Low-Thrust Trajectories to the Moon

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Abstract: - In this paper the problem of sending a spacecraft from Low Earth Orbit (LEO) to the Moon with minimum fuel consumption is considered. It is assumed that the "Two-Body model" approximation is valid in all phases of the mission and that the final orbit around the Moon is polar. The first part deals with impulsive maneuvers, and obtain a set of values for fuel expenditure and trip time for several trajectories. Two possible scenarios are considered: a single mission (only one spacecraft in orbit around the Moon) and a double mission (one main spacecraft and a sub-satellite around the Moon). The second part considers the use of low-thrust trajectories. The Euler-Lagrange equations are used to generate a set of differential equations that are numerically integrated to obtain the final orbit. The difficulty caused by a lack of initial values for all variables in the same point (Two Point Boundary Value Problem) is treated by making iterations in the initial values of the Lagrange multipliers. The results showed that large savings in fuel consumption can be obtained by using low thrust trajectories for the Earth-Moon part of the mission. The contribution of the present paper related to previous ones written by the same author is the use of the Powell's Quadratically Convergent method to solve the problem, that showed to have a better numerical behaviour regarding convergence.

Key Words: Astrodynamics, Orbital Maneuvers, Satellites, Space Trajectories, Optimal Control, Impulsive Transfer, Low Thurst.

1 Introduction

The establishment of a manned lunar base is certainly one of the next big step of the mankind in its journey to the space. To accomplish this goal, a more detailed study of the Moon, including its mineral resources and physical properties, has to be done. It is in this context that the Lunar Polar Orbiter mission appears. It is constituted by one or two spacecrafts in Moon's polar orbit, to make measurements in the Moon's surface and neighborhood. The data obtained will be used for several important tasks, like: site selection of the lunar base; improvements of trajectory calculation around the Moon, study of possible mineral exploitation, etc.

The objective of this paper is to make a preliminary study of the possible trajectories to be used to go to the Moon. It is assumed that the spacecraft begins its trip in Low Earth Orbit (LEO) and that the equations given by the Two-Body non-perturbed problem are valid for each phase of the mission. Two scenarios are studied: in the first one a single spacecraft will orbit the Moon; and in the second one the main spacecraft will have a sub-satellite in a higher orbit.

Special attention is given to the difference in fuel consumption obtained by using two options of thrust for the Earth-Moon transfer: Infinite (using Hohmann Transfer) and Low Thrust (using optimal control theory).

This kind of problem was analyzed before in Prado[1] and Prado and Rios-Neto[2], where the gradient projection method was used to solve the numerical problem. It showed to be a useful tool, but convergence was difficult in some situations. In the present paper, the Powell's Quadratically Convergent method[3] was used for the optimization steps of the algorithm. This method showed to be better, in terms of convergence, in the simulations performed. The problem considered here is very sensitive to initial guesses and it finds only local minimum, that are candidates for a global minimum. Then, from the engineering point of view, a numerical algorithm that is more efficient in terms convergence is an important tool for the mission designer.

2 Impulsive Maneuvers

In this case it is assumed that an infinite thrust acting during a negligible time can change the velocity of the spacecraft instantaneously. It is also assumed that the spacecraft will leave the Earth from a circular parking orbit with an altitude of 200 km. The main goal is to obtain the values of the velocity increment, trip time and mass of fuel required for many trajectories, to select one of them for a more detailed analysis. The two scenarios considered here are[4]:

1) A single mission with the spacecraft in a circular orbit around the Moon with an altitude of 100 km and 90 degrees of inclination;

2) A double mission with the two spacecrafts in different orbits:

- The main spacecraft in a circular orbit around the Moon with an altitude of 100 km and 90 degrees of inclination; and
- The sub-satellite (with no engines) in a elliptical orbit around the Moon with semi-major axis of 3000 km, eccentricity of 0.37, argument of perigee of 0.25 degrees West and inclination of 90 degrees.
- Using the "Two-Body model" approximation for the Earth-spacecraft system, it is possible to obtain the trip time for different trajectories, all of them assumed to be elliptical with perigee of 6570 km. The results are shown in Table 1.

