Some problems in water vapor tomography

PEEP MIIDLA	KALEV RANNAT	PEEP UBA
Institute of Mathematics,	Institute of Cybernetics	Estonian National Defence
University of Tartu	Tallinn University of Technology	College
Liivi 2, 50409 Tartu,	Akadeemia 21, 12618 Tallinn	Riia 12, 51013 Tartu,
ESTONIA	ESTONIA	ESTONIA

Abstract: The technical success and availability of numerical algorithms have promoted the implementation of tomography to atmospheric sciences. The principal specific character in initial constraints, data collection and assimilation methods, obtaining the final numerical results and interpretation of these makes the continuation of success story for GPS-tomography very challenging. The authors use numerical simulation as the most time- and money efficient way to study different processes connected with tropospheric water vapor tomography. This paper tends to give a short overview about some known difficulties in mathematical methods for detection, monitoring and modeling of the tropospheric water vapor. The possible mathematical approach to construct the virtual ground-based sensors (GPS-receivers) network for a real geographical location and discretisation of the troposphere, also some aspects of raw data filtering and analysis are described. Output of tomographical modeling of troposphere might to be applied to improve the results of large scale numerical weather prediction models. The questions of voxel geometry and methods of data processing are expected as key questions to construct an effective GPS-receiver network for water vapor tomography.

Key-Words: Global Satellite Navigation System, Troposphere water vapor, GPS-tomography, Kalman filter.

1 Introduction

GNSS (Global Satellite Navigation System) with known realisations as GPS, GLONASS and further coming GALILEO has become a part of everyday life. The GPS-signal for positioning is particularly used to obtain information on the concentration of tropospheric water vapor as one of the most important greenhouse gas and a carrier of latent heat in the atmosphere. The data about distribution of water vapour supports also numerical weather prediction (NWP) models. Related large-scale experiments and monitoring have been launched in Japan (GEONET), US (SuomiNet), Europe (E-GVAP), etc.

Tomographic methods have been developed to get a 3D distribution of water vapor in the troposphere [4,9,12,19]. The work on improvement and efficiency of the tomographic methods, depending on experimental setup and mathematical algorithms is a part of interest and a subject of research for many research groups.

The basic differences between medical and tropospheric tomography consist in the initial setup of the measurements and the behaviour and nature of the detectable environment. In medicine we have an exact dense constellation of signal transmitters and receivers and a full control on the movement of the test body. At the case of troposphere one cannot control the movements in the environment and the transmitter/receiver (GPS-satellite /GPS- receiver) constellation is variable and sparse. Even if knowing in principal the positions of the GPS-satellites, the real position is affected by orbital perturbations and the availability of precise orbits from IGS-service is delayed. The new signal transmitters (further-coming GALILEO) and new frequencies will help to get spatially more dense information from the atmosphere and to improve signal processing.

Although not so widely used yet, the tomographic method based on Slant Total Delay (STD) [27] has several essential advantages compared to traditional integrated precipitable water (IPW) estimation, based on Zenith Total Delay (ZTD) approach [21]. For high-resolution NWP the benefit of using STD is coming from the three main aspects: (i) the larger amount of observations available, as there are several satellites simultaneously visible from each point of observation, (ii) STD-observations can contain information on atmospheric anisotropy - the gradients of temperature and humidity (ZTD observations assume isotropic atmosphere), (iii) ZTD can be considered as a special case of STD (ZTD is derived from raw measurements along the slanted signal paths) [8].

STD observations provide potential for water vapor tomography – to obtain real 3D distribution of tropospheric water for kilometric scale modeling, but on the other hand, the procedure for production of STD observations is still in a stage of evolution. It is not persuasively clear if the accuracy of near real time STD observations is adequate for the operational demands of NWP systems. Due to the geometry of the slanted signal paths, assimilation of STD is not as straightforward as the assimilation of ZTD. Moreover, it is currently not known how to properly account for the complicated observation error correlations of STD [7,8].

From the NWP point of view, ZTD is a desirable observation since it is a linear function of IPW above the GPS receiver [1]. IPW is directly related to the model humidity variable and it is ideally suited to the model geometry. The relative complexity of usage of the slanted signal paths and common practice at NWP side has somehow suppressed the usage of water vapor tomography at operational level. The more it gives challenges for future research and modeling.

