A Study on Three-phase ZCZ Sequence Sets

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Abstract: The present paper investigates the application of three-phase ZCZ sequence sets to AS-CDMA systems that use coded TPSK. First, we clarify possible three-phase ZCZ sequence sets and the features of these ZCZ sequence sets. Next, we clarify the structure of the periodic correlation function between sequences that belong to three-phase ZCZ sequence sets. Finally, we propose a method for constructing a group of three-phase ZCZ sequence sets that have low cross-correlation properties. The results are useful in designing AS-CDMA systems that use coded TPSK.

Key–Words: ZCZ sequence sets, Three-phase sequences, Coded TPSK, AS-CDMA, Spreading sequences, Correlation property

1 Introduction

Code-division multiple-access (CDMA) has been widely applied in mobile communication systems such as digital cellular phone systems. Recently, approximately synchronized CDMA (AS-CDMA) systems have attracted a great deal of attention because these systems can avoid co-channel interference and multipath interference. In these systems, zero-correlation zone (ZCZ) sequence sets are used as spreading sequences in order to realize this advantage.

We have proposed coded ternary phase shift keying (coded TPSK), which uses TPSK modulation and a special class of convolutional codes over $GF(3)$ [12]–[15]. Coded TPSK has attractive properties for mobile communication systems, such as excellent bit error characteristics and low amplitude fluctuation. In particular, the latter property is very useful for mobile terminals, which must use non-linear amplifiers.

Three-phase ZCZ sequence sets are essential in order to design AS-CDMA systems that use coded TPSK. However, these ZCZ sequence sets have not been investigated thoroughly because their phase is not equivalent to the n'th power of 2. In the present paper, we investigate three-phase ZCZ sequence sets. After a brief explanation of the definitions of the terms used in the present paper, we clarify possible three-phase ZCZ sequence sets and the features of these ZCZ sequence sets. Next, we clarify the structure of the periodic correlation function between sequences that belong to three-phase ZCZ sequence sets. Finally, we propose a method for constructing a group of three-phase ZCZ sequence sets that have low cross-correlation properties.

2 Preliminaries

This section provides definitions of the terms used herein.

Let $C = \{C_0, C_1, \cdots, C_{M-1}\}$, $C_i = \left(c_0^i, c_1^i, \cdots, c_{P-1}^i\right)$, and c_j^i be a sequence set with family size M, a sequence of period P, and a sequence element, respectively. If all of the sequences in C satisfy the following correlation property, then C is referred to as a zero-correlation zone (ZCZ) sequence set.

$$\sum_{j=0}^{P-1} c_j^i \cdot c_{j+\tau}^{i'*} = 0 \text{ for } \begin{cases} (i = i',\ 1 \le |\tau| \le L) \\ (i \ne i',\ 0 \le |\tau| \le L) \end{cases}, \quad (1)$$

where $*$ denotes a complex conjugate. Note that $j+\tau$ is calculated by $\mathrm{mod} P$. L is referred to as the zero-correlation zone length of C. The ZCZ sequence set is represented as $Z(P, M, L)$. The zero-correlation zone length is evaluated by the following ratio:

$$\eta = \frac{LM}{P}, \quad (2)$$

which does not exceed 1 [6].

Let $S = (s_0, s_1, \cdots, s_{P-1})$ and s_j be a sequence of period P and a sequence element, respectively. If

S satisfies the following autocorrelation property, then S is referred to as a perfect sequence.

$$\sum_{j=0}^{P-1} s_j \cdot s_{j+\tau}^* = 0 \ \ (\tau \neq 0) . \tag{3}$$

Note that $j+\tau$ is calculated by $\bmod P$.

