A simplified analytical model of crosstalk in polarization division multiplexing system

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Abstract: - A simple method of the calculation of the crosstalk between two polarization multiplexed channel in an optical fiber link includes polarization dependence loss and polarization dependence gain is presented. This method allows a significant simplification of the calculation of crosstalk in polarization multiplexed transmission systems. The analytical results are compared with numerical results.

Key-Words: - Optical fiber, Polarization, Polarization division multiplexing, Crosstalk, Polarization dependence loss, Polarization dependence gain

1 Introduction

Polarization division multiplexing (PDM) is a well-known technique to increase spectral efficiency and can be used to double fiber capacity. In PDM system two signals are transmitted at the same wavelength with orthogonal states of polarization (SOP). In PDM systems polarization channels are demultiplexed at polarization beam splitter (PBS). Hence, the most important parameter of PDM systems is crosstalk between polarization channels. This crosstalk suffers from polarization effects: polarization dependence loss (PDL) and polarization dependence gain (PDG). In the case of single mode fibers orthogonal SOPs pairs at the input lead to orthogonal output SOPs pairs. But, when the optical fiber link includes PDL effect the SOPs are no longer orthogonal [1]. The crosstalk directly depends on the degree of orthogonality between polarization channels. Furthermore, due to PDL and PDG the polarized signals may be attenuated and amplified in a random manner as a function of time. In this paper, the simple analytical model of the impact of PDL and PDG effects on the crosstalk is presented. The analytical results are compared with numerical simulation results.

2 Analytical model

In the case of the orthogonality the angle between two SOPs vectors is equal to $\pi/2$. The PDL effect reduces the SOP vector and causes the loss of the orthogonality between SOPs vectors. The relationship between a given pair of input orthogonal SOPs vectors $\overrightarrow{S_a}$, $\overrightarrow{S_b}$ and output SOPs vectors $\overrightarrow{S'_a}$, $\overrightarrow{S'_b}$ can be written as:

$$\left[\overrightarrow{S'_a} \quad \overrightarrow{S'_b}\right] = \left[\overrightarrow{S_a} \quad \overrightarrow{S_b}\right] \cdot \begin{bmatrix} 1 & u \\ u & 1 \end{bmatrix}. \quad (1)$$

Where u is a parameter that describes impact of PDL on SOP vector. This parameter is equal to 1-α, where α is the magnitude of PDL (in dB) in linear units: $\alpha = 10^{\frac{-PDL_{dB}}{20}}$. The output SOPs vectors are given by:

$$\overrightarrow{S'_a} = \overrightarrow{S_a} + (1-\alpha)\overrightarrow{S_b}, \quad (2)$$

$$\overrightarrow{S'_b} = (1-\alpha)\overrightarrow{S_a} + \overrightarrow{S_b}. \quad (3)$$

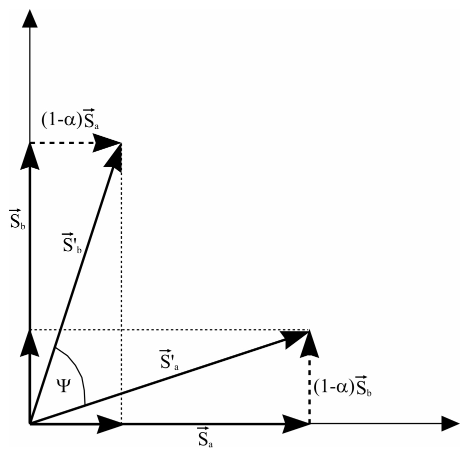

Fig. 1. Loss of orthogonality between SOPs vectors

Figure 1 shows impact of PDL effect on the orthogonality between SOPs vectors. The angle between $\overrightarrow{S'_a}$ and $\overrightarrow{S'_b}$ vectors is equivalent to the degree of orthogonality. The angle between $\overrightarrow{S'_a}$ and $\overrightarrow{S'_b}$ vectors is equal to:

$$\Psi = a\tan\left(\frac{K}{(1-\alpha)}\right) - a\tan((1-\alpha)K), (4)$$

where K= S_{0b}/S_{0a}. The values of S_{0a} and S_{0b} are the output intensities of polarization channels. Here, we assume that the value of K only depends on PDL and PDG. Next, we assume the following relationship between K and these polarization effects [2]:

$$K = \left(\frac{1+\alpha^2}{2}\right)\cdot\left(\frac{g^2+1}{2}\right). \qquad (5)$$

The value of g is related to PDG (in dB) through the following relationship [2]:

$$PDG \cdot d_{pol} = 20\log(g), \qquad (6)$$

where d_{pol} is degree of polarization. According to [3]:

$$d_{pol} = 0.11\sqrt{n} \cdot PDG, \qquad (7)$$

where n is number of birefringence segments. Next, we can compute the crosstalk (CX) between polarization channels by applying the following equation [4]:

$$CX = K\frac{\cos\left(\frac{\pi}{4}+\frac{\Psi}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\Psi}{2}\right)}. \qquad (8)$$