In fact the GPS-tomographic network does not need to be seen as an alternative to existing meteorological systems but something complimentary to it. GPS-based system can detect the water vapor and local extreme concentrations already before any radar detection is possible. In this sense the GPS-tomographic system can serve as a warning system.

To study the methods and large-scale experiment setups, the simulation is the most time- and cost effective way. The situation is modelled where the information on the water vapor content surveillance is obtained from the ground-based GPS-receivers supported by additional meteorological sensors. Those can be considered as stand-alone proactive agents with individual environment-dependent inputs and outputs. The IPW from a fixed geographical position is obtained by GAMIT/GLOBK software [14] and used as one of the links between simulation and reality. Real data from a fixed point is essential for calibration and evaluation of the model.

Kalman Filtering (KF) (and its modifications) has been investigated to minimize the noise in initial data.

2 The impact of the atmosphere

GPS signals propagating from GPS satellites to the ground-based receivers are affected by the atmospheric refraction which induces *slant delays* which is expressed by the excess path length of GPS radio signals (Fig 1). The ionosphere has dispersive effect on GPS-frequencies (L1=1575.42 MHz, L2=1227.60 MHz) and the troposphere has non-dispersive effect on the GPS-signal. It is possible to eliminate ionospheric influence by a linear combination of dual frequency data [15,17,29]. Troposphere is a non-dispersive medium and its impact cannot be determined by using the 2-frequency techniques.

The ionospheric refraction causes a delay of the information package, modulated on the carrier signal and the advance of carrier phase. The physical reason is that the wave group- and phase speed depend on the refraction index of the environment. The ionospheric delay can be removed in the model case by adding supporting 2-band receivers to the monitoring network and interpolation of the ionospheric constituents between gridpoints.

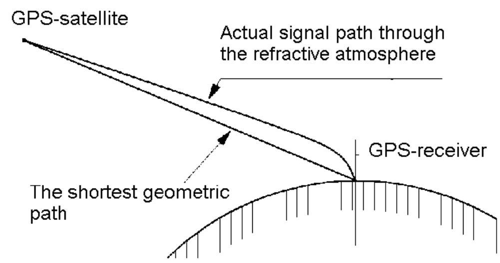

Figure 1: The signal path through the atmosphere.

The delay caused by the neutral atmosphere can be separated into two components: the zenith hydrostatic delay ("dry" component, ZHD) as a result of the induced dipole moment and a zenith wet delay ("wet" component, ZWD) due to the permanent dipole moment of Precipitable Water and liquid water (LW) present in the troposphere [23]. The wet component is spatially and temporally varying, therefore the errors in the models for the wet component are larger than the errors in the models for the "dry".

STD is induced both by dry air and water vapor. Slant dry delay can be estimated from surface pressure measurements or from numerical weather models [5]. Slant wet delay (SWD) can be estimated as the difference of total slant delay and slant dry delay.

The tropospheric delay can be expressed as follows [15]:

$$\Delta^{trop} = \int n \cdot ds - \int ds = \int (n-1)ds \quad (1)$$

where n(s) denotes the atmospheric refraction index on the signal path. Integration is performed over the total path in the troposphere and instead of refraction index a new quantity $N^{trop} = 10^6(n-1)$ will be introduced, often called as refraction. Equation 1 can be explained as

$$\Delta^{trop} = 10^{-6} \int N^{trop} ds \,. \quad (2)$$

Dividing the delay due to the tropospheric refraction into two components (*dry* and *wet*), the total delay can be expressed as

$$\Delta^{trop} = 10^{-6} \int N_d^{trop} ds + 10^{-6} \int N_w^{trop} ds \,. \quad (3)$$

This kind of dividing into components is coming from practical considerations. The dry component of the tropospheric delay is quite precisely described by different mathematical models (for example models of Hopfield, Niell, Saastamoinen and others) [20]. Prediction and modeling of the wet component is complicated. The problem comes from the lack of measurement data and the instability of the troposphere (tropospheric turbulence).