Let U and $u_{h_0,h_1}/\sqrt{N}$ be an $N \times N$ matrix and the (h_0, h_1)'th entry, respectively. If U satisfies the following relationship, then U is referred to as a unitary matrix.

$$\begin{aligned} \sum_{h=0}^{N-1} u_{h_0,h} \cdot u_{h_1,h}^* &= \sum_{h=0}^{N-1} u_{h,h_0}^* \cdot u_{h,h_1} \\ &= \begin{cases} N & (h_0 = h_1) \\ 0 & (h_0 \neq h_1) \end{cases} . \end{aligned} \tag{4}$$

3 Three-phase ZCZ Sequence Sets

As mentioned in Section 1, three-phase ZCZ sequence sets have rarely been studied. However, some existing methods for constructing ZCZ sequence sets can be applied to construct three-phase ZCZ sequence sets. In this section, we clarify possible three-phase ZCZ sequence sets and the values of the ratio η. We also classify these ZCZ sequence sets based on η. Hereinafter, 0, 1, and 2 represent $e^{2\pi\sqrt{-1}\cdot 0/3}$, $e^{2\pi\sqrt{-1}\cdot 1/3}$, and $e^{2\pi\sqrt{-1}\cdot 2/3}$, respectively.

Type 1-A: We have previously proposed a method for constructing ZCZ sequence sets [8], [10]. This method uses perfect sequences and unitary matrices. Therefore, we can obtain three-phase ZCZ sequence sets from three-phase perfect sequences and three-phase unitary matrices. To date, only three-phase perfect sequences of period 3 and period 9 have been found [1]–[3]. For example, (020) is a three-phase perfect sequence of period 3. The following matrix is a 3×3 three-phase unitary matrix:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \tag{5}$$

Using this perfect sequence and this unitary matrix, a three-phase ZCZ sequence set can be constructed as follows :

$$(020200002) , \ (011221020) , \ (002212011)$$

This ZCZ sequence set is $Z(9,3,1)$. In the same manner, we can also obtain $Z(27,3,3)$, $Z(27,9,1)$, $Z(81,9,3)$, $Z(81,27,1)$, $Z(243,27,3)$, and so on. η of these ZCZ sequence sets is $1/3$.

Type 1-B: Using the same method as that for the Type 1-A ZCZ sequence sets, we can obtain a different type of three-phase ZCZ sequence set from three-phase perfect sequences of period 9. For example, (000012021) is a three-phase perfect sequence of period 9. Using this perfect sequence and the unitary matrix presented in (5), a three-phase ZCZ sequence set can be constructed as follows:

$$\begin{gathered} (000012021000120210000201102) , \\ (021000012021111201021222120) , \\ (012021000012102222012210111) \end{gathered}$$

This ZCZ sequence set is $Z(27,3,7)$. In the same manner, we can also obtain $Z(81,3,21)$, $Z(81,9,7)$, $Z(243,9,21)$, $Z(243,27,7)$, $Z(729,27,21)$, and so on. η of these ZCZ sequence sets is $7/9$.

Type 2: We proposed another method for constructing ZCZ sequence sets [9], [10]. This method uses DFT matrices and unitary matrices. Only the 3×3 DFT matrix has three-phase entries. Therefore, we can obtain three-phase ZCZ sequence sets from the 3×3 DFT matrix and three-phase unitary matrices. The unitary matrix presented in (5) is the 3×3 DFT matrix. Note that this matrix can also be used as a three-phase unitary matrix. For example, using this matrix, a three-phase ZCZ sequence set can be constructed as follows:

$$(000021012) , \ (021012000) , \ (012000021)$$

This ZCZ sequence set is $Z(9,3,2)$. In the same manner, we can also obtain $Z(27,3,6)$, $Z(27,9,2)$, $Z(81,9,6)$, $Z(81,27,2)$, $Z(243,27,6)$, and so on. η of these ZCZ sequence sets is $2/3$.

