Transverse Full-Wave Analysis of Diffraction by a 2-D Metallic Strip

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Abstract: - The WCIP (Wave Concept iterative Process), a new iterative method based on wave concept, previously used to characterize scattering by cylindrical structures, is now extended to the investigation of electromagnetic diffraction in free space by planar structures using a cylindrical formulation . The basic case of scattering by a perfectly conducting plane is studied to validate this extension.

Key-Words: - Full-wave, iterative method, spatial domain, spectral domain, scattering.

1 Introduction

A tendency to full-wave simulations for microwave systems instead of analyzing components independently is growing [1]-[3].The WCIP, a transverse full-wave method, proposes efficient and general solutions involving cylindrical diffraction problems.

Many methods have been used to solve such problems. Among them, the electric integral field method EFIE in conjunction with the method of moments MoM or the finite element-method FEM are widely adopted. However, they cannot handle the entire structure due to its large electrical size which penalizes the computational time while inverting the MoM matrix.

The WCIP handles bounded operators and avoids the inversion of integral operator by treating, in an iterative way, the integral relations in the spectral domain and the continuity conditions in the spatial domain. It has also the advantage to circumvent treating singularities involving the green operator.

WCIP principle is the expression of boundary and closing conditions in term of waves (incident and scattered) .A system of equations is deduced from these conditions. It is resolved by an iterative process. The resolution is stopped when a desired precision is reached on the required value (input admittance, current, electromagnetic field).

In the next section, we describe the problem formulation and outline how the cylindrical formulation can be properly applied to a planar problem. Section 3 provides numerical results which validate the extension of a previous cylindrical formulation to the modeling of planar perfect electric conductor PEC scatterer. Suggestions for future research are given at the end of this section. Finally, conclusions are drawn in section 4.

2 Theoretical Formulation

The concept of wave is introduced through the analysis of a simple example. The analysis structure is composed by an interface Ω separating 2 media 1 and 2. The metallic scatterer is placed on the interface Ω.A source excitation is applied in the medium1.

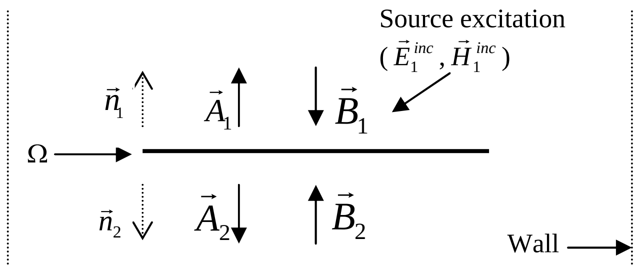

Fig.1Transverse section of the scattering interface.

The transverse incident waves $\vec{A}_i$ and the transverse scattered waves $\vec{B}_i$ at the electromagnetic discontinuity Ω are calculated from the diffracted tangential electric and magnetic fields $\vec{E}_i^d$ and $\vec{H}_i^d$ as:

$$\begin{cases} \vec{A}_i = \dfrac{1}{2\sqrt{Z_{0i}}}(\vec{E}_i^d - Z_{0i}\vec{J}_i^d) \\ \vec{B}_i = \dfrac{1}{2\sqrt{Z_{0i}}}(\vec{E}_i^d + Z_{0i}\vec{J}_i^d) \end{cases} \quad (1)$$

Where the index i refers to the medium 1 or 2 corresponding to the interface Ω.

$\vec{J}_i^d$ is the tangential surface current given by $\vec{J}_i^d = \vec{H}_i^d \wedge \vec{n}_i$ with $\vec{n}_i$ being the outward vector normal to Ω. Z_{0i} is the characteristic impedance of the medium i. Similarly, The incident excitation waves $\vec{A}_0$ and $\vec{B}_0$ are related to the incident electromagnetic fields $\vec{E}_i^{inc}$ and $\vec{H}_i^{inc}$ by:

$$\begin{cases} \vec{A}_0 = \dfrac{1}{2\sqrt{Z_{0i}}}(\vec{E}_1^{inc} - Z_{01}\vec{J}_1^{inc}) \\ \vec{B}_0 = \dfrac{1}{2\sqrt{Z_{0i}}}(\vec{E}_1^{inc} + Z_{01}\vec{J}_1^{inc}) \end{cases} \quad (2)$$

2.1 Relations in the spatial domain

The relations linking the incident waves to the scattered waves at the discontinuity interface are established by taking into account the continuity and the boundary conditions of the electromagnetic fields at the same interface.

The continuity conditions atΩ are the following:

$$\vec{E}_1^{\,d} + \vec{E}_1^{\,inc} = \vec{E}_2^{\,d} \quad (3)$$

$$\vec{n}_1 \wedge \vec{H}_1 + \vec{n}_1 \wedge \vec{H}_1^{inc} + \vec{n}_2 \wedge \vec{H}_2 = \vec{J}_{tot} \quad (4)$$

$\vec{J}_{tot}$ is the total surface current on the interface Ω.