Tropospheric refraction is often explained by empiric equations [15]:

$$N_{d,0}^{tropo} = \bar{c}_1 \frac{p}{T} \,,\; N_{w,0}^{tropo} = \bar{c}_2 \frac{e}{T} + \bar{c}_3 \frac{e}{T^2} \,, \quad (4)$$

where $\bar{c}_1 = 77.64 \quad K \cdot mb^{-1}$, $\bar{c}_2 = -12.96 \quad K \cdot mb^{-1}$ and $\bar{c}_3 = 3.718 \cdot 10^5 \quad K^2 \cdot mb^{-1}$, p is atmospheric pressure in mb, T is temperature in K and e is the partial pressure of water vapor. $\bar{c}_1, \bar{c}_2, \bar{c}_3$ are empiric constants. The results can be improved by measuring real meteorological data at the observation site. The real data is entered to some of the known refractivity models (Hopfield, Saastamoinen, etc.). It is possible to get an estimate for the dry component of tropospheric delay $< 1mm$ if the pressure at ground surface is measured with precision $0.3mb$ [6]. The wet component of tropospheric delay varies remarkably (from some millimeters at polar areas and up to 40 mm in tropics). Dry component estimation needs only the air pressure and temperature at the ground surface. For wet component also information on the relative humidity is needed.

To obtain the amount of water vapor in the atmosphere we need a relationship between the signal delays and IPW.

The amount of integrated water in the atmosphere (in zenith direction above the GPS-antenna) is equivalent to the height of a column of liquid water. It is found by dividing the zenith integrated water vapor by the density of liquid water. In a similar manner the slant water (SW) is defined, as the length of an equivalent column of liquid water on the ray path between GPS-receiver i and satellite m. SW_i^m is the Integrated Slant Water (ISW) vapor divided by the density of liquid water $SW_i^m = ISW_i^m / \rho$, where

$$ISW_i^m = \int_{rec_i}^{sat_m} \rho_w ds \,. \quad (5)$$

The ratio of SW to the Slant Water vapor Delay (SWD) is often known as a non-dimensional conversion factor Π, expressed as $\Pi = SW_i^m / SWD_i^m$. Here SWD_i^m can be derived from the STD measurements described in [8]. The IPW and SW can be easily found, as IPW=Π·ZWD and SW= Π·SWD (assuming the atmosphere isotropic).

$\Pi \neq const$, it depends on real environmental parameters [2]. Mostly it depends on the mean temperature of the atmosphere, T_m, defined in [23,26].

Water-vapor-weighted atmospheric mean temperature, T_m, is a key parameter in the retrieval of atmospheric precipitable water (PW) from ground-based GPS measurements of ZPD/SPD, as the accuracy of the GPS derived PW is proportional to the accuracy of T_m. The RMS (root mean square) error of GPS-derived PW ranges from smaller than 2 mm in North America to 3.7 mm in Japan. An uncertainty of 5 K in T_m corresponds to 1.6–2.1% uncertainty in PW. T_m has also temporal and spatial variations, described in [26].

For practical troposphere monitoring (for real reference data) the Trimble NetRS GPS-receiver was used at 58°23'30" N, 26°41'41" E, with antenna height 75.80 m above the reference ellipsoid. The data is captured from December 2006 on. The data post-processing is performed by GAMIT.

GAMIT-processing helps to obtain the real path and delay of the signal from each satellite to the receiver. The difference between real and – geometrical gives the initial information needed for tropospheric water estimation. The GAMIT software uses Global Mapping Function by default, developed by Boehm et al. [3] for atmospheric delays calculation. In the absence of *in situ* meteodata, the best choice of *a priori* pressure and temperature for a site comes from the global pressure and temperature (GPT) model [14].

On Figure 4 the 10 days interval is presented with the corresponding evolution of Integrated Precipitable Water, temperature, relative humidity and air pressure in Tartu.