Type 3: Fan *et al.* proposed a method for constructing ZCZ sequence sets from complementary sets [7]. Although the main purpose of this method is to generate binary ZCZ sequence sets or quadri-phase ZCZ sequence sets, the method can be easily generalized. Using this method, three-phase ZCZ sequence sets can be obtained from three-phase complementary sets. For example, $\{\{0,0,0\},\{0,1,2\},\{0,2,1\}\}$ is a three-phase complementary set. Using this complementary set, we can construct the following three-phase ZCZ sequence set:

$$\begin{gathered} (000000000012012012021021021) , \\ (000111222012120201021102210) , \\ (000222111012201120021210102) , \\ (012012012021021021000000000) , \\ (012120201021102210000111222) , \\ (012201120021210102000222111) , \\ (021021021000000000012012012) , \end{gathered}$$

Table 1: Classification based on η

η	Type
$1/3$	**1-A**
$2/3$	**2, 3**
$7/9$	**1-B**

$$(021102210000111222012120201),$$
$$(021210102000222111012201120)$$

This ZCZ sequence set is $Z(27,9,2)$. In the same manner, we can also obtain $Z(243,27,6)$, $Z(81,9,6)$, $Z(729,27,18)$, and so on. η of these ZCZ sequence sets is $2/3$.

These ZCZ sequence sets can be classified based on η, as shown in Table 1. AS-CDMA systems can avoid the influence of the time delays of all signals in the same cell by the zero-correlation property [4]. Therefore, the zero-correlation zone length of a ZCZ sequence set is a highly important factor that determines the tolerance of the time delays. From this point of view, a ZCZ sequence set with a large zero-correlation zone is preferred. In the case of three-phase ZCZ sequence sets, Type 1-B ZCZ sequence sets have the maximum ratio, that is to say $7/9$. These ZCZ sequence sets will be discussed in the next section.

4 Correlation Property of Type 1-B ZCZ Sequence Sets

In CDMA cellular systems, there are three types of interference: multipath interference caused by the reflection of a desired signal, co-channel interference caused by other terminals in the same cell, and inter-cell interference caused by terminals in adjacent cells. Since a ZCZ sequence set ensures the ideal auto-correlation and cross-correlation properties within a zero-correlation zone, multipath interference and co-channel interference do not exist in AS-CDMA systems if all time delays are held within the zero-correlation zone. However, even these systems cannot avoid inter-cell interference. Therefore, in order to reduce inter-cell interference, it is necessary to assign a group of ZCZ sequence sets that have low cross-correlation properties to the adjacent cells. In this section, we clarify the structure of the periodic correlation function between sequences that belong to Type 1-B ZCZ sequence sets. In addition, we propose a method for constructing a group of Type 1-B ZCZ sequence sets that have low cross-correlation properties.

First, the procedure for constructing Type 1-B three-phase ZCZ sequence sets is explained [8], [10]. Three-phase perfect sequences of period 9 can be represented as follows [3]:

$$\begin{aligned} S &= (s_0, \cdots, s_k, \cdots, s_8), \\ s_k &= \exp\left(\frac{2\pi\sqrt{-1}}{3}\left(f(k_1) + k_0 r(k_1)\right)\right), \end{aligned} \tag{6}$$

where $0 \le k_0 \le 2$, $0 \le k_1 \le 2$, and $k = 3k_0 + k_1$. $f(k_1)$ is an arbitrary function satisfying $f : \mathbf{Z_3} \to \mathbf{Z_3}$, and $r(k_1)$ is an arbitrary permutation satisfying $r : \mathbf{Z_3} \to \mathbf{Z_3}$. S is referred to as a starter. Let l be a natural number. A set of 3^l perfect sequences, Q, can be derived by shifting S cyclically.

$$\begin{aligned} Q &= \{Q_0, \cdots, Q_m, \cdots, Q_{3^l-1}\}, \\ Q_m &= (q_0^m, \cdots, q_k^m, \cdots, q_8^m), \\ q_k^m &= \begin{cases} s_{(3m+k) \bmod 9} & (l = 1) \\ s_{(m+k) \bmod 9} & (l > 1) \end{cases}. \end{aligned} \tag{7}$$

