On Active Filtering of Harmonics Pollution -Theoretical Aspects

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Abstract: - By harmonics pollution it is usually understood one of the basic problems of the electric energy supply quality – the presence of higher harmonics. In contemporary electric energy networks they are due to nonlinear loads of various natures. These higher harmonics have to be suppressed and this is done by filtering. Active filtering is done by feedback structures where the filter properties may be controlled. The present paper aims to discuss some theoretical aspects which are of the same nature as the harmonic balance applied to feedback control systems for the techniques of the describing function. Some connections to harmonics suppression in mechanical engineering (the feedback vibration absorbers) are also pointed out.

Key-Words: - Harmonic Pollution, Active Filtering, Feedback Control Systems, Describing Function

1 Basics and Problem Statement

A. A standard equivalent diagram of harmonics pollution is represented by a source and a nonlinear load to which we may add the feed line (normally inductive). A simple equivalent representation would be as follows (fig. 1)

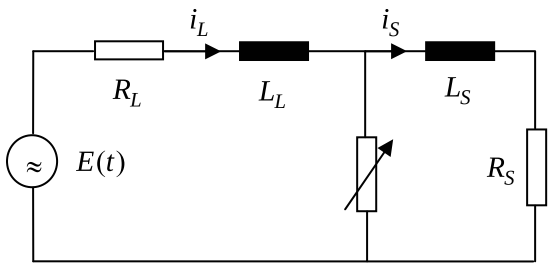

Fig. 1 Basic equivalent circuit

whose equations may be written as follows

$$\begin{aligned} L_L \frac{di_L}{dt} &= -R_L i_L - f(i_L - i_S) + E(t) \\ L_S \frac{di_S}{dt} &= -R_S i_S + f(i_L - i_S) \end{aligned} \qquad (1)$$

It appears quite clear that the nonlinear load which is current controlled i.e. it has a characteristic of the form $u = f(i)$ introduces a nonlinear feedback which appears to generate a model which is much alike to that of the so-called absolute stability problem (see, e.g. the book of Vidyasagar, [1]): a feedback composed of a linear block having transfer function a rational strictly proper, $H(s) = N(s)/D(s)$ and of a nonlinear static bloc with its characteristic defined by f (fig. 2).

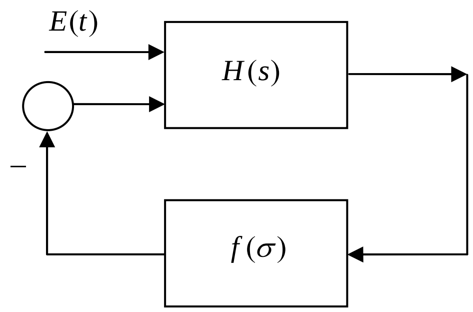

Fig. 2 Feedback structure

The difference with respect to the standard structure is the forcing term $E(t)$ which substitutes the stability problem by a problem of forced oscillations. This problem is less popular while it goes back to a paper of Yakubovich [2], being fully treated in a paper of Barbălat and Halanay [3]. It is worth recalling their result here.

Theorem 1. Consider the feedback structure of fig. 2 under the following basic assumptions:

i) the linear part is exponentially stable, in particular $H(s)$ has all its poles with strictly negative real parts;

ii) the nonlinear function $f(\sigma)$ is globally Lipschitz i.e. it satisfies the following inequality

$$0 \le \frac{f(\sigma_1) - f(\sigma_2)}{\sigma_1 - \sigma_2} \le L, \quad \forall \sigma_1 \ne \sigma_2 \qquad (2)$$

iii) the following Popov-like, with zero free parameter, frequency domain inequality holds

$$\frac{1}{L} + \operatorname{Re} H(j\omega) > 0 \tag{3}$$

Then the system has a unique limit regime which is of the type of the forcing exogenous signal $E(t)$ i.e. constant when $E(t)$ is constant, periodic when $E(t)$ is periodic and almost periodic when $E(t)$ is almost periodic. Moreover this limit regime is exponentially stable.

