Direct Mesh Method of Sewer Networks Optimal Design Based on Topography-Geomorphic

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Abstract: Utilize modified directness approach, consider sewer network structural system into model of Linear Programming, discrete design variable region with network, proceeding optimize compute use with Direct Work net Method, and compare with routine mathematical modeling. Compute and study indicate, this method more suitability on general Non-linear Programming problem, and having obviously optimal effectiveness.

Key words: Topography-Geomorphic, Imitate Meshnet Method, Sewer Networks, Optimal Design, Economy Parameter

1 Introduction

This paper discusses the role of topography-geomorphic in optimal design, takes the coordinate parameters charactering the topography-geomorphic features as constraint condition, introduces the new economic parameters closely related to the coordinates and the engineered meaning, gives formulas and assignment principle, and builts the linear programming model of sewer networks optimal design based on topography-geomorphic; combining the engineered experience, this model takes the problem of sewer networks as non-linear problem with the improved direct method, disperses the design variable area into grid, and uses the direct mesh method for optimal calculation; finally it is compared with the conventional mathematical model.

2 Conventional Design Model

2.1 Empirical Formula

The designing aim of underground sewer networks is low cost, well effective, long use-term (or low depreciation cost). All the variables to achieve these objectives are named design variable, the main effective variable[1] generally should be taken as the design variable. For specific engineering systems, design variables scope will be restricted because of the special requirement of engineering structure, such as the natural obstacles of topography-geomorphic, the restriction of mechanical behavior of material and so on. Optimal design is to identify main effective variable, objective function and various corresponding constraint conditions.

The cost of pipe network system is low, the formula for calculating the cost of per unit length [3] is as follows:

$$C_{ij} = (\alpha + BD_{ij}^{A}) \quad (1)$$

In Formula (1), A, B and α is empirical coefficient which can be checked from the design specification. The pipe length of i-j section is as follows:

$$\ell_{i.j} = [(x_1^{(i)} - x_1^{(j)})^2 + (x_2^{(i)} - x_2^{(j)})^2 + (x_3^{(i)} - x_3^{(j)})^2]^{\frac{1}{2}} = [(x_1^{(ij)})^2 + (x_2^{(ij)})^2 + (x_3^{(ij)})^2]^{\frac{1}{2}} \quad (2)$$

In Formula (2) $x_K^{(ij)} = x_k^{(i)} - x_k^{(j)}; x_k^{(i)}, x_k^{(j)}$

$(i, j = 0,1,\cdots N; l = 1,2,3)$

2.2 Design variable

The optimal design variables of sewer pipe are the coordinates of the survey points $x_1^{(m)}, x_2^{(m)}, x_3^{(m)} (m = 0,1,\cdots,N)$, the diameter of pipe D_{ij}, the material density ρ_{ij}, the pipe flow q_{ij}, the flow velocity V_{ij}, the length of pipe l_{ij} and so on.

2.3 Constraint condition

2.3.1 Constraints of dynamic equilibrium

① At a survey point, the sewage inflow and outflow should meet balance:

$$Q_i = \sum_{j=1}^{N_1} q_{ij} + q_i \quad (i = 1,2,\cdots,N_1) \qquad (3)$$

To sewer networks except the total survey point, $q_i = 0$.

② To a pipe section, its flow energy should meet balance:

$$x_3^{(i,j)} = k\ell_{ij}^n / D_{ij}^m \quad (i, j = 0,1,2,\cdots,\mathrm{N}_1) \qquad (4)$$

In formula (4), k, n and m are coefficient, they can be checked from the municipal engineering evaluation indicators.

2.3.2 Constraint of variables

The scope of variables should be determined:

$$\left.\begin{array}{l} q_{ij}^{(0)} \le q_{ij} \le \overline{q}_{ij}^{(0)}; \ell_{ij}^{(0)} \le \ell_{ij} \le \overline{\ell}_{ij}^{(0)} \\ D_{ij}^{(0)} \le D_{ij} \le \overline{D}_{ij}^{(0)}; Q_j^{(0)} \le Q_i \le \overline{Q}_i^{(0)} \end{array}\right\} \qquad (5)$$

In formula (5), the superscripts (0) mean the setting value of design.

2.3.3 Requirement of Hydraulics and design specification

1) $V_{\min} \le V_i \le V_{\max}$, $V_{\min}$ is the minimum static flow velocity and $V_{\max}$ is the maximal free flow velocity, V_i is the flow velocity.

2) Economic embedded depth of piping systems: $x_{3\min} \le x_3^{(i)} \le x_{3\max}$, $x_{3\min}$ is the minimum soil covering layer, $x_{3\max}$ is the maximal embedded depth.

3) The diameter of downstream is not less than that of upstream.