Let U be a $3^l \times 3^l$ three-phase unitary matrix, and let $u_{h_0,h_1}/\sqrt{3^l}$ be the (h_0, h_1)'th entry of U. Using Q and U, a sequence set C_1 is defined as

$$\begin{aligned} C_1 &= \left\{C_0^1, \cdots, C_i^1, \cdots, C_{3^l-1}^1\right\}, \\ C_i^1 &= \left(c_0^{1,i}, \cdots, c_j^{1,i}, \cdots, c_{3^{l+2}-1}^{1,i}\right), \\ c_j^{1,i} &= u_{i, j \bmod 3^l} \cdot q_{\lfloor j/3^l \rfloor}^{j \bmod 3^l}. \end{aligned} \tag{8}$$

$\lfloor j/3^l \rfloor$ denotes a maximum integer that does not exceed $j/3^l$. C_1 is a three-phase ZCZ sequence set with 3^l sequences of period 3^{l+2} and the zero-correlation zone length of C_1 is 7, i.e., C_1 is $Z\left(3^{l+2}, 3^l, 7\right)$.

C_1 can be expanded recursively. Let n be a natural number satisfying $n > 1$, and let e_n be an integer satisfying $0 \le e_n < l$. In addition, let U_d^n be a $3^{l-e_n} \times 3^{l-e_n}$ three-phase unitary matrix, where $0 \le d \le 3^{e_n} - 1$, and let $u_{g_0,g_1}^{n,d}/\sqrt{3^{l-e_n}}$ be the (g_0, g_1)'th entry of U_d^n. Then, using C_1 and U_d^n, a three-phase ZCZ sequence set C_n $(n \ge 2)$ can be obtained.

$$\begin{aligned} C_n &= \left\{C_0^n, \cdots, C_i^n, \cdots, C_{3^l-1}^n\right\}, \\ C_i^n &= \left(c_0^{n,i}, \cdots, c_j^{n,i}, \cdots, c_{P_n-1}^{n,i}\right), \\ c_j^{n,i} &= u_{\psi_n(i),\psi_n(j)}^{n,\phi_n(i)} \cdot c_{\phi_n(j)}^{n-1,\phi_n'(i)+\psi_n(j)}. \end{aligned} \tag{9}$$

P_n, $\phi_n(x)$, $\psi_n(x)$, and $\phi_n'(x)$ are defined as follows:

$$\begin{aligned} P_n &= 3^{nl+2} \prod_{v=2}^{n} 3^{-e_v}, \\ \phi_n(x) &= \lfloor x/3^{l-e_n} \rfloor, \\ \psi_n(x) &= x \bmod 3^{l-e_n}, \\ \phi_n'(x) &= \phi(x) \cdot 3^{l-e_n}. \end{aligned} \tag{10}$$

C_n is $Z\left(P_n, 3^l, P_n 7\cdot 3^{-l-2}\right)$. For details, see [8], [10].

Next, we clarify the correlation property between Type 1-B three-phase ZCZ sequence sets obtained from formula (8). Let C_1 and $\hat{C}_1$ be $Z\left(3^{l+2}, 3^l, 7\right)$. C_1 is defined by formula (8). Suppose that $\hat{C}_1$ uses $\hat{C}_i^1$, $\hat{c}_j^{1,i}$, $\hat{u}_{h_0,h_1}$, $\hat{q}_k^m$, $\hat{S}$, $\hat{s}_k$, $\hat{f}(k_1)$, and $\hat{r}(k_1)$ rather than C_i^1, $c_j^{1,i}$, u_{h_0,h_1}, q_k^m, S, s_k, $f(k_1)$, and $r(k_1)$, respectively. The periodic correlation function between $C_{i_0}^1$ and $\hat{C}_{i_1}^1$ is calculated as