Let consider the boundary conditions on the metallic domain of Ω:

$$\vec{E}_1^{\,d} + \vec{E}_1^{\,inc} = \vec{0} \text{ and } \vec{E}_1^{\,d} + \vec{E}_1^{\,inc} \neq \vec{0} \text{ elsewhere} \quad (5)$$

$$\vec{E}_2^{\,d} = \vec{0} \text{ and } \vec{E}_2^{\,d} \neq \vec{0} \text{ elsewhere} \quad (6)$$

The boundary conditions on the dielectric domain of Ω are written as:

$$\vec{J}_{tot} = \vec{J}_1^{\,d} + \vec{J}_2^{\,d} + \vec{J}_1^{\,inc} = \vec{0} \quad (7)$$

The sub-domains characterizing the interface Ω can be represented using Heaviside unit steps as

$$H_M = \begin{cases} 1 & \text{on the metal} \\ 0 & \text{elsewhere} \end{cases} \text{ and } H_I = \begin{cases} 1 & \text{on the dielectric} \\ 0 & \text{elsewhere} \end{cases} \quad (8)$$

Expressing equations (3)-(7) in term of waves, using the definition (8) and defining $n = \sqrt{\dfrac{z_{o1}}{z_{o2}}}$, the scattered waves are related to the incident waves as:

$$\begin{pmatrix} \vec{A}_1 \\ \vec{A}_2 \end{pmatrix} = [\hat{S}].\begin{pmatrix} \vec{B}_1 \\ \vec{B}_2 \end{pmatrix} + \begin{pmatrix} (-H_I - H_M)\vec{A}_0 + (\dfrac{1-n^2}{n^2+1}H_I + H_M)\vec{B}_0 \\ \dfrac{2n}{n^2+1}H_I\vec{B}_0 \end{pmatrix} \quad (9)$$

Where the scattering operator $\hat{S}$ is expressed as:

$$\hat{S} = \begin{pmatrix} \dfrac{1-n^2}{n^2+1}H_I - H_M & \dfrac{2n}{n^2+1}H_I \\ \dfrac{2n}{n^2+1}H_I & \dfrac{n^2-1}{n^2+1}H_I - H_M \end{pmatrix} \quad (10)$$

The scattering operator $\hat{S}$ is expanded in the spatial domain on the basis of pixels functions defined as Heaviside functions H_I and H_M . $\hat{S}$ can be seen as the “image” of the studied structure.

2.1 Relations in the spectral domain

By use of (1) and the closing condition $\vec{J}_i^{\,d} = \hat{Y}_i\vec{E}_i^{\,d}$, the waves are linked to each other by the external equation:

$$\vec{B}_i = \hat{\Gamma}_i\vec{A}_i \quad (11)$$

Where the reflection operator $\hat{\Gamma}_i$ is derived from the transverse admittance operator $\hat{Y}_i$ as:

$$\hat{\Gamma}_i = \sum_{n,\alpha} \left| \vec{f}_n^{\,\alpha} \right\rangle \frac{1 - z_{oi}.Y_n^{\alpha,i}}{1 + z_{oi}.Y_n^{\alpha,i}} \left\langle \vec{f}_n^{\,\alpha} \right|$$

(12)

With

$\vec{f}_n^{\alpha}$: The cylindrical mode functions basis of the lateral wall boarding the structure.

α =TE,TM.

The external equation (11) can be rewritten in a matrix representation as:

$$\begin{pmatrix} \vec{B}_1 \\ \vec{B}_2 \end{pmatrix} = [\hat{\Gamma}].\begin{pmatrix} \vec{A}_1 \\ \vec{A}_2 \end{pmatrix} \quad (13)$$

With:

$$\hat{\Gamma} = \begin{pmatrix} \hat{\Gamma}_1 & 0 \\ 0 & \hat{\Gamma}_2 \end{pmatrix} \quad (14)$$

The bounded diffraction operator $\hat{\Gamma}_i$ denotes the interaction between the interface and the environment of the medium i.

2.3 The iterative process

The mechanism of the iterative procedure is initialized by the excitation waves $\vec{A}_0$ and $\vec{B}_0$ .The application of (9) to $\vec{A}_0$ and $\vec{B}_0$ generates diffracted waves. These latter will be also reflected, in the spectral domain, respecting to (11) giving also incident waves on the electromagnetic discontinuity Ω .This process is reiterated until the convergence is reached (In this paper, the convergence criterion is applied on the magnitude of the current distribution).The mathematical WCIP convergence proof is given in [1].