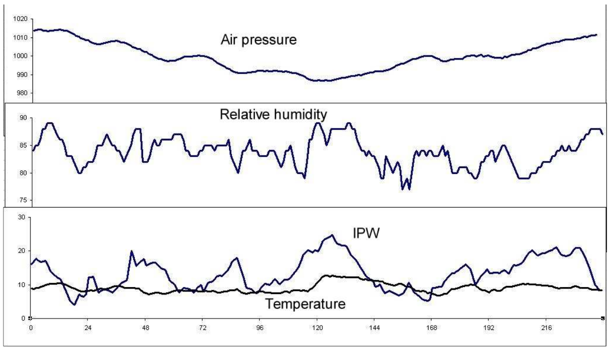

Figure 4: Pressure, relative humidity, precipitable water and temperature in 01-10. Dec. 2006. Horizontal resolution 1 hr.

The presented results are un-smoothed. The short-term evolution of relative humidity at ground level and the IPW are quite unstable. It can be interpreted as the atmospheric turbulence, but the result contains also all possible discrepancies at this stage.

3. Error management and numerical filtering of the initial data

Although the GPS-measurements are believed to be extremely accurate mostly for every application, they can exhibit significant errors depending on environmental conditions and technical setup of the experiments. From the modeling point of view and geodetic accuracies the results are mostly affected by the orbit errors and clocks drift.

Assimilation of the results into NWP models needs accuracies better than 2 mm of PWV (Precipitable Water Vapor) within 1 h of data collection [13]. For now-casting (often referred as the forecasting of the weather in the 0-12 hrs timeframe) the TOUGH project report [25] recommends the time resolution of nearly real time (NRT) GPS data as high as 2-4 observations/hour with an update every hour, with a potential for faster updates in the future.

The first obstacle for geodetic data processing is related to the delayed availability of GPS-satellites precise orbits. For real- and near real time applications only predicted or Ultra Rapid products can be used. Improvement can be achieved by sliding-window technique (e.g. [10]).

The orbits errors are systematic (except the occasions of the Sun bursts), therefore they are easier to handle. The receiver clock errors can be mostly eliminated by double-differencing method. The systematic errors can be eliminated by combining and comparing different data of observations.

Atmospheric tomography (and the related modeling) has additionally some specific sources of errors, impossible to ignore. The *first* is related to the voxels (the meaning of the term is presented later), not giving any information because of not intercepted by any signal ray. The *second* is related to the real data acquisition at the point of measurement - sudden malfunction of some of the sensors or loss of the data.

The *first* category of problems is expected to be smoothed by Kalman Filtering (KF). KF and smoothing have been used also in GPS-campaign at Onsala, described in [9]. The successful exploitation of KF or its modifications [16,18,28] is not always straightforward due to the limitations coming from the receiver's network geometry. KF needs also the observability and controllability conditions to be satisfied [11], otherwise the algorithm will not converge. One possibility to overcome the geometry limitations is exploiting the Wet Refractivity Kalman Filter [12].

The *second* category of errors is overcome by using the paradigm of *agents*. By this concept each node of the GPS-receivers network is considered an autonomous proactive agent, with the main task to capture the data on GPS-observations and 3 additional meteoparameters. Additionally, each node (agent) is responsible on the data link with a central data processing unit. If for example one meteoparameter (let's say ground surface temperature) is missing, then it can be asked from the adjacent agents, the result interpolated to the malfunctioning node and "complete" data sent to the central unit. This makes it technically possible to distribute some operational tasks to the agents in the nodes leaving more computational resources to the data processing.

The random measurement errors are also processed by KF. The KF addresses the general problem of trying to estimate the state X of a discrete-time controlled process that is governed by the linear equation on its development from the timepoint k-1 to the instant k:

$$X_k = AX_{k-1} + w_{k-1} . \quad (6)$$

Assume also that the measurement Z is expressed by the state vector X:

$$Z_k = HX_k + v_k . \quad (7)$$

Here X and Z are the vectors of n- and m-dimensional real Euclidean space respectively. The random variables w_k and v_k represent the process and measurement noise (respectively). They are assumed to be independent of each other and auto-independent, white, with normal probability distributions. In practice, the process noise covariance and measurement noise covariance matrices Q and R might change at each time step, however here assumed constant. The $n \times n$ matrix A in the equation (6) relates theoretically the state at the previous time step $k-1$ to the state at the current time step k, in the absence of either a driving function or process noise. Note that in practice A and H might change with each time step, but here we assume those are constant matrices. The $m \times n$ matrix H in the measurement equation (7) relates the state to the measurement Z_k .