$$\begin{aligned} &R_{i_0,i_1}^1(\tau) \\ &= \sum_{j=0}^{3^{l+2}-1} c_j^{1,i_0}\cdot \hat{c}_{(j+\tau) \bmod 3^{l+2}}^{1,i_1 *} \\ &= \sum_{j=0}^{3^{l+2}-1} u_{i_0, j \bmod 3^l}\cdot q_{\lfloor j/3^l\rfloor}^{j \bmod 3^l} \\ &\quad \cdot \hat{u}^*_{i_1,(j+\tau) \bmod 3^{l+2} \bmod 3^l} \\ &\quad \cdot \hat{q}^{(j+\tau) \bmod 3^{l+2} \bmod 3^l *}_{\lfloor (j+\tau) \bmod 3^{l+2}/3^l\rfloor} \\ &= \sum_{j_0=0}^{8}\sum_{j_1=0}^{3^l-1} u_{i_0,j_1}\cdot q_{j_0}^{j_1}\cdot \hat{u}^*_{i_1,j_1+\tau_1-3^l\epsilon_1} \\ &\quad \cdot \hat{q}^{j_1+\tau_1-3^l\epsilon_1 *}_{(j_0+\tau_0+\epsilon_1) \bmod 9} \\ &= \sum_{j_1=0}^{3^l-1} u_{i_0,j_1}\cdot \hat{u}^*_{i_1,j_1+\tau_1-3^l\epsilon_1} \\ &\quad \cdot \sum_{j_0=0}^{8} s_{(\epsilon_2 j_1+j_0) \bmod 9} \\ &\quad \cdot \hat{s}^*_{\left(\epsilon_2\left(j_1+\tau_1-3^l\epsilon_1\right)+j_0+\tau_0+\epsilon_1\right) \bmod 9}, \end{aligned} \tag{11}$$

where τ is a time shift variable. Note that the integers, j_0, j_1, τ_0, and τ_1, are defined as

$$\begin{aligned} &j = 3^l j_0 + j_1,\ 0\le j_0\le 8,\ 0\le j_1\le 3^l-1, \\ &\tau = 3^l\tau_0+\tau_1,\ 0\le\tau_0\le 8,\ 0\le\tau_1\le 3^l-1. \end{aligned} \tag{12}$$

In addition, the integers, ϵ_1 and ϵ_2, are defined as

$$\begin{aligned} \epsilon_1 &= \begin{cases} 0 & \left(j_1+\tau_1 < 3^l\right) \\ 1 & \left(j_1+\tau_1 \ge 3^l\right) \end{cases}, \\ \epsilon_2 &= \begin{cases} 3 & (l=1) \\ 1 & (l>1) \end{cases}. \end{aligned} \tag{13}$$

Note that $(j+\tau) \bmod 3^{l+2}$ can be represented as

$$\begin{aligned} (j+\tau) \bmod 3^{l+2} &= 3^l\left((j_0+\tau_0+\epsilon_1) \bmod 9\right) \\ &+ \left(j_1+\tau_1-3^l\epsilon_1\right). \end{aligned} \tag{14}$$

Formula (11) shows that $R_{i_0,i_1}^1(\tau)$ can be represented as a function of the correlation function between S and $\hat{S}$. The correlation function between S and $\hat{S}$ can be classified by the relationship between r and $\hat{r}$ [2], [3]. When $r=\hat{r}$, this correlation function can be represented as

$$\begin{aligned} &R_{S,\hat{S}}(\sigma) \\ &= \begin{cases} 3\displaystyle\sum_{\alpha=0}^{2} W_3^{f(\alpha)-\hat{f}(\alpha)-\sigma_0 r(\alpha)} & (\sigma_1=0) \\ 0 & (\sigma_1\ne 0) \end{cases}, \end{aligned} \tag{15}$$