4) The elevation of the top of downstream pipe is not higher than that of upstream pipe.

2.4 Systematic and conventional mathematical model

The best aim of the design of sewer networks is low cost, the cost of sewer networks is:

$$C_y = \sum_{i=1}^{N} (\alpha + BD_i^A)\ell_i \qquad (6)$$

According to formula (2) and (3), formula (6) leads to

$$C_y = C_y(x_1^{(0)} x_2^{(0)} x_3^{(0)}; \cdots x_1^{(i)} x_2^{(i)} x_3^{(i)}; \cdots; x_1^{(n)} x_2^{(n)} x_3^{(n)}; q_{ij})$$

$$= \sum_{i=1}^{N} (\alpha + B[2(\frac{q_{ij}}{\pi(2gx_3^{(i,j)} + V_i^2)^{1/2}})^{\frac{1}{2}}]^A).$$

$$[(x_1^{(i,j)})^2 + (x_2^{(i,j)})^2 + (x_3^{(i,j)})^2]^{1/2}$$

$$= \sum_{i=1}^{N} (\alpha + B(2(\frac{q_{ij}}{\pi(2g(x_3^{(i)} - x_3^{(j)}) + V_i^2)^{1/2}})^{1/2})^A).$$

$$((x_1^{(i)} - x_1^{(j)})^2 + (x_2^{(i)} - x_2^{(j)})^2 + (x_3^{(i)} - x_3^{(j)})^2)^{1/2}$$

(7)

3 Non-conventional mathematical models

According to [2], the non-conventional mathematical models can be established as:

$$\exists X = \left\{ \begin{array}{l} x_1^{(0)}, x_2^{(0)}, x_3^{(0)}; \cdots; x_1^{(i)}, x_2^{(i)}, x_3^{(i)}; \cdots; \\ x_1^{(n)}, x_2^{(n)}, x_3^{(n)} \end{array} \right\}^T$$

Let $F(X)=\sum_{j=0}^{n}[C^{(1)}X^{(1)}+C^{(2)}X^{(2)}+C^{(3)}(X^{(3)}-h)] = \sum_{j=0}^{n} CX - C^{(3)}h \rightarrow \min$ (8)

In formula (8), h is the height from a survey point to the origin of coordinate along $x^{(3)}$ direction, $X^{(3)}-h$ is the embedded depth and $C^{(3)}h$ is constant S.T. (for sewer networks)

Dynamical equilibrium:

$$Q_i = \sum_{\substack{j=1 \\ (i\neq j)}}^{N1} q_{ij} + q_i \ \text{(i=1,2……N1)} \quad (9a)$$

Relationship among V, q and D:

$$\left.\begin{aligned} V_{\max} \ge V_j \ge V_{\min} \\ q_{ij,\max} \le \pi D_{ij,\min}^2 V_j \\ D_{\max} \ge D_{ij} \ge D_{,\min} \end{aligned}\right\} \quad (9b)$$

Topography-Geomorphic:

$$a_k^{(i)} \le X_k^{(i)} \le b_k^{(i)} (i=0,1,\cdots N, k=1,2,3, i\neq j) \quad (9c)$$

The embedded depth:

$$x_{3\max} \le x_3^{(i)} \le x_{3\min}, \quad (9d)$$

4 Example

Here, as shown in figure 1, its background is a typical district in the south-western of Nanyang urban for optimal calculation.

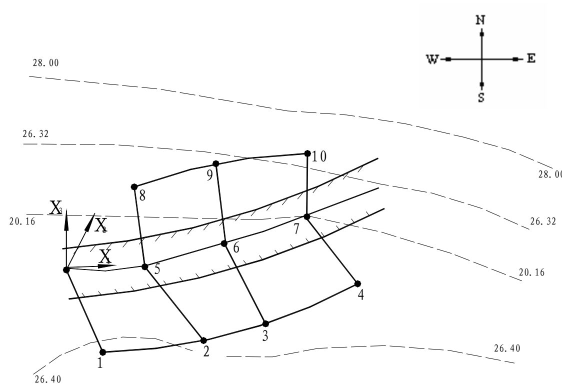


Figure 1

4.1 Given data and primary data

Given data of engineering requirements: Empirical coefficient—α=0, B=161.13, A=1.53.

inflow and outflow Q^{q_i} (m^3/s): Q_o=0.41, q_o=12.22；

Q_1=0.7, q_1=0 ； Q_2=0.17, q_2=0 ； Q_3=0.1, q_3=0; Q_4=0.15, q_4=0 ； Q_5=0.35, q_5=0 ； Q_6=0.3, q_6=0; Q_7=0.02, q_7=0 ； Q_8=0.8, q_8=0 ； Q_9=10.0, q_9=0; Q_{10}=0.05, q_{10}=0

Primary data: the coordinate of survey points $(x_1^{0(i)}, x_2^{0(i)}, x_3^{0(i)})(m)$

0(0，0，0，); 1(150，-100，10.5);
2(200，-2，1.5); 3(280，-89，11.5); 4(366,-104,12);
5 (58,-12,12.5);
6 (134,-4,5.8), 7 (201,-1,7.6);
8 (39,94,10.4); 9 (64,102,11);
10 (129,106,10.6).

