

# Time- Space Trellis Coding for Multi-Antennas Transmission Systems

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*Abstract:* - This work deals with the performance analysis of improved space-time trellis coding used for ultra high data rates radio systems. A comparison of classic STBC and STTC and the proposed super orthogonal STTC is achieved in order to evaluate the performances. The new technique seems to behave better in the same channel conditions with a slightly higher complexity in the trellis design.

*Key-Words:* - space-time diversity, trellis, block code

## 1 Introduction

The increasing need for ultra high data bit rate required by new multimedia services such as internet, Visio conference and TV for mobiles better quality wireless systems urges for new techniques to improve channel capacity. The exciting systems associate modulation and appropriate coding in point to point links so to reach the theoretical limits imposed by Shannon. However they gained spectral efficiency seems to falls short as far as future demand is concerned. The trend is to use MIMO or multi antennas systems.

Recent works [1] show that the capacity of multi antenna systems increases in linear manner with the number of antenna used, the Shannon limit seems even considerably surpassed. Therefore, such systems appear to offer much higher data rates as well as overcome interference and fading. A number of algorithms using space-time diversity are thus proposed in association with the CDMA and OFDM access techniques [2]. The drawback of such proposals appears to be circuit complexity.

In this text a time-space coding called STTC is presented that combine space-time diversity and convolution (trellis) coding. This type of code is an alternative of space-time block code called STBC. The trellis coding introduces a certain correlation in the transmitted symbols that improves the performances which is paid by higher complexity. The aim of this work is to present the criteria for better performance STTC design as far as frame rate is concerned.

## 2 Space-Time Trellis Coding

The basic STTC system consists of two antennas and a repetition length of two as shown in figure 1.

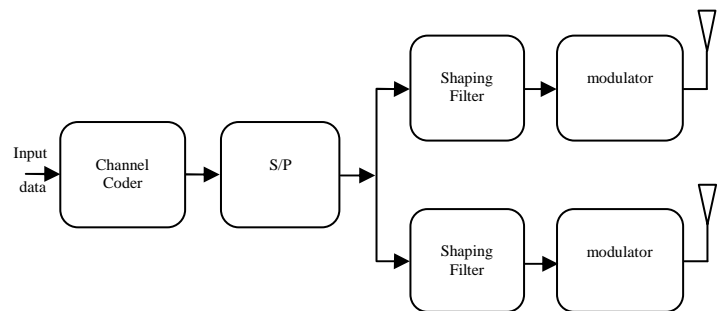


Fig.1 Transmission scheme considered

It has been shown that such an arrangement affects frequency selectivity of the radio channel [3]. A maximum likelihood detection based receiver will easily make sense of the transmit diversity. In this light, the idea is to extend the principle beyond  $\frac{1}{2}$  coding. The work done by Tarokh showed that STTC systems using 4-PSK, 8-PSK and 16-MAQ modulation techniques perform 2 to 3dB high than the channel outage capacity [4]. This work has been revisited in this text, as well as other codes are proposed called SOSTTC which stands for super orthogonal STTC, which combine STTC and STBC advantages. The design is similar to M-TCM, (multi dimensional TCM).

Usually STTC codes use determinant and rank criteria for the design. Optimal STTC may be obtained so it makes a compromise between transmission rate, diversity gain, trellis complexity and the modulation constellation size.

### 3 Super-Orthogonal Space-Time Trellis Code

Alternatives have been proposed to improve the STTC first initiated by Tarokh et al in [4] and are available in [5][9]. The main drawback appears to be the compromise between diversity gain for a given trellis complexity. This disadvantage may be overcome by using Ungerboeck's partitioning methods.

In this case, a transmit diversity of order two is considered. The transmission matrices are based on Alamouti scheme and therefore are in the form:

$$C(x_1, x_2, \Theta) \begin{pmatrix} x_1 e^{j\Theta} & x_2 \\ -x_2^* e^{j\Theta} & x_1^* \end{pmatrix} \quad (1)$$

The value of  $\theta$  is chosen so that for each pair of the words  $x_1$  and  $x_2$  of the initial constellation the transmitted signals all belong to the same constellation. In other words, for an L-ary PSK modulation,  $x_1$  and  $x_2$  signals may be formulated in the form of  $e^{2\pi j l / L}$  With  $l=0, 1, \dots, L-1$ .

