G}_i + \tilde{G}_{i+1/2} + \tilde{G}_{i+1}), \\ \tilde{f}_i = \tilde{f}_i^- + \tilde{f}_i^+, \\ G_{-1/2} = G_{N+1/2} = 0. \quad (19)$$

There are two cases for the expressions of the coefficients $\tilde{G}_{-1}$, $\tilde{G}_{N+1}$, $\tilde{u}_{-1}$, $\tilde{u}_{N+1}$:

1. If $\lambda_0 \neq 0$ (and $\lambda_1 \neq 0$), then

$$G_{-1} = 2v_0 / \lambda_0, \quad G_{N+1} = 2v_1 / \lambda_1, \\ u_{-1} = \Phi_0 / \lambda_0, \quad u_{N+1} = \Phi_1 / \lambda_1; \quad (20)$$

2. If $\lambda_0 = 0$ (and $\lambda_1 = 0$), then

$$G_{-1} = 2v_0 - G_0, \quad G_{N+1} = 2v_1 - G_N, \\ u_{-1} = \Phi_0 + u_0, \quad u_{N+1} = \Phi_1 + u_N. \quad (21)$$

We underline an interesting moment regarding the GIPS: the system of the linear algebraic equations for the calculations of the spline's coefficients contains only the "parabolic part" coefficients. The linear part characteristics are represented trough the coefficients (19).

Second important aspect: the GIPS naturally transforms to IPS [5], [7], [11] when all $z_{i-1/2} = z_i$. The same property holds for the explicit representation for the coefficients $\tilde{e}_i$ of the GIPS, details are given in [12]:

$$\tilde{e}_i = \gamma_i^{(0)} \tilde{f}_0^- \tilde{u}_{-1} + \gamma_i^{(1)} \tilde{f}_N^+ \tilde{u}_{N+1} + \sum_{j=0}^{N} \tilde{\beta}_{i,j} \tilde{u}_j. \quad (22)$$

3.2 The reduction of the 2-D Problem to the 1-D System by the Conservative Averaging Method

Similarly as in the paper [12] we introduce $N+1$ values $u_i(x,t)\left(v_i(x,t)\right)$ of the function $U_i\left(x,z,t\right)\left(V_i\left(x,z,t\right)\right)$ over sub-segments

$[z_i, z_{i+1/2}]$, i.e., over the main layers:

$$u_i(x,t) = \frac{1}{\tilde{H}_i}\int_{z_i}^{z_{i+1/2}} U_i(x,z,t)dz,$$

$$v_i(x,t) = \frac{1}{\tilde{H}_i}\int_{z_i}^{z_{i+1/2}} V_i(x,z,t)dz, \quad (23)$$

$\tilde{H}_i = z_{i+1/2} - z_i,\ i = \overline{0,N}.$

The integration over the appropriate main layer, approximation the mass fluxes on lines between layers and interlayers and usage of the representation (22) finally gives (details are given in the papers [11], [12]):

$$\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x}\left(D_i^{(x)}\frac{\partial U_i}{\partial x}\right) + \beta_i\left(w_i - u_i\right) +$$

$$\frac{2}{\tilde{H}_i}\sum_{j=0}^{N}\left[\gamma_j^+\left(u_{j+1} - u_i\right) + \gamma_j^-\left(u_{j-1} - u_j\right)\right], \quad (24)$$

$$\frac{\partial v_i}{\partial t} = \beta_i\left(u_i - w_i\right),$$

$$v_i = F_i\left(w_i\right).$$

The initial conditions take the form:

$$u_i\big|_{t=0} = u_i^0(x),$$

$$v_i\big|_{t=0} = v_i^0(x). \quad (25)$$

Finally, must be added the concrete boundary conditions.