where $W_3=\exp\left(2\pi\sqrt{-1}/3\right)$, $\sigma_0=\lfloor\sigma/3\rfloor$, $\sigma_1=\sigma \bmod 3$, and σ is a time shift variable. The absolute value of the summation in formula (15) takes 0, $\sqrt{3}$, or 3. However, it is impossible for all values of σ_0 to take 0. Therefore, the lower bound of $\max\limits_{0\le\sigma_0\le 2}\left\{\left|3\sum_{\alpha=0}^{2} W_3^{f(\alpha)-\hat{f}(\alpha)-\sigma_0 r(\alpha)}\right|\right\}$ is $3\sqrt{3}$. Hereafter, we assume that this lower bound is satisfied. On the other hand, when $r\ne\hat{r}$, the correlation function can be represented as

$$\begin{aligned} &R_{S,\hat{S}}(\sigma) \\ &= 3W_3^{f(\beta)-\hat{f}((\beta+\sigma_1) \bmod 3)-\left(\sigma_0+\lfloor\frac{\beta+\sigma_1}{3}\rfloor\right)r(\beta)}, \end{aligned} \tag{16}$$

where β is an integer satisfying $r(\beta)=\hat{r}((\beta+\sigma_1) \bmod 3)$. Here, $\left|R_{S,\hat{S}}(\sigma)\right|$ is equivalent to 3 for all σ. Using $R_{S,\hat{S}}(\sigma)$, $R_{i_0,i_1}^1(\tau)$ can be represented as

$$\begin{aligned} R_{i_0,i_1}^1(\tau) &= \sum_{j_1=0}^{3^l-1} u_{i_0,j_1}\cdot\hat{u}^*_{i_1,j_1+\tau_1-3^l\epsilon_1} \\ &\quad \cdot R_{S,\hat{S}}(\tau_0+\epsilon_1+\tau_1\epsilon_2). \end{aligned} \tag{17}$$

Note that $3^l\epsilon_1\epsilon_2 = 0 \pmod 9$. We can obtain the upper bound of $\left|R_{i_0,i_1}^1(\tau)\right|$ from (15), (16), and (17).

$$\begin{aligned} &\max_{0\le\tau\le 3^{l+2}-1}\left\{\left|R_{i_0,i_1}^1(\tau)\right|\right\} \\ &\le \max_{0\le\tau\le 3^{l+2}-1}\left\{\sum_{j_1=0}^{3^l-1}\left|u_{i_0,j_1}\right|\cdot\left|\hat{u}^*_{i_1,j_1+\tau_1-3^l\epsilon_1}\right|\right. \\ &\qquad \left.\cdot\left|R_{S,\hat{S}}(\tau_0+\epsilon_1+\tau_1\epsilon_2)\right|\right\} \\ &= \max_{0\le\tau\le 3^{l+2}-1}\left\{\sum_{j_1=0}^{3^l-1}\left|R_{S,\hat{S}}(\tau_0+\epsilon_1+\tau_1\epsilon_2)\right|\right\} \\ &= \begin{cases} 3^{l+1}\sqrt{3} & (r=\hat{r}) \\ 3^{l+1} & (r\ne\hat{r}) \end{cases}. \end{aligned} \tag{18}$$

Note that $(\tau_0 + \epsilon_1 + \tau_1\epsilon_2) = 0 \pmod 3$ for all j_1 when $\tau_0 = 0 \pmod 3$ and $\tau_1 = 0$.

Next, we propose a method for constructing a group of Type 1-B three-phase ZCZ sequence sets that have low cross-correlation properties. From formula (18), it is clear that we can construct a pair of ZCZ sequence sets that have a low cross-correlation property by using two perfect sequences satisfying $r \neq \hat{r}$ as starters. Since the number of f is 3^3 and the number of r is 3!, the number of three-phase perfect sequences of period 9 is 162. However, these sequences include the same perfect sequences. We can obtain the following six different three-phase perfect sequences of period 9 by removing the same perfect sequences [3].

$$
\begin{aligned}
S_{00} &= (000012021)\,, \\
S_{01} &= (001010022)\,, \\
S_{02} &= (002011020)\,, \\
S_{10} &= (000021012)\,, \\
S_{11} &= (001022010)\,, \\
S_{12} &= (002020011)\,.
\end{aligned}
$$