3 Numerical model

In order to validate described analytical model for evaluation of the crosstalk (CX) between polarization channels numerical simulations are performed. An optical fiber link as a cascade of birefringence segments is modeled. Here, each birefringence segment is composed of one PDG element, one PDL element and one PMD element. Amplified spontaneous emission noises are neglected. The single birefringence segment (M_{WP}) can be written as:

$$M_{WP}=M_{R(-\theta g)}\cdot M_{PDG}\cdot M_{R(\theta g)}\cdot M_{R(-\theta)}\cdot M_{PDL}\cdot M_{DGD}\cdot M_{R(\theta)}. \ (9)$$

The rotation matrix $M_{R(\theta)}$ is given by [5]:

$$M_{R(\Theta)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\Theta) & \sin(2\Theta) & 0 \\ 0 & -\sin(2\Theta) & \cos(2\Theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (10)$$

where Θ is the angle between the so called „fast axis" and x-axis of the reference frame. The retarder matrix M_{DGD} is given by [5]:

$$M_{DGD} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\omega\tau) & -\sin(\omega\tau) \\ 0 & 0 & \sin(\omega\tau) & \cos(\omega\tau) \end{bmatrix}, (11)$$

where ω is the angular frequency and τ is the differential group delay. In turn, the PDL element matrix M_{PDL} is equal to [6]:

$$M_{PDL} = \begin{bmatrix} \frac{1+\alpha_i^2}{2} & \frac{1-\alpha_i^2}{2} & 0 & 0 \\ \frac{1-\alpha_i^2}{2} & \frac{1+\alpha_i^2}{2} & 0 & 0 \\ 0 & 0 & \alpha_i & 0 \\ 0 & 0 & 0 & \alpha_i \end{bmatrix}. (12)$$

According to [2] we can model the PDG similar to PDL, except that the direction of maximum gain should be chosen self-consistently with the signal polarization. Here, the rotation of PDG element (the value of Θg) is determined by the SOP, which is obtained by superposing SOPs of both polarization channels. The PDG element matrix M_{PDG} is described by the following equation:

$$M_{PDG} = \begin{bmatrix} \frac{g_i^2+1}{2} & \frac{g_i^2-1}{2} & 0 & 0 \\ \frac{g_i^2-1}{2} & \frac{g_i^2+1}{2} & 0 & 0 \\ 0 & 0 & g_i & 0 \\ 0 & 0 & 0 & g_i \end{bmatrix}. (13)$$

The transmission matrix of optical fiber link, consisting of n birefringence segments, is obtained as the product of the n birefringence segment matrices. Computing the crosstalk between two output polarization channels is equivalent to calculating their projections on the PBS x-y coordinate system [4, 7]. Using the well-known relationships between the Stokes parameters and the spherical coordinates we can calculate value of the angle between output polarization channels and their output intensities [5]. The simulation conditions:

1) The simulated optical fiber link consists of n=50, 100, 150 and 200 birefringence segments.
2) The angular frequency ω=1216.1 rad/ps corresponding to the wavelength of 1550 nm.
3) The differential group delay value is uniformly distributed between 0 and 0.05 ps.
4) The angle Θ is uniformly distributed between 0 and π.
5) The SOP of input polarization channels: linear horizontal and linear vertical.
6) The value of PDL_i (single birefringence segment) equals 0.1 dB, 0.2 dB and 0.3 dB.
7) The value of PDG_i (single birefringence segment) equals 0.1 dB, 0.2 dB and 0.3 dB.
8) For any given values of PDL_i and $PDG_{i,}$ the simulation is repeated 500 times.

4 Results

The crosstalk is calculated using the analytical model and the numerical simulation. In order to calculate the crosstalk, using the analytical model, the mean PDL (PDL_{mean}) and PDG (PDG_{mean}) values are used. Because the PDL and PDG distribution can be approximated by a Maxwellian function [8, 9] we have:

$$PDL_{mean} = \sqrt{\frac{8}{3\pi}}\left(\sum_{i=1}^{n} PDL_i^2\right)^{\frac{1}{2}}, \quad (14)$$

$$PDG_{mean} = \sqrt{\frac{8}{3\pi}}\left(\sum_{i=1}^{n} PDG_i^2\right)^{\frac{1}{2}}. \quad (15)$$

In the numerical simulation the mean value of the angle between output polarization channels and intensities of polarized signals (polarization channels) are determined. Relationship between the analytical result and the numerical result is characterized by the following ratio:

$$R_{CX} = \frac{CX_a}{CX_n} \cdot 100\%, \qquad (16)$$

where CX_a is the value of crosstalk from the analytical model and CX_n is the value of crosstalk from the numerical simulation. We compare the analytical model with the numerical simulations in the case when we have the same value of individual PDL_i and PDG_i. The crosstalk CX resulting from PDL and PDG effects is presented in fig. 2.