The iterative scheme can be summarized as:

$$\left\| \begin{array}{l} \vec{A}_0, \vec{B}_0 \\ \begin{pmatrix} \vec{A}^n_1 \\ \vec{A}^n_2 \end{pmatrix} = [\hat{S}].\begin{pmatrix} \vec{B}^{n-1}_1 \\ \vec{B}^{n-1}_2 \end{pmatrix} + \begin{pmatrix} (\vec{A}_0, \vec{B}_0) \\ (\vec{A}_0, \vec{B}_0) \end{pmatrix} \\ \begin{pmatrix} \vec{B}^n_1 \\ \vec{B}^n_2 \end{pmatrix} = [\hat{\Gamma}].\begin{pmatrix} \vec{A}^n_1 \\ \vec{A}^n_2 \end{pmatrix} \end{array} \right. \quad (15)$$

n: the iteration order.

Hence, the WCIP technique is fully characterized in spatial and spectral domains.

In previous works, WCIP has been applied successfully to the modelling of electromagnetic diffraction by cylindrical structures [4]-[5].The mode functions $\vec{f}_n^{\alpha}$ involved in (12) are developed in the cylindrical coordinate system in order to implement the external equation (13). In this paper, we propose an extension of the cylindrical formulation to the planar case.

The basic idea is that the planar PEC strip can be approximated by a metallic cylindrical strip of infinite radius.

To validate this assumption, the reflection coefficient between two neighbouring pixels of the PEC, as it can be calculated in (12), is plotted against the radius of the approximated cylindrical strip. The plot in Fig.2 shows the validity of this approximation. In fact, the magnitude of the reflection coefficient is independent of the radius value when this latter is taken to infinity.

In the next section, the approximation proven above will be taken into account in all WCIP calculations involving planar cases.

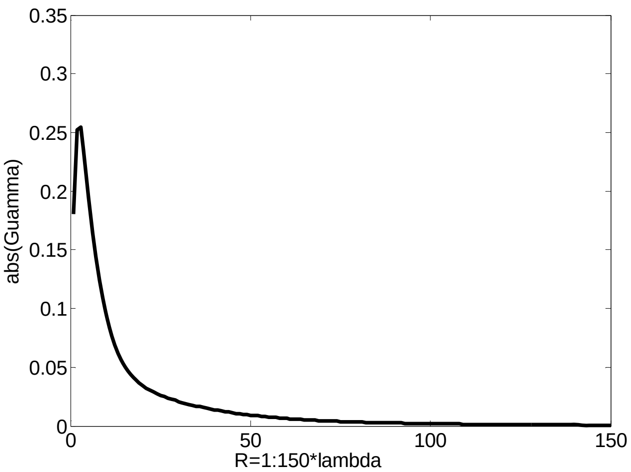

Fig.2Variation of the magnitude of the reflection coefficient versus the approximated cylinder radius R

3 Numerical results

The example presented here in Fig.3 deals with diffraction by a 2-D metallic strip illuminated by a TM polarized source excitation under normal incidence. The scatterer is uniform and infinite along z direction.

The strip length w is 10λ. The scatterer is meshed into 480 piecewise segments called also pixels.

In case (a), a plane wave is taken as a source excitation while in case (b), the scatterer is illuminated by a line electric current parallel to z axis.

The distance *d* between the line electric current and the strip is 2λ as chosen in [6].

The numerical results of the induced current distributions on the scatterer are shown in Fig.4 and Fig.5.

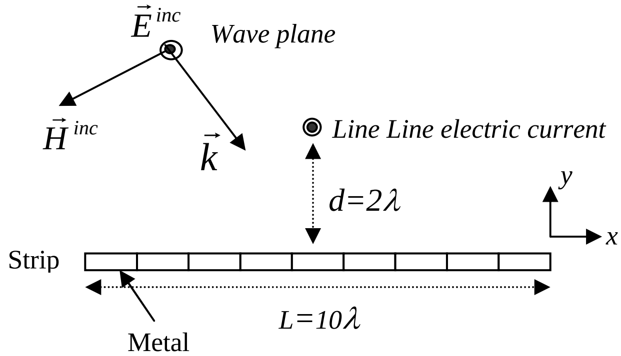

Fig.3 EM scattering problem for a 2-D strip illuminated by a wave plane (case a) or by a line electric current (case b)

The computed results are given after 150 iterations for both cases of source excitation.In term of computing time, the iterative process spends few seconds to reach the convergence.

The simulated results by the WCIP approach are confronted to the analytical solutions calculated by the EFIE method in conjunction with the matching-point technique.

As depicted in Fig.4 and Fig.5, the computed results agree well with the simulated EFIE results with imperfection on the edges.

