Behavioral Model Generation of Analog Circuits

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Abstract: - Behavioral models are used for top-down design and for bottom-up verification. The efficient simulation of large–scale dynamical system needs a systematic procedure for order reduction of the original circuit. A method based on the moment matching for behavioral model generation of analog integrated circuits and by comparison some approaches to obtain an accurate approximation of the circuit function are presented. It is shown that we can handle the MIMO system using the semi-state method and Padé approximation. An illustrative example is given and some conclusions are pointed out.
Key-Words: - Analog integrated circuits, Simulation, Transfer function, State equations, Order reduction, Behavioral modeling.

1 Introduction

Behavioral models capture certain functional properties without relying on specific structural representations. Using behavioral models a segnificant speedup of higher level simulation is achived. Also, they allow early design verification during top-down circuit design improving design efficient [1-3, 10]. The most important objective in analog circuit design is establishing the relationship between the input-output variables. This relationship depends on the device type and geometry, phenomenon and behavior, and on the circuit configuration, so that, especially in the large-scale system case, for an efficient simulation, an equivalent reduced size model of the original circuit is needed. There are different approaches of this problem: some of them are based on the sensitivity computation in order to simplify the circuit structure by noncritical element elimination, others reduce the insignificant terms in the circuit function expression, or the internal node of the circuit by equivalent electrical transforms; there are approximation methods based on Taylor series, on Padé Approximations, on Lagrange Polynomials, on Spline functions etc. No matter what is the principle they are build on, the aim of all these methods is getting an approximated transfer function, with a reduced number of poles. The reduced model can be then used for circuit response prediction in time or frequency domain, in a predicted range of the signal frequencies. The choice of one reducing order method or other one depends on the specific problem we have to solve it.

The major problems involved in obtaining reliable reduced-order models are [1-3]: a good accuracy with enough small size, numerical stability and the necessity to keep the original system passivity. The most recently algorithms used for the order reduction of the large-scale analog circuits could be classified in two categories:

- Methods based on the explicit (direct) matching of the moments to a reduced model – like *Asymptotic Waveform Evaluation* – AWE;
- Methods based on implicit (indirect) moment matching – developed in Krylov space – like *Padé via Lanczos* (PVL) and *Arnoldi algorithm*.

Developed by Pillage and Rohrer in 1990, AWE is the standard method for the analysis of large scale analog circuits. This technique uses the Padé approximation based on explicit moment matching to extract the dominant poles and the residues of the circuit. It consists in two steps: *moment computation* – meaning computation of the coefficients of MacLaurin series of the transfer function $H(s)$; *moment matching* – matching these moments with those of a reduced model using the Padé approximation.

The reduced-order model is characterized by an approximate transfer function $\hat{H}(s)$ expressed by a number of rational terms equal with the number of the approximated poles.

In this paper we present two systematic procedures for order reduction by moment matching. The moment computation is based both on state equation formulation [4-6] and on modified nodal equation in dynamic behavior formulation [7], implemented in two very efficient programs [8, 9]. Some examples are used to illustrate the methods.

2 Transfer function generation

In state space the equations of a linear analog lumped circuit can be written in the following normal form:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{cases}, \quad (1)$$

where: $\boldsymbol{x}(t)$ is the state vector 1xn, $\dot{\boldsymbol{x}}(t)$ - the state derivative vector, $\boldsymbol{u}(t)$ – the input vector 1xm, $\boldsymbol{y}(t)$ – the output vector (the circuit response) 1xl, $\boldsymbol{A}$ – the state matrix of the circuit nxn, and $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ – matrices having corresponding dimensions.

Applying the Laplace transform with null initial conditions, we obtain the independent variable vector $\boldsymbol{X}(s)$ and the transfer function vector $\boldsymbol{H}(s)$ corresponding to the selected ports:

$$\boldsymbol{X}(s) = (s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B}\boldsymbol{U}(s), \quad (2)$$

$$\boldsymbol{H}(s) = \frac{\boldsymbol{Y}(s)}{\boldsymbol{U}(s)} = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}. \quad (3)$$