Table 1 - Orbital parameters and trip time for different trajectories to the Moon

Orbit	Semi-major axis (km)	Eccentricity	Trip Time in hr. (days)
1	500000	0.986	58.5 (2.43)
2	400000	0.983	61.0 (2.54)
3	300000	0.978	67.0 (2.80)
4	250000	0.974	73.9 (3.08)
5	230000	0.971	77.0 (3.21)
6	220000	0.970	83.2 (3.47)
7	200000	0.967	100.0 (4.17)
8	195485	0.960	119.6 (4.98)

Using impulsive approximation, it is possible to evaluate the velocity increment necessary to put the spacecraft into Lunar Transfer Orbit (LTO). Four different orbits were chosen for detailed calculations: 2, 6, 7 and 8. The results are shown in Table 2.

Table 2 - Velocity increment for LTO

Orbit	Apogee distance (km)	ΔV (km/s)
2	793430	3.18
6	433430	3.14
7	393430	3.13
8	384400	3.11

The spacecraft arrives at the sphere of influence of the Moon with hyperbolic excess velocity and it is necessary to apply a retrograde impulse to achieve an elliptic lunar orbit. Using the basic equations from the Two-Body model for the Moon-spacecraft system, it is possible to obtain the velocity of the spacecraft with respect to the Moon (assumed to be in circular orbit around the Earth) and the velocity decrement required. At this point it is necessary to consider the maneuvers in two scenarios, because the velocity decrement depends on how many spacecrafts are in the mission. If there is only one spacecraft, it is possible to assume that with small mid-course corrections it can achieve a hyperbolic arrival at the Moon with the desired perigee altitude (1840 km) and orbit inclination (90 degrees). Then it is necessary to apply only one impulse, at the perigee, to obtain the desired circular orbit. The velocity decrements are shown in Table 3.

Table 3 - Velocity decrement to insert one probe into Moon's orbit

Orbit	ΔV (km/s)
2	1.07
6	0.91
7	0.88
8	0.79

If there are two spacecrafts, it will be necessary to use a more complex maneuver, because the sub-satellite has no engine. In this case, the insertion into Moon's orbit will be done with both spacecrafts together, in the orbit desired for the sub-satellite. After the separation of the two spacecrafts, the primary spacecraft will be transferred to its final orbit.

Assuming that the optimal maneuver (insertion at the perigee of the elliptical orbit) is used and that, after separating from the sub-satellite, the main spacecraft will be transferred to its final orbit using a bi-impulsive Hohmann Transfer, the results for the ΔV required can be calculated. They are shown in Table 4.

Table 4 - Velocity decrement to insert two probes in Moon's orbit

Orbit	ΔV_i (km/s)	ΔV_t (km/s)
2	0.78	0.29
6	0.63	0.29
7	0.60	0.29
8	0.50	0.29

where ΔV_i is the velocity decrement for lunar insertion of both spacecrafts together and ΔV_t is the total ΔV to transfer the main spacecraft to its final orbit.

The results of both approaches (ΔV_1 for a single mission and ΔV_2 for a double mission) are summarized in Table 5.

Table 5 - Total ΔV for both missions

Orbit	ΔV_1 (km/s)	ΔV_2 (km/s)
2	4.68	4.68
6	4.45	4.47
7	4.41	4.42
8	4.30	4.30

To consider errors in the models involved and midcourse corrections, 10% was added to the ΔVs. With the total ΔVs obtained, it is possible to calculate the fuel mass required for the mission. Three different specific impulses (I_{sp}) were assumed and 10% was added to include the hardware required for storage. The results are shown in Tables 6 (single mission) and 7 (double mission). In this last case, the difference in mass between the set of the two spacecrafts and the main spacecraft itself was neglected, since the mass of the sub-satellite will be very small.

Table 6 - Mass of fuel required for a single mission

I_{sp}(s)\Orbit	2	6	7	8
240	6.95	6.20	6.07	5.75
290	4.61	4.17	4.09	3.90
340	3.38	3.08	3.03	2.90

Table 7 - Mass of fuel required for a double mission

I_{sp}(s)\Orbit	2	6	7	8
240	6.95	6.26	6.10	5.75
290	4.61	4.20	4.11	3.90
340	3.38	3.12	3.04	2.90

Some alterations in the trajectory maybe necessary for many reasons. If it is necessary to change the orbit during the mission or, if a different parking orbit (or no parking orbit) around the Earth is used, the optimal trajectory may change. However, these results are still valid because these alterations do not affect the ΔV, trip time or mass of fuel consumed by a significant amount. The literature[5,6] also confirms these values.