Some comments are necessary. The fact that the limit regime i.e. the steady state motion is reproducing the exogenous signal is a standard property of the linear systems. In the nonlinear case the above theorem gives conditions for the same kind of properties in some nonlinear cases. This is called almost linear behavior [4]. We may check the fulfillment of the assumptions of Theorem 1 for system (1). It is not difficult to see that for (1) the transfer function $H(s)$ is the transfer function of the linear system

$$\begin{aligned} L_L \frac{di_L}{dt} &= -R_L i_L + u(t) \\ L_S \frac{di_S}{dt} &= -R_S i_S - u(t) \\ \sigma &= i_L - i_S \end{aligned} \tag{4}$$

and reads

$$H(s) = \frac{1}{L_L s + R_L} + \frac{1}{L_S s + R_S} \tag{5}$$

with two real negative poles and satisfying

$$\operatorname{Re} H(j\omega) = \frac{R_L}{R_L^2 + L_L^2\omega^2} + \frac{R_S}{R_S^2 + L_S^2\omega^2} > 0, \quad \forall \omega \tag{6}$$

Concerning the types of the exogenous signals it is obvious that they describe d.c. sources (constant signals), a.c. sources (periodic signals) and modulated sources (almost periodic signals).

B. The interest in the harmonics pollution problem is in connection with the case of a.c. sources. The main conclusion of the theorem for this case ensures a periodic steady state for periodic (in particular harmonic) source, also the preservation of the period, but says nothing about the harmonics contents. Or the specific technical problem is here as follows: the nonlinear load is generating higher harmonics while the linear subsystem is filtering them. Unfortunately the standard grid structure has not good filtering properties and they have to be corrected by passive/active filters.

2 About the Harmonics Contents

In order to compute the harmonics contents subject to the basic grid filtering properties only, we shall apply to the structure of fig. 2 the method of the harmonic balance ([5], [6]). With this aim we shall analyze a general case suggested by the feedback structure of fig. 2; for this reason we consider a state representation of the transfer function $H(s)$ namely the triple (A, b, c) to obtain a system which generalizes in fact (1)

$$\dot{x} = Ax - b\varphi(c^T x) + f(t) \tag{7}$$

satisfying the conditions of Theorem 1. Under these assumptions, if $f(t)$ is T-periodic system (7) has a unique exponentially stable periodic solution $\tilde{x}(t)$. A rather straightforward computation suggested by Pervozvonski [6] gives the following representation for $\tilde{x}(t)$

$$\tilde{x}(t) = -\int_0^T G_T(t-\tau) b\varphi(c^T \tilde{x}(\tau)) d\tau + \int_0^T G_T(t-\tau) f(\tau) d\tau \tag{8}$$

where $G_T(t)$ is the Green matrix function extended by T periodicity and thus defined by

$$\begin{aligned} &G_T(t) = e^{At}(I - e^{AT})^{-1}, 0 \le t \le T, \\ &G_T(t+T) \equiv G_T(t) \end{aligned} \tag{9}$$

Using (9) we obtain the integral equation of the periodic output of the system

$$\begin{aligned} \tilde{\sigma}(t) \equiv c^T \tilde{x}(t) = -&\int_0^T h_T(t-\tau)\varphi(\tilde{\sigma}(\tau)) d\tau + \\ &\int_0^T c^T G_T(t-\tau) f(\tau) d\tau \end{aligned} \tag{10}$$

where $h_T(t) = c^T G_T(t) b$.

Using approximation techniques for solving operator equation (see, Krasnosel'ski et al., [7]) we may solve this nonlinear integral equation, but here we aim to a qualitative estimate of the harmonic contents.