S_{w0}, S_{w1}, and S_{w2} can be obtained from the same permutation r_w, respectively. On the other hand, r_0 and r_1 differ from each other. Therefore, we can obtain a pair of ZCZ sequence sets that have a low cross-correlation property by using S_{0x} and S_{1y} as starters. However, we need at least three ZCZ sequence sets in order to cover all cells so that two bordered cells do not use the same ZCZ sequence set. Therefore, we add a ZCZ sequence set derived from S_{0z} $(z \neq x)$ or S_{1z} $(z \neq y)$. The conditions, $z \neq x$ and $z \neq y$, are necessary in order to satisfy the lower bound in formula (15), i.e., $3\sqrt{3}$.

For example, suppose that S_{00}, S_{10}, and S_{11} are selected as starters. Using these perfect sequences and the unitary matrix presented in (5), the following ZCZ sequence sets are obtained.

$$
\begin{aligned}
C_{00} &= \left\{C_0^{00}, C_1^{00}, C_2^{00}\right\}, \\
C_0^{00} &= (000012021000120210000201102)\,, \\
C_1^{00} &= (012021000012102222012210111)\,, \\
C_2^{00} &= (021000012021111201021222120)\,, \\
C_{10} &= \left\{C_0^{10}, C_1^{10}, C_2^{10}\right\}, \\
C_0^{10} &= (000021012000210120000102201)\,, \\
C_1^{10} &= (012000021012222102012111210)\,, \\
C_2^{10} &= (021012000021201111021120222)\,, \\
C_{11} &= \left\{C_0^{11}, C_1^{11}, C_2^{11}\right\}, \\
C_0^{11} &= (000021120000210201000102012)\,, \\
C_1^{11} &= (012000102012222210012111021)\,, \\
C_2^{11} &= (021012111021201222021120000)\,.
\end{aligned}
$$

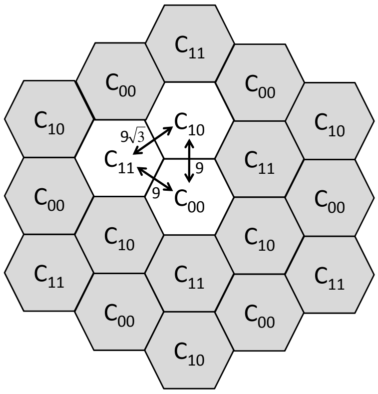

Figure 1: Allotment of the ZCZ sequence sets

Note that C_{00}, C_{10}, and C_{11} are derived from S_{00}, S_{10}, and S_{11}, respectively. The maximum absolute value of the cross-correlation function between C_λ^{00} $(0 \leq \lambda \leq 2)$ and C_μ^{10} $(0 \leq \mu \leq 2)$ is 9. Similarly, the maximum absolute value of the cross-correlation function between C_λ^{00} and C_ν^{11} $(0 \leq \nu \leq 2)$ is 9, and that between C_μ^{10} and C_ν^{11} is $9\sqrt{3}$. Thus, we can confirm that the upper bound shown in (18) agrees with these maximum absolute values. Note that $l = 1$ in this example. Figure 1 shows the concept of the allotment of the ZCZ sequence sets to a cellular system.

Finally, we discuss the correlation property between Type 1-B three-phase ZCZ sequence sets obtained from formula (9). The periodic correlation function between $C_{i_0}^n$ and $\hat{C}_{i_1}^n$ is represented as