If the semi-state approach is used, the equations will be

$$\begin{cases} \boldsymbol{W}\dot{\boldsymbol{x}}(t) + \boldsymbol{G}\boldsymbol{x}(t) = \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{L}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{cases}, \quad (4)$$

from where

$$\boldsymbol{X}(s) = \left(\boldsymbol{I} - s\boldsymbol{P}\right)^{-1}\boldsymbol{R}\boldsymbol{U}(s), \quad (5)$$

and

$$\boldsymbol{H}(s) = \frac{\boldsymbol{Y}(s)}{\boldsymbol{U}(s)} = \boldsymbol{L}\left(\boldsymbol{I} - s\boldsymbol{P}\right)^{-1}\boldsymbol{R} + \boldsymbol{D}, \quad (6)$$

with

$$\boldsymbol{P} = -\boldsymbol{G}^{-1}\boldsymbol{W}, \ \boldsymbol{R} = \boldsymbol{G}^{-1}\boldsymbol{B}. \quad (7)$$

The goal of the order reduction techniques is to find approximate expressions for the transfer functions (3) or (6) in order to decrease the computation effort for large scale circuit simulation.

3 Circuit moments

The MacLaurin series of a transfer function $H(s)$ is

$$H(s) \approx H(0) + s\frac{H^{(1)}(0)}{1!} + \ldots + s^n\frac{H^{(n)}(0)}{n!} = \sum_{i=0}^{n} s^i m_i, \quad m_i = \frac{H^{(i)}(0)}{i!}. \quad (8)$$

The coefficients m_i of the series are called circuit *moments*.

In the case of a single input-single output circuit, the transfer function can be represented as a ratio of two polynomials in s (9), or as a sequence of expressions depending on the poles p_i and the corresponding residues k_i (10):

$$H(s) = \frac{P(s)}{Q(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_0}, \ m < n \quad (9)$$

$$H(s) = c + \sum_{i=1}^{n_p}\frac{k_i}{s - p_i}. \quad (10)$$

In the case of large scale circuits because of the huge number of poles, obtaining the form (10) is computationally very expensive even impossible.

The techniques for order reduction avoid this computation by finding an approximate transfer function having a reduced order (q) denominator polynomial $Q(s)$, $q << n_p$ being the number of the dominant poles.

To obtain a reduced-order model with high accuracy matching as much as possible moments is necessary. This number defines the order of approximation. In order to generate an approximate *q-pole* transfer function, $\hat{H}(s) \cong H_q(s)$, $2q$ moments are needed.

4 Moment computation

Moment computation can be done either by expanding in Taylor series the independent variable vector (Eq. (2) or (5)) or the transfer function (Eq. (3) or (6)) about $s = s_0$. A critical view of moment-matching approach points out that Padé approximation is accurate near the point of expanssion (the point use to generate the moments). If that neighborhood contains most of the signal energy (large residues and poles close to the origin) then the time domain approximation will be accurate, otherwise inaccuracies will appear in the transient simulation.

If moments from both $s = 0$ and $s = \infty$ are matched, then the approximation will be accurate in those regions, and may be inaccurate in unknown regions between those two points. A single Padé approximation cannot guarantee the detection of all resonant frequencies in a given frequency band. These problems are addressed by a multi-point moment-matching.

In the particular cases when $s = 0$ and $s = \infty$ the moments will be computed with the expressions given in Table 1, and 2, respectively.

Table 1

	State variable	method
	$\boldsymbol{X}(s)$ expanding	$H(s)$ expanding
$s=0$	$\boldsymbol{AM}_0 = -\boldsymbol{B}$ $\boldsymbol{AM}_k = \boldsymbol{M}_{k-1}$	$m_0 = -\boldsymbol{CA}^{-1}\boldsymbol{B} + \boldsymbol{D}$ $m_k = -\boldsymbol{CA}^{-(k+1)}\boldsymbol{B}$
$s=\infty$	$\boldsymbol{AM}_0 = -\boldsymbol{B}$ $\boldsymbol{AM}_k = \boldsymbol{M}_{k-1}$	$m_0 = \boldsymbol{D}$ $m_k = \boldsymbol{CA}^{k-1}\boldsymbol{B}$

Table 2

	Semi-state	method
	$\boldsymbol{X}(s)$ expanding	$H(s)$ expanding
$s=0$	$\boldsymbol{GM}_0 = \boldsymbol{B}$ $\boldsymbol{GM}_k = -\boldsymbol{WM}_{k-1}$	$m_0 = \boldsymbol{CR} + \boldsymbol{D}$ $m_k = \boldsymbol{CP}^k\boldsymbol{R}$
$s=\infty$	$\boldsymbol{GM}_0 = \boldsymbol{B}$ $\boldsymbol{GM}_k = -\boldsymbol{WM}_{k-1}$	$m_0 = \boldsymbol{D}$ $m_k = -\boldsymbol{CP}^{-k}\boldsymbol{R}$