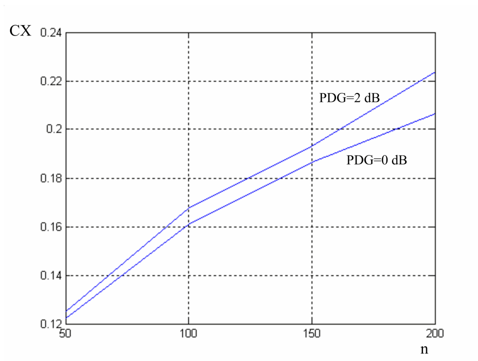

Fig. 2. Crosstalk evolution as a function of number of birefringence segments n; PDL=0.2 dB

In turn, fig. 3 and 4 show the value of R_{CX} ratio as a function of PDL_{mean} and PDG_{mean}.

Figures 3 and 4 reveal that the analytical predictions of the crosstalk are in good agreement with the numerical simulations only for low values of PDL_i and PDG_i. If the mean values of PDL and PDG are less than 2 dB then the value of R_{CX} is greater than 90 %.

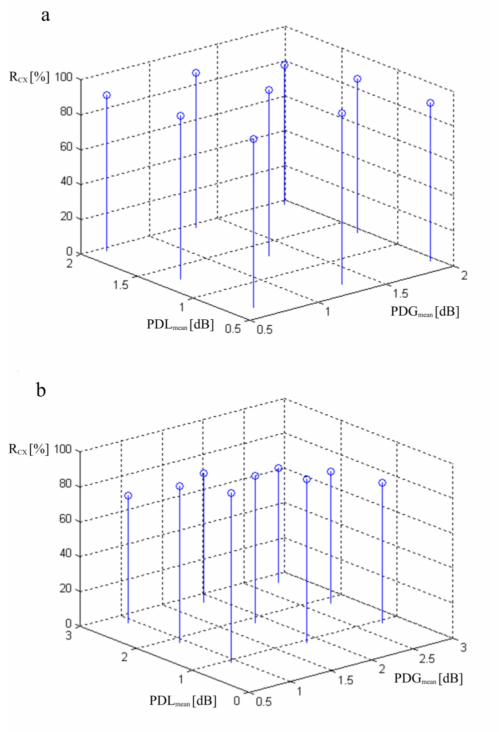

Fig. 3. Impact of PDL_{mean} and PDG_{mean} on Rcx ratio; n=50 (a), n=100 (b)

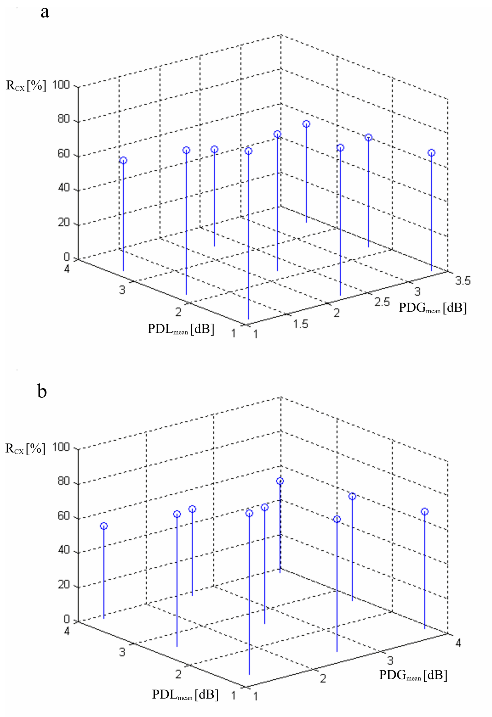

Fig4. Impact of PDL_{mean} and PDG_{mean} on Rcx ratio; n=150 (a), n=200 (b)

Next, the impact of the pure PDL effect (PDG_i=0 dB) on R_{CX} is investigated. Figure 5 show relationship between R_{cx} and PDL_{mean}.

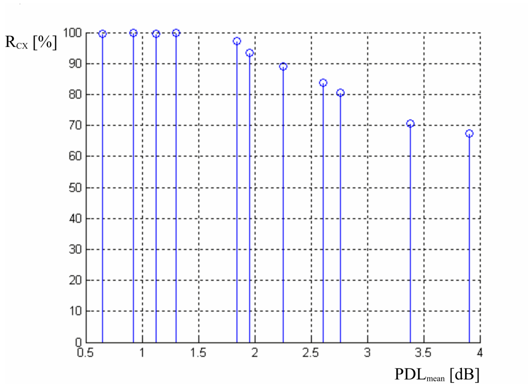

Fig. 5. Parameter R_{cx} as a function of PDL_{mean}

These results confirm good agreement between analytical and numerical results for PDL_{mean}<2 dB.

4 Conclusion

A simplified analytical model to evaluate the crosstalk between two polarization channel has been presented. It allows a significant simplification of the crosstalk calculation in polarization multiplexed transmission systems. This model gives good qualitative predictions of the crosstalk for PDL and PDG mean values < 2 dB.

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