If, as in the main application of Section 1, the forcing signal is harmonic and therefore, the second term in the right hand side of (10) will result also harmonic. The higher harmonics, as known from the describing function technique, are generated by the nonlinear function $\varphi(\cdot)$; if we follow the way of the method of the harmonic balance we find the following Fourier expansion

$$\tilde{v}(t) = \sum_{-\infty}^{\infty} \hat{v}_k e^{jk\omega t}, \quad \omega = \frac{2\pi}{T}$$
$$\hat{v}_k = \frac{1}{T} \int_0^{2\pi/\omega} \varphi(\tilde{\sigma}(t)) e^{-jk\omega t} dt \tag{11}$$

These harmonics are filtered by the convolution with the T-periodized Green impulse response $h_T(t)$. The periodic impulse response has its own Fourier series

$$h_T(t) = \sum_{-\infty}^{\infty} \hat{h}_k e^{jk\omega t}$$
$$\hat{h}_k = \frac{1}{T} \int_0^T h_T(t) e^{-jk\omega t} dt \tag{12}$$

It is interesting to compute the Fourier coefficients of $h_T(t)$ what reduces to the computation of the Fourier matrix coefficients of the matrix Green function $G_T(t)$. We shall have

$$\hat{h}_k = c^T \left(\frac{1}{T} \int_0^T e^{At} (I - e^{AT})^{-1} e^{-jk\omega t} dt \right) b = -\frac{1}{T} H(jk\omega) \tag{13}$$

where $H(s) = c^T (sI - A)^{-1} b$ is the transfer function of the linear part of (7).

Now, the two terms of the right hand side of (10) are both cyclic convolutions of periodic functions. The result is periodic and the corresponding Fourier coefficients are the products of the coefficients of the two functions entering in cyclic convolution (see, e.g. Oppenheim and Willsky [8]); since the cyclic convolution is an integral of the form of (10) multiplied by $1/T$, we have to multiply the coefficient product by T to find

$$-\int_0^T h_T(t-\tau) \varphi(\tilde{\sigma}(\tau)) d\tau = \sum_{-\infty}^{\infty} H(jk\omega) \hat{v}_k e^{jk\omega t} \tag{14}$$

i.e. each harmonic is attenuated according to the gain frequency domain characteristic of system's transfer function (associated to the linear part). The same is true for the other convolution:

$$\int_0^T c^T G_T(t-\tau) f(\tau) d\tau = \sum_{-\infty}^{\infty} c^T (A - jk\omega I)^{-1} \hat{f}_k e^{jk\omega t} \tag{15}$$

with $\hat{f}_k$ being the Fourier coefficients of the forcing periodic function. This will give the equations of a harmonic balance

$$\hat{\sigma}_k = c^T (A - jk\omega I)^{-1} (-b\hat{v}_k + \hat{f}_k) \tag{16}$$

Now, if the forcing is harmonic $\hat{f}_k = 0, \quad k \neq 1$ and if φ is an odd function then $\hat{v}_0 = 0$.

The *harmonics pollution* is expressed by the Fourier coefficients

$$\hat{\sigma}_k = -H(jk\omega)\hat{v}_k, \quad k > 1 \tag{17}$$

The filtering means reduction of the coefficients up to an acceptable level.

3 Passive and Active Filtering

Passive filtering is the simplest standard way of filtering. It requires a series linear system with the transfer function $H_F(s)$ such that (17) should become

$$\hat{\sigma}_k = -H_F(jk\omega) H(jk\omega) \hat{v}_k, \quad k > 1 \tag{18}$$

with $|H_F(jk\omega)H(jk\omega)| \ll 1$. This simple solution is applicable with difficulty in power grids because of two reasons: high power levels and relatively low frequencies to filter (unlike in electronic circuits); these circumstances will lead to such disadvantages as high costs and bulky devices.

Active filtering is also based on low pass filters but, from the system point of view is more like a parallel connected filter what transforms the structure of fig. 2 in the structure of fig. 3.

Using the standard rules of diagram reduction we find that the structure of fig. 1 will be preserved with a new transfer function

$$H_F(s) = \frac{H(s)}{2 + F(s)} \tag{19}$$

where $F(s)$ is a low-pass filter. This filtering structure has mainly technological advantages.