3.3 The Alternating Direction Finite Difference Scheme for 1-D System (24)

We construct the finite difference scheme which is similar with the classical Peaceman-Rackford scheme. In our case the role of the second space direction will play the index (number) of layer. This finite difference scheme can be written in the following form. In the first stage we solve following three-diagonal system of difference equations by factorization method:

$$\frac{u_{i,k}^{n+1/2} - u_{i,k}^{n}}{\tau/2} = D_i^{(x)}\Lambda_{\bar{x}x}u_{i,k}^{n} + \beta_i\left(w_{i,k} - u_{i,k}\right)^{n+1/2}$$

$$+\left[\gamma_i^+\left(u_{i+1,k} - u_{i,k}\right) + \gamma_i^-\left(u_{i-1,k} - u_{i,k}\right)\right]^{n+1/2} +$$

$$\frac{2}{H_i}\sum_{j\neq i}\left[\gamma_i^+\left(u_{i+1,k} - u_{i,k}\right) + \gamma_i^-\left(u_{i-1,k} - u_{i,k}\right)\right]^{n},$$

$$\frac{v_{i,k}^{n+1/2} - v_{i,k}^{n}}{\tau/2} + \beta_i\left(w_{i,k} - u_{i,k}\right)^{n+1/2} = 0,$$

$$v_{i,k}^{n+1/2} = F_i\left(w_{i,k}^{n+1/2}\right).$$

Here we have introduced traditional for finite difference method notations:

$$u_{i,k}^n = u_i(x_k, t_n),$$

$$\left(w_{i,k} - u_{i,k}\right)^n = w_{i,k}^n - u_{i,k}^n,$$

$$\Lambda_{\bar{x}x}u_{i,k}^n = \frac{u_{i,k+1}^n - 2u_{i,k}^n + u_{i,k-1}^n}{h_x^2}.$$

For the linear sorption kinetic the practical solution of this system of the linear algebraic equations is clear. In case of the sorption kinetic with saturation we propose as first step rewriting the second equation in the following form:

$$w_{i,k}^{n+1/2} = u_{i,k}^{n+1/2} - \frac{v_{i,k}^{n+1/2} - v_{i,k}^{n}}{\beta_i\tau/2}.$$

In the second step we substitute this representation for $w_{i,k}^{n+1/2}$ in the first and the last difference equations.

In the second stage we solve by factorization method following the three-diagonal system in the $x-$direction of the difference equations:

$$\frac{u_{i,k}^{n+1} - u_{i,k}^{n+1/2}}{\tau/2} = D_i^{(x)}\Lambda_{\bar{x}x}u_{i,k}^{n+1} + \beta_i\left(w_{i,k} - u_{i,k}\right)^{n+1/2}$$

$$+\left[\gamma_i^+\left(u_{i+1,k} - u_{i,k}\right) + \gamma_i^-\left(u_{i-1,k} - u_{i,k}\right)\right]^{n+1/2} +$$

$$\frac{2}{\tilde{H}_i}\sum_{j\neq i}\left[\gamma_j^+\left(u_{j+1,k} - u_{j,k}\right) + \gamma_j^-\left(u_{j-1,k} - u_{j,k}\right)\right]^{n},$$

$$\frac{v_{i,k}^{n+1} - v_{i,k}^{n+1/2}}{\tau/2} + \beta_i\left(w_{i,k} - u_{i,k}\right)^{n+1/2} = 0,$$

$$v_{i,k}^{n+1/2} = F_i\left(w_{i,k}^{n+1/2}\right).$$

The adding and using of the concrete initial and boundary conditions are self evident and can be omitted.

Also evident is the other possibility to propose the one stage finite difference scheme, based on the matrix factorization: the representation of the system of partial differential equations as the differential equation vector.

For finding the unknown functions for all main layers we can use the generalized integral parabolic spline (12) and reconstruct the distribution of the concentration fields in the $z-$direction. After this step we can reconstruct the linear distribution of the concentration fields $U_{i+1/2}(x_k, z, t_{n+1})$ in the interlayers.

For the underground processes the system of the partial differential equations (3) (or (4)) can contain additional convective terms, see, e.g. [11], [12],

[19]. Self evident is generalization of the conservative averaging method for this case.

4 Conclusion

The original conservative averaging method and the generalized integral parabolic spline allows to transform the 2-D diffusion-sorption problem for the layered stratum with interlayers to the 1-D system of the PDE with continuous coefficients and with order of the system of the differential equations equal to the number of productive layers. The finite difference scheme for the solution of the obtained system of partial differential equations is proposed.

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