Finding the best modification of KF is one of the next challenges of the modeling, as numerical precision and stability is counterbalanced with computational load, directly related to the applicability of the monitoring concept and surveillance network.

One important aim to obtain the information about water vapor distribution in troposphere is to use the results as input for the large-scale NWP models. Output for GPS-tomography is the concentrations of water vapor in every single voxel introduced. This means that observers have the set of virtual humidity sensors below the whole considerated area. Those are from one side dynamically moving and for some parts of (for some voxels) troposphere maybe interpolated, but from other side, also do not need service and do not disturb aviation. The use of obtained information as feedback for better estimation of the state of weather is possible through the KF.

4. GPS-receivers network and discretization of the troposphere

The installation of the GPS-receivers network for tropospheric tomography is modelled. Tomographic methods to obtain the 3D distribution of tropospheric water has been a subject for numerous investigations [4,9,12,19].

The initial idea is coming from conventional tomography, known in medicine. The information on ray propagation (delaying or attenuation) in a certain part of environment is obtained from different angles and later the image of the investigated constituents is reconstructed by tomographic algorithms. In the model case the delay is equivalent to signal path (additional delay corresponds to the prolongation of the path).

From atmospheric point of view the GPS-signal time delays and the contributions of atmospheric water on the delays in every discrete part of the troposphere (hereafter called *voxel*) are under consideration. The discretization of the troposphere into rectangular voxels is explained at Figure 5. There exist different geometrical setups for building the voxel system, but for the explained simulation experiment the rectangular grid is chosen as the simplest one (assuming the monitoring area relatively small).

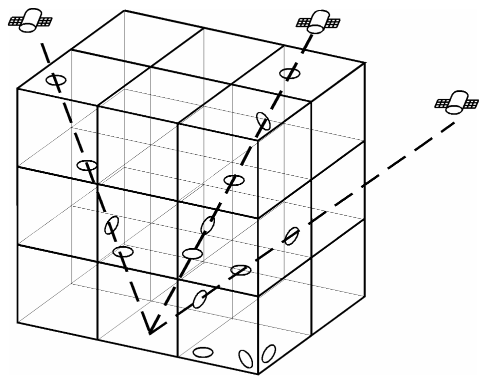

Figure 5. Ray paths from the satellites to a GPS-receiver through rectangular voxels.

For large-scale tomography, for areas in some hundreds of quadratic kilometres different discretization models are preferable, taking into account the Earth curvature. On Figure 6 another possibility of construction of voxels is shown [22].

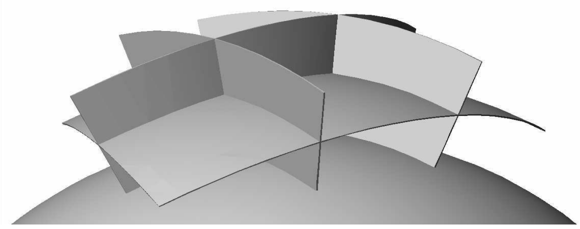

Figure 6. Curved voxels.

The voxel boundaries are constructed in spherical system of coordinates. Orthogonal Earth-centered coordinates (x,y,z) and spherical coordinates (ρ,φ,λ) are mutually transformed by formulas x = ρ cosφ sinλ ; y = ρ sinφ sinλ ; z = ρ cosλ .

Below we consider the rectangular voxel system.The tropospheric water is mostly distributed in a layer with height up to 7 – 10 km. The horizontal resolution 10-40 km of the grid is usually taken in similar experiments (station separations ~50 km or less recommended in the final report of the TOUGH-project [24]). Detecting the water vapor takes into account all the GPS-signal ray paths (where the signal has high quality enough) from the visible satellites into each location of GPS-receiver. The maximum number of ray paths is $n \cdot m$, where n corresponds to the number of satellites and m to the number of receivers. Hereby must be noticed that n corresponds to a number of those satellites only that can be technically used in the experiment (having acceptable signal/noise ratio at a point of observation). This means that n can be smaller than the number of visible satellites and additionally, the number of usable ray paths can be reduced due to technical conditions.