Let us consider the matrix  $B(c_1, c_2)$  also called the differences of code words  $c_1$  and  $c_2$  and  $B^H(c_1, c_2)$  its transposed conjugate. It can be shown that the diversity of the code may be defined as the minimum rank of the matrix  $B(c_1, c_2)$ .

For a maximum diversity code, the minimum of the determinant of the matrix  $A(c_1, c_2) = B(c_1, c_2) B^H(c_1, c_2)$  for all code words  $(c_1, c_2)$  pairs, corresponds to the coding gain. Thus one can define the coding gain distance between the two code words as  $d^2(c_1, c_2) = \text{Det}(A(c_1, c_2))$ . Generally, when the code diversity is less than  $r$  which is the transmitting antennas number, the distance may be defined as the harmonic mean of non null eigen values of the matrix  $A(c_1, c_2)$ . The coding gain distance allows for the partitioning of a constellation in sub-constellations as depicted in figure 2 for m-PSK and STBC.

In order to maximize the coding gain without sacrificing the data transmission rate, the procedure is as follows. A code is affected to each transition from a given state according to the STBC given in eq.1. The adjacent states of the trellis are given different codes. On the other hand, a same code may be given to the branches of trellis that converge to the same point. In other words, each couple of code words converges or diverges to or from a state that has a maximum diversity because the words belong to the same STBC code.

The obtained codes are called super orthogonal STTC. The figure 2a shows an example of such an approach with 4 states  $r = 2b/s/Hz$ , 4-PSK, and figure 2b

illustrates the case when  $r=3b/s/Hz$  8-PSK. The minimum coding gain distance obtained from figure 2a equals 16, which is higher than that obtained by Tarokh [4]. For figure 2b, the minimum coding gain distance equals 2.69, there is no corresponding STTC code 4 states with 8-PSK [4].