The two graphs outline the part played by the excitation source. In fact, the EFIE current solution depends on the excitation vector of the problem [6].Similarly, the WCIP solution is governed by the excitation waves $\vec{A}_0$ and $\vec{B}_0$ initializing the iterative procedure and affecting then the final solution. However, the advantages of our full-wave approach could be not clear when applied to these two simple examples: It has been proven in previous works, that the number of operations needed by the WCIP algorithm is remarkably less than the one required by MoM for circuits requiring an important number of metal pixels[1],[3]. Furthermore, no matrix inversion is required and the convergence is insured independently of the source excitation and the interfaces of the studied structure.

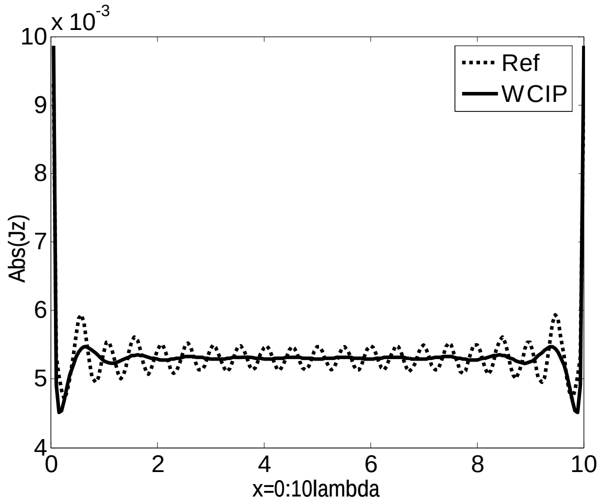

Fig.4 Induced current distribution on the 2-D strip in case a (TM plane wave illumination)

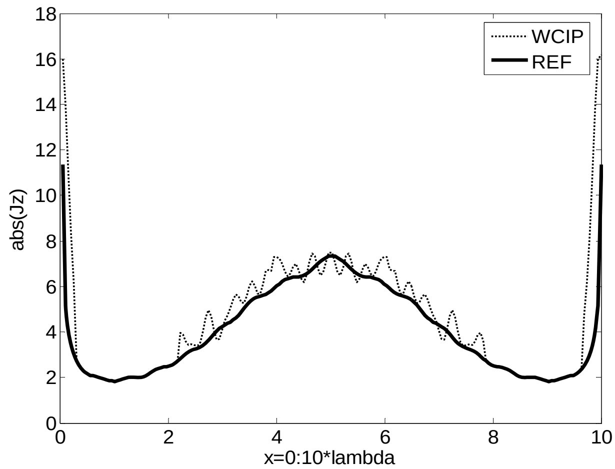

Fig.5 Induced current distribution on the 2-D strip in case b (TM electric line current illumination)

Although the satisfactory results, a rigourous treatment will be carried on the calculation of the refelection coefficients in free space for planar case in 2-D and 3-D problems in next work. We can prove that the kernel function of the reflection operator in (12) can be evaluated analytically without performing the summation of the TE and TM mode coefficients in the spectral domain. This feature will help ameliorating drastically the computing time of the WCIP algorithm.

4 Conclusion

The wave concept method WCIP as applied to the analysis of 2-D diffraction problem in free space is presented and successfully adapted to a planar geometry using cylindrical formulation.The computed results are in good agreement with the analytical solutions.

This extension enhances the adaptability of the WCIP approach to the modeling of various diffraction problems involving planar,cylindrical structures and more generally arbitrarily shaped bodies.

References:

[1] S. Wane, D. Bajon, H. Baudrand, and P. Gamand, “a new full-wave hybrid differential–integral approach for the investigation of multilayer structures including non uniformly doped diffusions”, *IEEE Trans.MTT*, vol.53, No.1, pp.200–214,Jan. 2005.

[2] S. Wane, D. Bajon, H. Baudrand, and P. Gamand, “Full wave analysis of isolated pockets to improve isolation performances in silicon based technology”, *IEEE MTT-S Int. Microwave Symp. Dig*., pp.987–990. Jun. 2002

[3] R.S. N’Gongo and H. Baudrand, “Application of. wave concept iterative procedure,” *Recent Res. Develop Microwave Theory and Tech*,,vol.1 pp.187-197,1999

[4] N. Raveu , T. P. Vuong , I. Terrasse , G.-P. Piau and H.Baudrand “ Near fields evaluated with the wave concept iterative procedure method for an E-polarisation plane wave scattered by cylindrical strips”,*Microwave and Optical Technology Letters*, Volume 38, Issue 5, pp 403 – 406,July 2003

[5] N.Ammar, T.Aguili and H.Baudrand : “Study of electromagnetic waves diffraction using the Wave Concept iterative Process (W.C.I.P) in spatial domain”, *MMS 2006*,Genoa , Italy.

[6] T.Shijo,T.HIirano and M.Ando: “Large-Size Local-Domain basis functions with phase detour and fresnel Zone Threshold for Sparse Reaction Matrix in the Method of Moments”, *IEICE trans.Electron* , vol E88-C, pp 2208-2215 , Dec 2005.