5. The model and modeling

Each signal on the path between a receiver and the satellite penetrates a certain amount of voxels. Each voxel (with water vapor) has its individual contribution to the total delay of the signal corresponding to Eq. (3). The full path of the signal is a sum of the paths in the intercepted voxels. The model can be interpreted as a system of linear equations

$$\sum_{i=1}^{v} x_i \cdot s_{i,j} = f_j, \quad (8)$$

where j = 1,…, K, K is the total number of signal paths from all visible satellites in the GPS-network at a fixed time instant, v is the total number of voxels, $s_{i,j}$ is the length of the jth ray in voxel i, the x_i are considered as weights of a corresponding voxel to the slant path delay and interpreted as indicators of the PW concentration. Interpretation of the solution of the system of equations Eqs. (8) gives the distribution of water vapor by the voxels. The differences between the geometrical shortest path and the extra path, induced by the tropospheric water (depending on the temperature, humidity and air pressure at the point of measurement), are specified by the absolute term f_i in an underdetermined system of linear equations (Eqs. 8). Based on precise trajectories of the satellites (obtained from IGS-service) the ray path length in each voxel is found, making the system (Eqs. 8) composed.

The system is resolved by the Least Squares (LS) method. It can be resolved also by Kalman Filter, using the prediction and correction step alternately.

The model software has two main tasks – (i) to construct the voxel system according to initial model parameters with handling of the voxel related data and (ii) the monitoring of the signal ray paths through the voxels. Hereby should be mentioned that the model (also the GPS-receivers network) obtains the slant delays directly from the GPS signal phase observations, similarly like described in [19].

According to the satellite constellation at a certain time instant the geometrical shortest paths (Figure 1) are constructed from all satellites to all of the receivers. For each signal path the intercepted voxels will be found with geometrical points of ingoing and outgoing (to fix the shortest possible ray path inside the voxel). These partial lengths are used as coefficients in Eqs. (8). Each ray will penetrate only a minor part of voxels and the situation changes at each time step.

The simulation of monitoring consists of generation of initial data for a certain mathematical model and the data processing related to a specific scenario. The synthetic data consists of the outputs of GPS-receivers (including both (i) the data sent by a satellite and (ii) the data generated by a receiver).

The *first category* represents mostly the positions of the satellites, and the time parameters of atomic clocks. This data consists in GPS-satellites navigation message and can be obtained also from GPS Ground Control Stations or some public databases via internet. The *second category* consists of data about the time instants the signal was received (receiver time), carrier phase, the position of satellites (from receivers point of view) and some supporting information.

The simulation program, compiled on the basis of the mathematical model, is responsible on generation of a situation as realistic as possible. The model situation must be described by a real geographical location and the real constellation of GPS-satellites at a certain time instant. Based on this information additional analysis is performed to find suitable locations for GPS-receivers in the monitoring network. The criterion is the solution of Eqs. (8) – the result must be realistic in meteorological sense.

The simulation helps to specify at which initial conditions the system (Eqs. 8) can be resolved and when the result is reliable.

As a rule, the system of Eqs. (8) has less equations than unknowns (the number of signal rays is smaller than the number of voxels). Depending on the temporary constellation of GPS-satellites and receivers in the ground-based network, the system can be either unresolved or the solution is not precise enough (unrealistic). The results are also affected by the allocations of the receivers relative to each-other, the number of the receivers and geographical position (regarding the constellation of satellites). Depending on the abovementioned reasons and configuration of the voxel's system, the system of linear equations (Eqs. 8) can be over-, under- or normally determined.

Based on analysis of different initial conditions and the solutions of model equations, it can be concluded – the more homogeneous the GPS-receivers network, the better the result. In fact, the reality is not as simple regarding installation of the network — the sensors are installed on positions considering practical possibilities. Simulation helps to find the best available spatial setup for the receiver's network.

For ideal conditions the GPS-receivers need open horizon that is impossible to guarantee in reality (the nearby forests, high buildings, etc. mask a certain part of the sky or initiate additional inaccuracy in measurements).