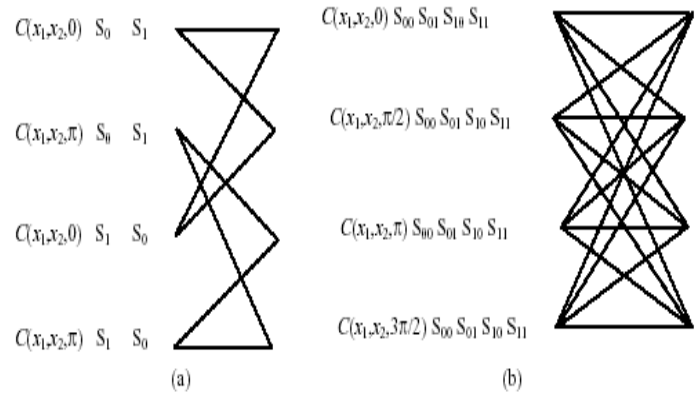


Fig.2: 4 states SOSTTCT codes

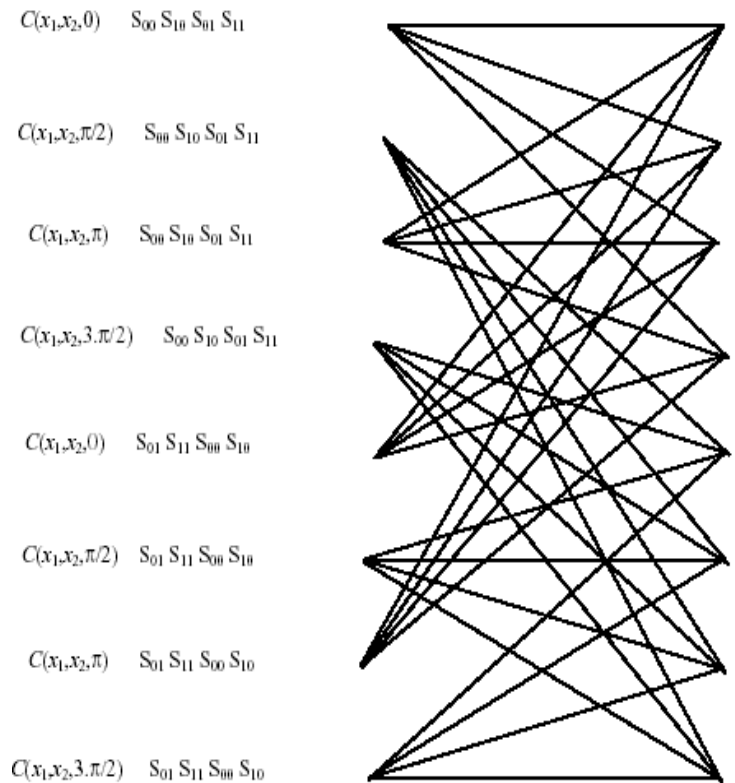


Fig.3: 8 states SOSTTCT codes

Before looking into the performances of SOSTTC and the improvement that are made in regard to STTC codes, the mathematical tools that allows the partitioning of the constellation are given. The aim is to maximize the distance between sub constellations as in the case of Ungerboeck TCM modulation. If an L-ary PSK is considered each signal may written in the form of  $S = e^{2\pi j l/L}$ , or  $S = e^{j l \omega}$ , with  $\omega$  being  $2\pi/L$ . When a two transmission antenna system is considered then, distinct symbol pairs are obtained in the transmitted constellation from eq. 1.

$$(s_1^1 = e^{jk_1 \omega}, s_2^1 = e^{jl_1 \omega}) \text{ and } (s_1^2 = e^{jk_2 \omega}, s_2^2 = e^{jl_2 \omega}) \quad (2)$$

Therefore; for the transmitted STBC codes used symbols ( $\theta=0$ ). The matrices A and B already mentioned can be calculated. For parallel branches of the trellis the result is:

$$B = \begin{pmatrix} e^{jk_1 \omega} - e^{jk_2 \omega} & e^{jl_1 \omega} - e^{jl_2 \omega} \\ e^{-jl_2 \omega} - e^{-jl_1 \omega} & e^{-jk_1 \omega} - e^{-jk_2 \omega} \end{pmatrix} \quad (3) \text{ and } (4)$$

$$A = \begin{pmatrix} 4 - 2\cos(\omega(k_2 - k_1)) - 2\cos(\omega(l_2 - l_1)) & 0 \\ 0 & 4 - 2\cos(\omega(k_2 - k_1)) - 2\cos(\omega(l_2 - l_1)) \end{pmatrix}$$

Using the expression of the matrix A, the determinant will be :

$$Det(A) = \{4 - 2\cos[\omega(k_2 - k_1)] - 2\cos[\omega(l_2 - l_1)]\}^2 \quad (5)$$

Thus, if for the first code word of the set of symbols of the sub-constellation has:

$$Det(A) = \left\{ \sum_{p=1}^P 4 - 2\cos(\omega(k_2^p - k_1^p)) - 2\cos(\omega(l_2^p - l_1^p)) \right\}^2 \quad (6)$$

The equation eq.6 combines the sum of P terms all positive, it results the following inequality:

$$Det(A) = \left\{ \sum_{p=1}^P 4 - 2\cos(\omega(k_2^p - k_1^p)) - 2\cos(\omega(l_2^p - l_1^p)) \right\} \geq$$

$$\left\{ \sum_{p=1}^P \{ 4 - 2\cos(\omega(k_2^p - k_1^p)) - 2\cos(\omega(l_2^p - l_1^p)) \} \right\}^2 \quad (7)$$

This inequality eq.7 allows the building of the sub-constellation as defined in figure 4.