$$
\begin{aligned}
R_{i_0,i_1}^n(\tau) &= \sum_{j=0}^{P_n-1} c_j^{n,i_0} \cdot \hat{c}_{(j+\tau) \bmod P_n}^{n,i_1 *} \\
&= \sum_{j_1=0}^{3^{l-e_n}-1} u_{\psi_n(i_0),j_1}^{n,\phi_n(i_0)} \cdot \hat{u}_{\psi_n(i_1),\psi_n(j_1+\tau_1)}^{n,\phi_n(i_1)*} \\
&\quad \cdot \sum_{j_0=0}^{\phi_n(P_n)-1} c_{j_0}^{n-1,\phi'_n(i_0)+j_1} \\
&\quad \cdot \hat{c}_{j_0+\tau_0+\phi_n(j_1+\tau_1)}^{n-1,\phi'_n(i_1)+\psi_n(j_1+\tau_1)*} \\
&= \sum_{j_1=0}^{3^{l-e_n}-1} u_{\psi_n(i_0),j_1}^{n,\phi_n(i_0)} \cdot \hat{u}_{\psi_n(i_1),\psi_n(j_1+\tau_1)}^{n,\phi_n(i_1)*} \\
&\quad \cdot R_{i'_0,i'_1}^{n-1}(\tau')\,. \qquad (19)
\end{aligned}
$$

The integers, j_0, j_1, τ_0, τ_1, i'_0, i'_1, and τ' are defined as

$$
\begin{aligned}
j_0 &= \phi_n(j)\,,\ j_1 = \psi_n(j)\,, \\
\tau_0 &= \phi_n(\tau)\,,\ \tau_1 = \psi_n(\tau)\,, \\
i'_0 &= \phi'_n(i_0) + j_1,
\end{aligned}
$$

$$
\begin{aligned}
i_1' &= \phi_n'(i_1) + \psi_n(j_1+\tau_1), \\
\tau' &= \tau_0 + \phi_n(j_1+\tau_1). \qquad (20)
\end{aligned}
$$

We can obtain the upper bound of $\left|R^n_{i_0,i_1}(\tau)\right|$ by the repetition of formula (19).

$$
\begin{aligned}
&\max_{0\le\tau\le P_n-1}\left\{\left|R^n_{i_0,i_1}(\tau)\right|\right\} \\
&\le \max_{0\le\tau\le P_n-1}\left\{\sum_{j_1=0}^{3^{l-e_n}-1}\left|u^{n,\phi_n(i_0)}_{\psi_n(i_0),j_1}\right|\cdot\right. \\
&\qquad \left.\left|\hat{u}^{n,\phi_n(i_1)*}_{\psi_n(i_1),\psi_n(j_1+\tau_1)}\right|\cdot\left|R^{n-1}_{i_0',i_1'}(\tau')\right|\right\} \\
&= \max_{0\le\tau\le P_n-1}\left\{\sum_{j_1=0}^{3^{l-e_n}-1}\left|R^{n-1}_{i_0',i_1'}(\tau')\right|\right\} \\
&\le \max_{0\le\tau\le P_n-1}\left\{\sum_{\tilde{j}_1=0}^{P_n/9-1}\left|R_{S,\hat{S}}(\theta(\tau,j))\right|\right\} \\
&\le \left\{\begin{array}{cc} P_n\sqrt{3}/3 & (r=\hat{r}) \\ P_n/3 & (r\ne\hat{r}) \end{array}\right. . \qquad (21)
\end{aligned}
$$

$\tilde{j}_1$ is an integer, and θ is a function of τ and j. The formula (21) shows that in order to construct a group of Type 1-B ZCZ sequence sets that have low cross-correlation properties, it is essential to select a set of perfect sequences that have low cross-correlation property as starters, even if $n \ge 2$. Therefore, the proposed method can also be used in this case.

5 Conclusion

In the present paper, we have investigated three-phase ZCZ sequence sets. We clarified possible three-phase ZCZ sequence sets and the values of the ratio η. We also classified these ZCZ sequence sets based on η. In addition, we clarified the structure of the periodic correlation function between sequences that belong to Type 1-B ZCZ sequence sets. Finally, we have proposed a method for constructing a group of Type 1-B ZCZ sequence sets that have low cross-correlation properties. The results are useful in designing AS-CDMA systems that use coded TPSK.

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