The moisture per each voxel can be derived by interpreting the numeric solution of the model. In the simulation package this is done by a stand-alone module.

Using the delays in voxels and the conversion factor Π, the amount of water vapor is found. Information on all voxels gives a 3D distribution of water vapor in the troposphere and IPW additionally. The IPW can be found also from real measurements and GAMIT-processing at a certain point of the monitoring area (for current experiment the station at 58°23'30" N, 26°41'41" E). This gives a possibility for validation of the model. As a favorable by-product from GAMIT-processing, the STD can be derived and compared to the total delays calculated by the simulation model. The temporal and geographical variation of Π is unknown, but using the estimated value around 0.15 satisfies the simulations at current stage.

It is foreseen that the simulation leads to practical monitoring of the tropospheric water. Based on modeling results (the best geometry), the GPS-receivers network can be installed with the supporting meteorological sensors and data communication devices. Quite an independent task will be the optimization of the communication network and protocols for real-time (or near real-time) application.

Conclusions

The modeling and simulation approach shortly described afore, is the initial phase in developing a ground-based GPS-network for meteorological purposes by authors. The collaboration can be foreseen with Estonian Land Board and Estonian Meteorological and Hydrological Institute for establishing the common monitoring network and database, but also on international level.

The modeling environment works well for receiver-network geometry analysis. The concept of agents helps to model situations closer to reality and to decentralize the data processing load. The future effort is targeted to the data filtering as it is not clear what kind of KF will give the best result and how to resolve the questions of poor voxel geometry in an optimal way. These two topics will probably be the key questions to construct an effective GPS-receiver network for water vapor tomography.

This paper is supported by the grant No. 7097 of Estonian Science Foundation.

References:

[1] Bevis, M., S. Businger, T. A. Herring, C. Rocken, R. A. Anthes, and R. H. Ware. GPS meteorology: Remote sensing of atmospheric water vapor using the Global Positioning System. *J. Geophys. Res.,*Vol. 97, 1992, pp 15,787-15, 801.

[2] Bevis, M., S. Businger, S. Chiswell, T. A. Herring, R. A. Anthes, C. Rocken, and R. H. Ware. GPS meteorology: Mapping zenith wet delays onto precipitable water. *J. Appl. Meteorol.,*Vol. 33, 1994, pp 379-386.

[3] Boehm, J., Niell, A., Tregoning, P., Schuh, H. Global Mapping Function (GMF): A new empirical mapping function based on numerical weather model data, *Geophysical Research Letters*, Vol. 33, 2006, L07304, doi:10.1029/2005GL025546.

[4] Braun, J.J. and C. Rocken. Water vapor tomography within the planetary boundary layer using GPS. *Proceedings of the International Workshop on GPS Meteorology, GPS Meteorology: Ground-Based and Space-Borne Applications* (Tsukuba, Japan, January 14-17, 2003).

[5] Chen, G., and T. Herring, Effects of atmospheric azimuthal asymmetry on the analysis of space geodetic data." *J. Geophys. Res.*, Vol. 102, 1997, pp 20,489-20,502.

[6] Duan. J.P. and Coauthors, GPS Meteorology: Direct Estimation of the Absolute Value of Precipitable Water. *Journal of Applied Meteorology*, Vol.35, 1996, No.6, pp 830-838.

[7] Elosegui, P. Davis, J. Accuracy assessment of GPS slant-path determinations. In: *Proceedings of the InternationalWorkshop on GPS Meteorology*, Tsukuba, Japan, 14–17 Jan 2003 (eds. T. Iwabuchi and Y. Shoji).

[8] Eresmaa, R., Järvinen, H. An Observation operator for ground based GPS slant delays". *Tellus A*, 58, No 1, 2006, pp 131-140.

[9] Flores, A., L.P. Gradinarsky, P. Elosegui, G. Elgered, J.L. Davis and A. Rius. Sensing atmospheric structure: tropospheric tomographic results of the small-scale GPS campaign at the Onsala Space Observatory. *Earth Planets Space*, No. 52, 2000, pp 941-945.

[10] Foster, J., Bevis, M., Businger, GPS Meteorology: Sliding-Window Analysis. *Journal of Atmospheric and Oceanic Technology*, AMS, Vol. 22, 2005, pp 687-695.