We can make some important remarks:

1. The moment computation by $\boldsymbol{X}(s)$ expanding requires a recursive procedure and as computing effort only a LU decomposition and few forward-backward substitutions are needed.
2. Expanding the transfer function means computing the moments by matrix ($\boldsymbol{A}$ or $\boldsymbol{P}$) inversion that, for large k, produces ill-conditioned moment matrix.
3. The state variable method doesn't work for circuits containing excess elements as E-L loops and/or J-C cut sets, because in this case the state matrix ($\boldsymbol{A}$) of the circuit is singular.
4. While the state variable method works only for SISO (single input-single output) systems, the semi-state method can be used in MIMO (multiple inputs-multiple outputs) systems.

5 Moment matching technique

The above calculated circuit moments are used to compute the coefficients of the approximated transfer function. For example expanding about $s=0$ we obtain a q-pole approximation of a transfer function in rational form (11), where the coefficients a_0, aj, b_1, bq are unknowns:

$$H(s) \approx \frac{\hat{P}(s)}{\hat{Q}(s)} = \frac{a_j s^j + a_{j-1}s^{j-1} + ... + a_0}{b_q s^q + b_{q-1}s^{q-1} + ... + 1} =$$

$$m_0 + m_1 s + m_2 s^2 + ... + m_{j+q}s^{j+q}, \quad j < q \qquad (11)$$

The selection of the values j and q is a problem of analysis, but a natural option is j=q-1. Cross multiplying and equating the coefficients of similar powers of s on both sides of (10), we can evaluate the unknown coefficients. For j=q-1 we get

$$\begin{bmatrix} m_0 & m_1 & ... & m_{q-1} \\ m_1 & m_2 & ... & m_q \\ ... & ... & ... & ... \\ m_{q-1} & m_q & ... & m_{2q-2} \end{bmatrix} \begin{bmatrix} b_q \\ b_{q-1} \\ ... \\ b_1 \end{bmatrix} = -\begin{bmatrix} m_q \\ m_{q+1} \\ ... \\ m_{2q-1} \end{bmatrix},$$

$$a_0 = m_0$$

$$a_1 = m_1 + b_1 m_0$$

$$... \qquad . \qquad (12)$$

$$a_j = m_j + \sum_{i=1}^{\min(j,q)} b_i m_{j-i}$$

The coefficients a_0, a, b_1, be will give the approximation transfer function from Esq. (11).

6 Example

Find the reduced-order model of the circuit in figure 1.

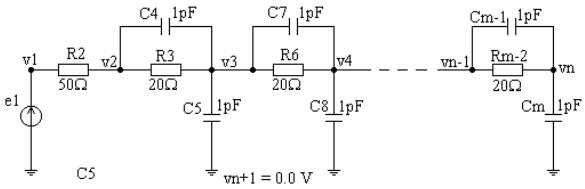

Fig. 1. RC lader circuit.

We want to find an optimal reduced order model for the circuit shown in Figure 1, in which the branch number is b=85 and node number is n=30. Because there are some excess capacitors (C-loops, for example loop made up of: {C_5, C7, C8}), the state equation number is equal to 29.

Bode characteristic (magnitude-frequency) of the transfer function of original circuit, *Aei*, together with the approximated ones: *Aei_rez*, *Aei_rezm*, *Aei_frac*, *Aei_red*10_11, *Aei_red*8_9, *Aei_red*4_5, *Aei_red*23, *Aei_red*24, and *Aei_red*26 (the first number is the numerator order and the second one represents the denominator order) and the pole/residue model are represented in Figures 2, 3 and 4.

The behavioral model generation with the pole/residue procedure was performed in two alternatives. In the first one we used AWE method (denoted by *Aei_rez* in figure 2) and in the second one we used our procedure (denoted by *Aei_rezm* in figure 3). It is easy to denote that the AWE model differs from to the original circuit, while our behavioral model is practically identic with the one of the original circuit (figures 2 and 3). Starting

from the *Aei_red*10_11 reduced model it was also generated a behavioral model obtained by decomposition into simple fractions - *Aei_par_frac.* The Bode *Aei, Aei_par_frac,* and *Aei_red*_10_11 represented in figure 2 are near overlaped. The great errors appear in *Aei_red*4_5 approximation.