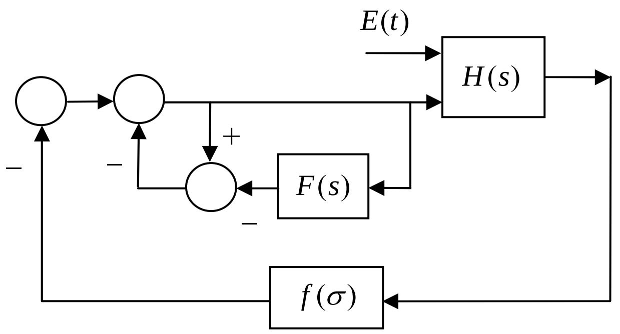

Fig. 3 Active filtering (systemic view)

The design techniques for $F(s)$ are somehow a consequence of the technological choice. Moreover the technical realization introduces new nonlinear elements and it is not very clear if these nonlinearities compensate or not the existing ones of the load. Should it be so, this would lead to a resulting “weak nonlinearity” what would considerably reduce the harmonic contents. Another idea could be the application of the feedback structures occurring in mechanical vibration absorbers. It is known, for instance, that the standard Frahm absorber (see e.g. the books of Den Hartog [9] or Bulgakov [10]) can completely suppress an harmonic perturbation. This application would require an electrical model of the absorber and its implementation.

Proliferation of nonlinear loads in power systems has increased harmonic pollution and deteriorated power quality. To improve power quality when transient harmonics appear, the dominant harmonics identified from Prony analysis are used as the harmonic reference for harmonic selective active filters [11].

Some recent methods for design the control of active power filters are presented in [12] and [13].

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References:

[1] M. Vidyasagar, *Nonlinear Systems Analysis*, Second Edition, SIAM, Philadelphia, 2002.

[2] V. A. Yakubovich, Methods of matrix inequalities in nonlinear control systems stability. I. Absolute stability of forced oscillations (*in Russian*), *Avtom. I Telemekhanika*, Vol. 25, No. 7, 1964, pp. 1017-1029.

[3] J. Barbălat, A. Halanay, Conditions pour un comportement presque lineaire des systemes automatiques, *Rev. Roum. Sci. Techn. Electrot. et Energ.*, Vol. 19, No. 4, 1974, pp. 321-341.

[4] Vl. Răsvan, Almost linear behavior in systems with sector restricted nonlinearities, *Proc. Romanian Academy Series A: Mathem., Physics, Techn. Sci., Inform Sci.*, Vol. 2, No. 3, 2004, pp. 127-135.

[5] M. A. Ayzerman, *Lectures on Automatic Control Theory* (*in Russian*), Nauka, Moskow, 1966.

[6] A. A. Pervozvonski, *A Course in Automatic Control Theory* (*in Russian*), Nauka, Moskow, 1986.

[7] M. A. Krasnosel’ski, G. M. Vagnikko, P. P. Zabreyko, *Approximate solutions of operator equations* (*in Russian*), Nauka, Moskow, 1969.

[8] A. V. Oppenheim, A. S. Willsky, *Signals and Systems*, Prentice Hall, Englewood Cliffs, 1997.

[9] J. P. Den Hartog, *Mechanical Vibrations*, Dover Publications, 1985.

[10] B. V. Bulgakov, *Oscillations* (*in Russian*), Gostekhizdat, Moskow, 1954.

[11] L. Qi, L. Qian, S. Woodruff, D. Cartes, Prony analysis for power system transient harmonics, *Journal on Applied Signal Processing*, no. 1, 2007, pp. 170-179.

[12] A. Szromba, Sampled Methods of Active Power Filter Control, *Electrical Power Quality and Utilization Journal*, vol. XII, no. 1, 2006, pp. 29-39.

[13] R. Măgureanu, D. Creangă, V. Bostan, M. Priboianu, Recursive Control for Active Power Filters, 9th *International Conference Electrical Power Quality and Utilization*, Barcelona, 9-11 October 2007.