[11] Gelb, A. (ed.) Applied Optimal Estimation, MIT Press, Cambridge, MA., 1984.

[12] Gradinarsky, L.G. Sensing Atmospheric Water Vapor Using Radio Waves. PhD thesis, *Chalmers University of Technology, Technical report* No 436, 2002.

[13] Gutman, S.I., Benjamin, S.G. The role of ground-based GPS meteorological observations in numerical weather prediction. *GPS Solutions*, Vol. 4, 2001, pp 16-24.

[14] Herring, T. A., King, R. W., McClusky, S. C. *GAMIT Reference Manual*, Release 10.3, Department of Earth, Atmospheric, and Planetary Sciences, MIT, 2006.

[15] Hofmann-Wellenhof, B., H. Lichtenegger and J. Collins. *GPS Theory and Practice*, Springer-Verlag Wien, NewYork, NY, USA, fifth edition, 2001.

[16] Kalman, R.E., A new Approach to Linear Filtering and Prediction Problems". *Transaction of the ASME – J.of Basic Engineering*, March 1960, pp.35-45.

[17] Klobuchar, J., Ionospheric effects in GPS. In: B. Parkinson, J. Spilker (eds) *Global Positioning System: Theory and applications*, AIAA, Vol. 1, Chapter 12, 1996.

[18] Lange, A.A., Simultaneous Statistical Calibration of the GPS signal delay measurements with related meteorological data, *Physics and Chemistry of the Earth*, Part A: Solid Earth and Geodesy, Vol. 26, No. 6-8, 2001, pp 471-473.

[19] Nilsson, T., Assessment of Tomographic Methods for Estimation of Atmospheric Water Vapor Using Ground-Based GPS. *Thesis for the degree of Licentiate of Engineering*. Chalmers University of Technology, 2005.

[20] Shrestha, S.M., Investigations into the Estimation of Tropospheric Delay and Wet Refractivity Using GPS Measurements. *A Thesis in partial fulfilment of the requirements for the degree of master of science*. Galgary, Alberta, 2003.

[21] Stoew, B., Jarlemark, P., Johansson, J., Elgered, G., Real-time processing of GPS data delivered by SWEPOS. *Phys. Chem. Earth (A)* Vol. 26, 2001, pp 493–496.

[22] Tammo, P., Simulation of the output of GPS receiver (in Estonian). Bachelor thesis, University of Tartu, Institute of Applied Mathematics, 2007.

[23] Tregoning, P., Accuracy of absolute precipitable water vapor estimates from GPS observations. *Jornal of Geophys. Res.*, Vol. 103, 1998, pp 28701-287109.

[24] Vedel, H. et al. Final report for the TOUGH project". In: H. Vedel (Co-ordinator), *Targeting Optimal use of GPS Humidity Measurements in Meteorology*, EVG1-CT-2002-00080, 6th Periodic Report, 1 February 2003 – 31 January 2006, Section 6, http://tough.dmi.dk, (last visited 04.11.2007).

[25] Vedel, H. et al., GPS Data Recommendations for European Numerical Weather Prediction". In: H. Vedel (Co-ordinator), *Targeting Optimal use of GPS Humidity Measurements in Meteorology*, Deliverable D73, http://tough.dmi.dk, 2006 (last visited 04.11.2007).

[26] Wang, J., Zhang, L., Dai, A., Global estimates of water-vapor-weighted mean temperature of the atmosphere for GPS applications. *Journal of Geophysical Research*, Vol. 110, 2005, D21101, doi:10.1029/2005JD006215

[27] Ware, R., C. Alber, C. Rocken, F. Solheim. 1997. "Sensing Integrated Water Vapor along GPS Ray Paths." *Geophys. Res. Lett.*, Vol. 24, 1997, pp 417-420.

[28] Welch, G., Bishop, G., *An Introduction to the Kalman Filter*. University of North Carolina. Shapel Hill, TR 95-41, 2003.

[29] Xu, G., GPS. *Theory, Algorithms and Applications*. Springer Verlag Berlin, Heidelberg, 2003.