Flow of i-j pipe section q_{ij}(m^3/s)(i→j)：

$q_{1,0}^{(0)} = 0.0997, q_{5,0}^{(0)} = 0.0903, q_{2,1}^{(0)} = 0.0297; q_{3,2}^{(00} = 0.1463,$

$q_{2,5}^{(0)} = 0.0006, q_{4,3}^{(0)} = 0.0111; q_{3,6}^{(0)} = 0.0426,$

$q_{7,4}^{(0)} = 0.1389, q_{6,5}^{(0)} = 0.2361; q_{7,6}^{(0)} = 0.2967,$

$q_{8,5}^{(0)} = 0.0176, q_{9,8}^{(0)} = 0.1968; q_{7,10}^{(2)} = 0.0444,$

$q_{9,10}^{(0)} = 0.056, q_{9,6}^{(0)} = 0.1968$

It is noteworthy that: in some of the pipeline network system q_{ij} chooses the maximal value $q_{ij,\max}$， q_{ij} of above data is given ones. In order to be simple and test the reliability and versatility of optimization, q_{ij} should be taken as given data of engineering requirement.

4.2 Conventional optimal calculation

4.2.1 Specific form of constraint conditions

Flow constraint(i→j):

$q_{ij}=q_{ij}^{(0)}$ (Values of i and j, e.g. primary data) (10)

Energy constraint: $\left|x_3^{(i,j)}\right| = k\ell_{ij}\, q_{ij}^{n} / D_{ij}^{m}$ (k, n and m are checked from municipal engineering indicators) (11)

Geometric constraint:

$$50 \le \ell_{ij} \le 300 \quad (12)$$

Topography-Geomorphic (survey points) constraint:

$$\left|x_K^{(i)} - x_K^{o(i)}\right| \le 5 \text{ (i=1~4,8 ~ 10,}k\text{=1,2,3)}$$

$$\left|x_k^{(i)} - x_k^{o(i)}\right| \le 2 \text{ (i=1~0,5~7;}k\text{=1,2,3)} \quad (13)$$

4.2.2 Optimal design calculation

Here, direct mesh method is used for optimal design calculation.

1. Change of variable:

Take $t_k^{(i)} = x_k^{(i)} - x_k^{(i)}$ (i=0,1,…,10;k=1,2,3) (14)

as new variable, the variable scope is as follows:

$$-5 \le t_k^{(i)} \le 5(i=1\sim 4,8\sim 10; k=1,2,3) \quad (15)$$

It can be written reunification as:

$$a_k^{(i)} \le t_k^{(i)} \le b_k^{(i)} (i=0,1,\cdots,10; k=1,2,3)$$

$$-2 \le t_k^{(i)} \le 2(i=0,5\sim 7; k=1,2,3) \quad (16)$$

If $t_k^{(i)}$ is given, $x_k^{(i)}$ will be obtained by formula (14).

2. Discrete grid: First, use coarse grid to explore possible minimum region, take

$$\varepsilon_k^{(i)} = \frac{b_k^{(i)} - a_k^{(i)}}{S} (i=0,1,\cdots,10; k=1,2,3) \quad (17)$$

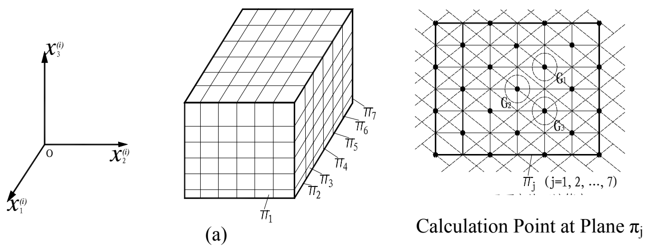
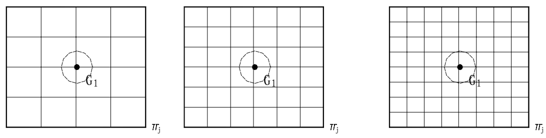


Figure 2: Discrete grid

as grid spacing. Nodes [4-5] along the diagonal are accessed to calculate the comparing objective functions and determine the relative minimum region. S is a positive even integer, if S =6, it is shown in figure 2. When S=10, the minimum region estimated are extremely reliable.