The advantageous of a short trip time are that it requires less time of tracking and has more safety (because there is some excess in velocity). The disadvantageous is that it requires more fuel. Considering these facts and the data available it is necessary to find the most economical orbit inside the range of safety. After preliminary studies, orbit 6 was considered to be a good choice for a more detailed study.

3 Low-Thrust Trajectory

This study was done with the objective of comparing the differences in fuel consumption for an Earth-Moon transfer with low thrust.

The spacecraft is supposed to be in planar Keplerian motion controlled only by the thrust, whenever it is active. This thrust is assumed to have the following characteristics:

i) Fixed magnitude;
ii) Constant Ejection Velocity;
iii) Free angular motions;
iv) Operation in on-off mode.

The solution is given in terms of the time-histories of the thrust (pitch angle), fuel consumed and duration of the propelled phase.

This is a typical optimal control problem, and it is formulated as follows:

Objective Function: M_f,

where M_f is the final mass of the vehicle and it has to be maximized with respect to the control **u**(.);

Subject to: Equations of motion, constraints in the state (initial and final orbit) and control (limits in the angles of "pitch" and "yaw", forbidden region of thrusting and others);

And given: All parameters (gravitational force field, initial values of the satellite and others).

The equations of motion are the ones suggested by Biggs[7], that avoid the singularities in circular and/or planar orbits. They are given by:

$$dX_1/ds = f_1 = SiX_1F_1 \quad (1)$$

$$dX_2/ds=f_2= Si\{[(Ga+1)\cos(s)+X_2]F_1+\nu F_2\sin(s)\} \quad (2)$$

$$dX_3/ds = f_3 = Si\{[(Ga+1)\sin(s)+X_3]F_1-\nu F_2\cos(s)\} \quad (3)$$

$$dX_4/ds = f_4 = Si\nu F(1-X_4)/(X_1W) \quad (4)$$

$$dX_5/ds = f_5 = Si\nu(1-X_4)m_0/X_1 \quad (5)$$

$dX_6/ds=f_6=-SiF_3[X_7\cos(s)+X_8\sin(s)]/2$ (6)

$dX_7/ds=f_7=SiF_3[X_6\cos(s)-X_9\sin(s)]/2$ (7)

$dX_8/ds=f_8=SiF_3[X_9\cos(s)+X_6\sin(s)]/2$ (8)

$dX_9/ds=f_9=SiF_3[X_7\sin(s)-X_8\cos(s)]/2$ (9)

where:

$Ga = 1 + X_2\cos(s) + X_3\sin(s)$ (10)

$Si = (\mu X_1^4)/[Ga^3 m_0(1-X_4)]$ (11)

$F_1 = F\cos(\alpha)\cos(\beta)$ (12)

$F_2 = F\sin(\alpha)\cos(\beta)$ (13)

$F_3 = F\sin(\beta)$ (14)

and F is the magnitude of the thrust, W is the velocity of the gases when leaving the engine, ν is the true anomaly of the spacecraft.

In those equations the state was transformed from the Keplerian elements (a = semi-major axis, e = eccentricity, i = inclination, Ω = argument of the ascending node, ω = argument of periapsis, ν = true anomaly of the spacecraft), in the variables X_i, to avoid singularities, by the relations:

$X_1 = [a(1-e^2)/\mu]^{1/2}$ (15)

$X_2 = e\cos(\omega-\phi)$ (16)

$X_3 = e\sin(\omega-\phi)$ (17)

$X_4 = (\text{Fuel consumed})/m_0$ (18)

$X_5 = t = \text{time}$ (19)

$X_6 = \cos(i/2)\cos((\Omega+\phi)/2)$ (20)

$X_7 = \sin(i/2)\cos((\Omega-\phi)/2)$ (21)

$X_8 = \sin(i/2)\sin((\Omega-\phi)/2)$ (22)

$X_9 = \cos(i/2)\sin((\Omega+\phi)/2)$ (23)

$\phi = \nu + \omega - s$ (24)

and s is the range angle of the spacecraft.