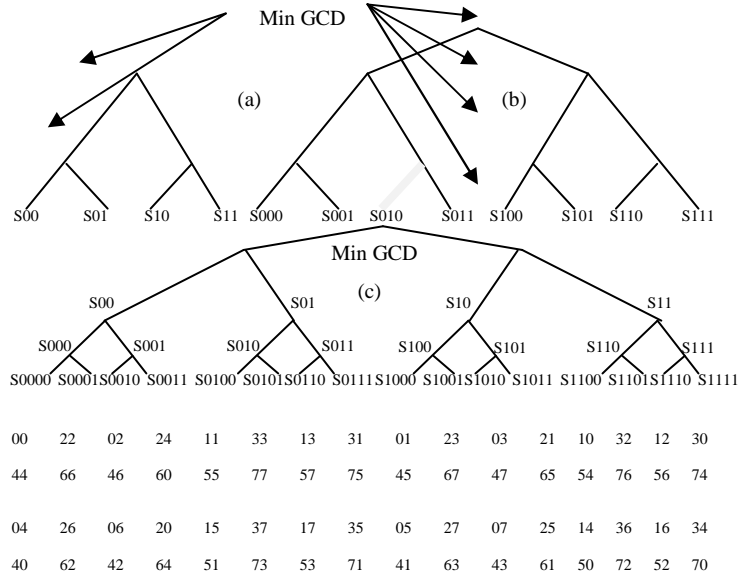


Fig.4: Partitioning principle

#### 4 Performances Analyses

The STTC codes are compared to SOSTTC. Figures illustrate the performances for two transmit antennas and one or two receive antennas system. The transmit packets have a size of 260 symbols (130 on each antenna). The code is that presented in figure 2a, with 4-PSK. The coefficients are simulated as a Gaussian complex noise of zero mean and 0.5 variance.

The codes have been chosen as in [11] to [14]. It can be easily seen from figures 5 and 6 that the SOSTTC codes behave better than STTC, as far as Frame rate of  $10^{-2}$  is concerned which is in the order of 1 to 1.5 dB in the case of two receive system and 2dB in the case of only one receive antenna system. Consequently, the advantages of SOSTTC decrease for higher number of receive antenna systems. It can be also noted that YB, BBH, CYV and STTC codes behave similarly, in the case of one antenna systems. The advantage appears only in a two receive antennas system.

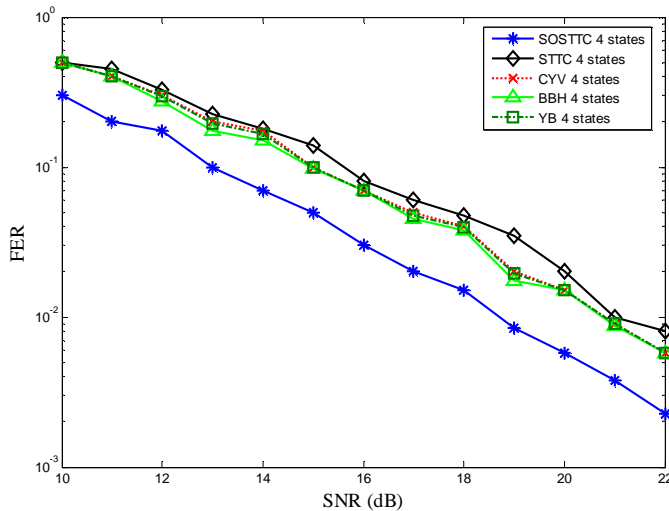


Fig.5: Performances of SOSTTC codes in the case of 2 transmit antennas and 1 receive antenna

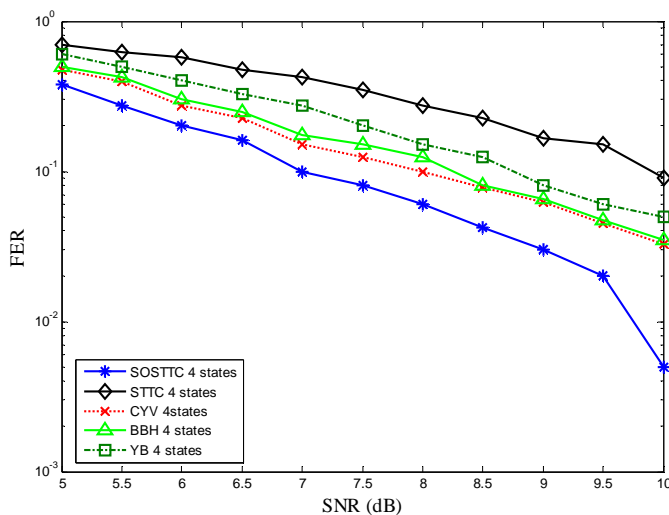


Fig.6: Performances of SOSTTC codes in the case of 2 transmit antennas and 2 receive antennas

### 5 Conclusion

This text dealt with a new approach in coding technique called super orthogonal space time trellis diversity code to be used for ultra high bit rate digital radio communication systems. Actually, this technique makes use of trellis coding and multi-antenna diversity systems. The gain obtain seems higher than those of classic STBC coding. The simulations showed that the performances can be improved by 1 to 2dB in terms of same frame outage when compared to STTC codes mentioned by Tarokh for multipath propagation channel. However, the resulting gain-complexity trade-off has to be further investigated.

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