Evidently, the Padé approximation is accurated near the point of expansion. If that neighborhood contains most of the signal energy (large residues and poles are closed to the origin) then the time domain approximation will be accurate, otherwise inaccuracies will appear in the transient simulation.

Changing the matrices $\boldsymbol{B}$ and $\boldsymbol{L}^t$, the semi-state method can be used to obtain the behavioral model of MIMO (multiple inputs-multiple outputs) circuits.

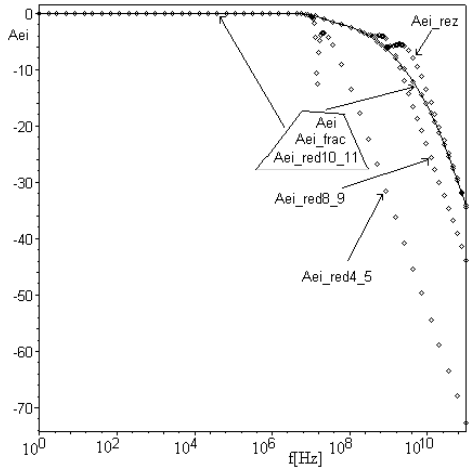

Fig. 2. Bode characteristics: *Aei*, *Aei_rez*,*Aei_frac*, *Aei_red*10_11, *Aei_red*8_9, and *Aei_red*4_5.

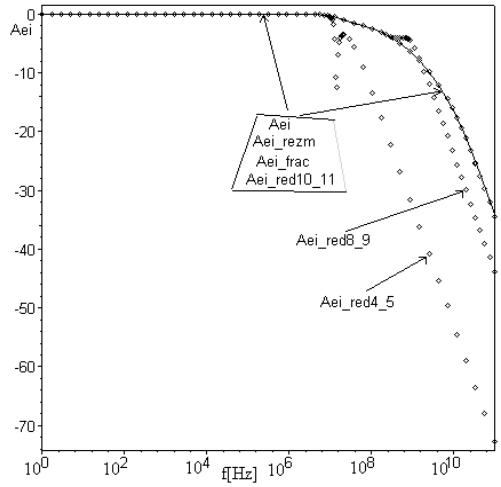

Fig. 3. Bode characteristics: *Aei*, *Aei_rezm*, *Aei_frac*, *Aei_red*10_11, *Aei_red*8_9, and *Aei_red*4_5.

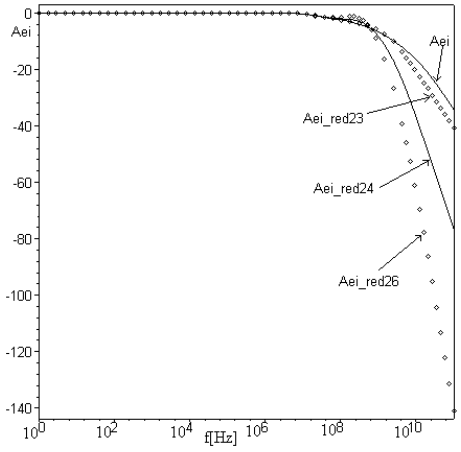

Fig. 4. Bode characteristics: *Aei*, *Aei_red*23, *Aei_red*24, and *Aei_red*26.

7 Conclusion

Based on the state equation formulation a technique for behavioral model generation by moment matching is developed in order to obtain an accurate approximation of the original transfer function. Performing this procedure a set of reduced models were obtained. Comparing their characteristics (magnitude/phase and poles of transfer function), the best approximation can be selected. Moreover, after applying a synthesis procedure, a symbolic transfer function in reduced form is available for the equivalent reduced-size circuit. By simple changing of the matrices $\boldsymbol{B}$ and $\boldsymbol{L}^t$, the semi-state method can be used in MIMO (multiple inputs-multiple outputs) circuits. Padé approximation often produces unstable poles on the right-hand side of the complex plan and the accuracy deteriorates when move away from the expansion point. Because direct moment matching techniques such as AWE, discussed in this paper, have some disadvantages (the moment-matrix is ill-conditioned and the passivity of the model is not guaranteed) other algorithms bassed Krylov subspace formulation and congurent transformation will be used in the future.

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