The number of state variables defined above is greater than the minimum required to describe the system, which implies that they are not independent and relations between than exist, like: $X_6^2 + X_7^2 + X_8^2 + X_9^2 = 1$. This system is also subject to the constraints in state and some of the the Keplerian elements of the initial and the final orbit. All the parameters (gravitational force field, initial values of the satellite, etc...) are assumed to be known.

4 Numerical Method

In the references cited before[1,2] the gradient projection method was used to solve the numerical problem. It showed to be a useful tool, but convergence was difficult in some situations. In the present paper, the Powell's Quadratically Convergent method[3] was used for the optimization steps of the algorithm. This method showed to be better, in terms of convergence, in the simulations performed. It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla\mathbf{f}^{\mathbf{T}}.\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{\mathbf{T}}\right]^{-1}\mathbf{f} \quad (25)$$

where f is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \bar{\alpha}\frac{\mathbf{d}}{|\mathbf{d}|} \quad (26)$$

where:

$$\bar{\alpha} = \gamma\frac{J(\boldsymbol{u})}{\nabla J(\boldsymbol{u}).\mathbf{d}} \quad (27)$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla\mathbf{f}^{T}\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{T}\right]^{-1}\mathbf{f}\right)\nabla J(\boldsymbol{u}) \quad (28)$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in equations (25) to (28) are avoided by choosing the error margins for tolerance in convergence large enough.

This procedure continues until $\left|\boldsymbol{u}_{i+1} - \boldsymbol{u}_i\right| < \varepsilon$ in both equations (25) and (26), where ε is a specified tolerance.

4 Results

The low thrust propulsion system was studied only for the Earth-Moon trajectory and not for the lunar insertion phase. The satellite is supposed to leave the Earth from a circular orbit with semi-major axis of 6570 km and to go to an orbit with eccentricity of 0.97 and semi-major axis of 220000 km, that is the Lunar Transfer Orbit desired (Orbit 6 in Table 1). This transfer is considered to be planar, because this is the less expensive case (in terms of fuel consumed) and it can be obtained with an adequate choice of the launch time. Two values were considered for the mass of the satellite after the low thrust maneuver: 150 and 180 kg. These values are compatible with a final mass of 100 and 120 kg in lunar orbit, respectively. The motor/fuel combination is supposed to have a specific impulse of 3500 s, and a thrust magnitude of 200 and 20 N was simulated. Figs. 2 to 5 show the optimal control (pitch angle) for all combinations studied. Table 8 shows the fuel consumption and the duration of the propelled phase for all cases.

Table 8 - Fuel consumed and duration of the propelled phase for all trajectories simulated

Mission	E-M (kg)	Ins. (kg)	Total (kg)	Time (h)
Imp 1	205.78	31.40	237.18	0.00
Imp 2	247.69	38.16	285.85	0.00
L. T. 1	22.37	31.40	53.77	10.67
L. T. 2	14.75	31.40	46.15	0.65
L. T. 3	27.05	38.16	65.21	12.89
L. T. 4	18.02	38.16	56.18	0.86

Impulsive 1: Single mission using an engine with specific impulse of 340 s;
Impulsive 2: Double mission using an engine with specific impulse of 340 s;
L. T. 1: Single mission using an engine with 20 N and specific impulse of 3500 s, and assuming a mass of 150 kg for the spacecraft after the Low-Thrust maneuver;
L. T. 2: Single mission using an engine with 200 N and specific impulse of 3500 s, and assuming a mass of 150 kg for the spacecraft after the Low-Thrust maneuver;
L. T. 3: Double mission using an engine with 20 N and specific impulse of 3500 s, and assuming a mass of 180 kg for the spacecraft after the Low-Thrust maneuver;
L. T. 4: Double mission using an engine with 200 N and specific impulse of 3500 s, and assuming a mass of 180 kg for the spacecraft after the Low-Thrust maneuver;
E-M: Fuel consumed for the Earth-Moon trajectory;
Ins.: Fuel consumed for the lunar insertion, assuming an engine with specific impulse of 340 s;
Total: Total mass of fuel required (Earth-Moon trajectory + lunar insertion).
Time: Duration of the propelled arc;