Secondly, if G_1 is the minimum point (the point of the figure 3), fractionize the region around the point, as illustrated in figure 3. If there are two minimum point G_1 and G_2 (namely the difference between $C_y(G_1)$and $C_y(G_2)$is less than 5%), fractionize the region around the G_1 and G_2, as illustrated in figure 4.

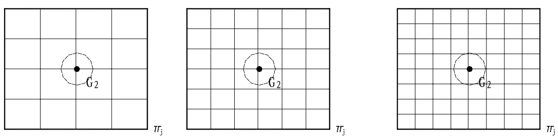


halved refinement trisected refinement quarterly refinement

(on plane j=5,7,9)

Figure 3 uniform refinements

For the refinement grid of π_j plane, there is a correspondence to establish branch mesh(Figure 2(a)).



halved refinement trisected refinement quarterly refinement

(on plane j=5,7,9)

Figure 4 G_2 uniform refinements

3. Optimal calculation

According to figure 2, calculable nodes are

$$S_n\left[\frac{a_1+a_2}{2}n - a_n\right]\times(S+1) = [(a_1+a_n)n - a_n]\times 7$$
$$=175$$

In formula, S=6, n=4, a1=1, an=S+1=7. There are total 175 schemes for comparison; the whole optimization process is the sum of the process of each program calculation. Specific programming will not give unnecessary details here.

4. Results: the best program is the program corresponding to G_3 point in π_3 plane:

$$X^* = \begin{Bmatrix} x_1^{*(0)}, x_2^{*(0)}, x_3^{*(0)}; \cdots; x_1^{*(i)}, x_2^{*(i)}, x_3^{*(i)}; \cdots; \\ x_1^{*(10)}, x_2^{*(10)}, x_3^{*(10)} \end{Bmatrix}^T,$$

In formula,

$$x_2^{*(i)} = 4\varepsilon_2^{(i)} + x_2^{0(i)}; x_k^{*(i)} = 2\varepsilon_k^{(i)} + x_k^{0(i)}$$

$$(i = 0,1,\cdots,10; k = 1,2,3); \varepsilon_k^{(i)} = \frac{1}{6}(b_k^{(i)} - a_k^{(i)})$$

$$b_k^{(i)} = 5, a_k^{(i)} = -5 (i = 0,1 \sim 4,8 \sim 10; k = 1,2,3);$$

$$b_k^{(i)} = 2, q_k^{(i)} = -2 (i = 0,5 \sim 7; k = 1,2,3)$$

$$(x_1^{0(i)}, x_2^{0(i)}, x_3^{0(i)}) = f_1, \quad (i = 0,1,\cdots,10)$$

The minimum cost is 82.1% of the cost of original program (corresponding $\varepsilon_k^{(i)} = 0$), optimized result reduces cost by 17.9%.

4.3 Linear programming calculation based on new economic parameter

The requirement data is same to the given data 3.1, the constraint condition is same to formula (9), The process of optimal design is same to 4.2, only the result is the program corresponding to G_2 point in π_4 plane:

$$X^* = \begin{Bmatrix} x_1^{*(0)}, x_2^{*(0)}, x_3^{*(0)}; \cdots; x_1^{*(i)}, x_2^{*(i)}, x_3^{*(i)}; \\ \cdots; x_1^{*(10)}, x_2^{*(10)}, x_3^{*(10)} \end{Bmatrix}^T,$$

In formula,

$$x_k^{*(i)} = 3\varepsilon_k^{(i)} + x_k^{0(i)}; (i = 0,1,\cdots,10; k = 1,2,3);$$

$$\varepsilon_k^{(i)} = \frac{1}{6}(b_k^{(i)} - a_k^{(i)}), b_k^{(i)} = 5, a_k^{(i)} = -5$$

$$(i = 0,1 \sim 4,8 \sim 10; k = 1,2,3);$$

$$b_k^{(i)} = 2, q_k^{(i)} = -2 (i = 0,5 \sim 7; k = 1,2,3);$$

$$(x_1^{0(i)}, x_2^{0(i)}, x_3^{0(i)}) = f, (i = 0,1,\cdots,10)$$

The minimum cost is 76.3% of the cost of original program (corresponding $\varepsilon_k^{(i)} = 0$), optimized result reduces cost by 23.7%.

5 Conclusion

1. These two results prove that the corresponding mathematical model is fit to the engineering reality of sewer networks, optimal method is reliable, and the optimal effect is quite obvious.

2. The original program($\varepsilon_k^{(i)} = 0$) is from the traditional design experience of sewer system, optimal result reduces the cost of sewer networks significantly.

3. These results show that the nonlinear programming model of the new economic parameter optimal design of sewer networks based on topography-geomorphic has greater improvement than the traditional optimal design method.

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