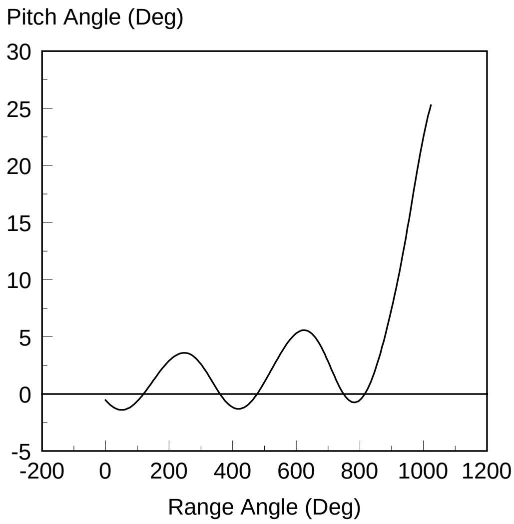

Fig. 2 - Pitch angle (deg) X Range angle (deg) for Low Thrust 1 maneuver.

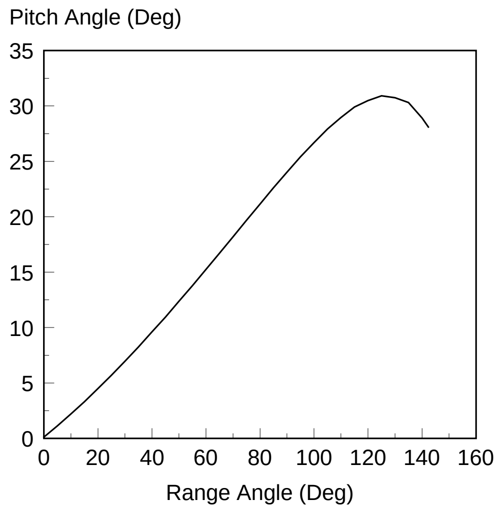

Fig. 3 - Pitch angle (deg) X Range angle (deg) for Low Thrust 2 maneuver.

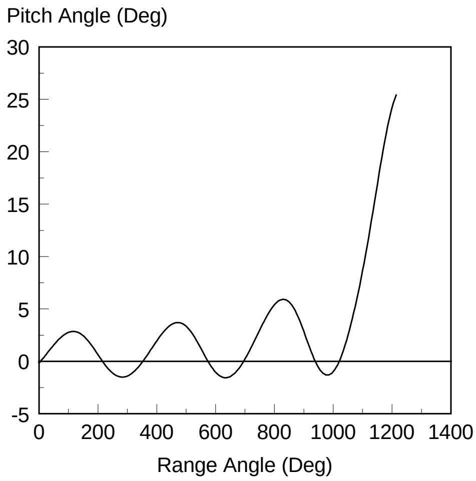

Fig. 4 - Pitch angle (deg) X Range angle (deg) for Low Thrust 3 maneuver.

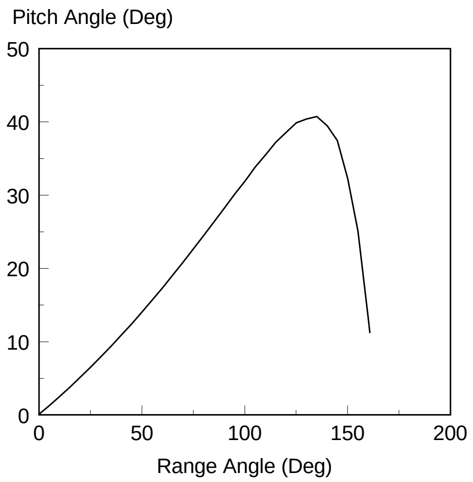

Fig. 5 - Pitch angle (deg) X Range angle (deg) for Low Thrust 4 maneuver.

6 Conclusion

This paper showed the basic parameters for several trajectories to send, insert and keep a spacecraft in lunar orbit. The numerical results also showed that, with the error parameters used, there is no necessity to use an underburn technique for lunar insertion. It also showed that the use of low thrust maneuver for the Earth-Moon trajectory can make very large savings in fuel expenditure;

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