Modeling study on a forward recursive construction approach of dynamical equations of mechanism systems with flexible articulated joints

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Abstract: This paper discusses modeling approach of a forward recursive construction of mechanical system with the flexible hinge joints. A forward recursive construction approach for dynamical equations of articulated flexible multibody systems with open loop construction are presented in the Wittenberg interconnected matrix systems. The motion of the flexible multibody systems with articulated joints has been resolved using the floating reference frames which are defined on the rigid states of flexible bodies. Vibration modes of components and model coordinates are used to express the deformation of the flexible multibody systems with articulated joints. Forward recursive construction approaches are used in kinetic analyzing. Kinetic equations and dynamics equations of the flexible body system with roll hinge and slide hinge are derived. The equations are fairly direct-vision and simple in resolution of the motion, straddling of elastic deformation of component and description of construction of system.

Key words: Flexible articulated joints; A forward recursive construction approach ; Dynamical equations of mechanism systems; Dynamics Analysis

1 Introduce

Dynamics model for Space complex mechanical system is controlled multi-body system. Taking into account the modern structure of the growing and high-speed operation, their components and joints must be considered as flexible body. It is the basis for Digital simulation of mechanical system dynamics that facilitate the mathematical model of flexible system with function of formality and computer automatically. In theory, It is not too difficult that derived a mathematical model of dispersion system according to the basic principles of mechanics, is more complicated.

In recent decades, multibody system dynamics rapid development as the fastest growing area of applied mechanics. On one hand, multi-body system is more and more used in the actual system model as such as robots, spatial structure and dynamics of biological systems, on the other hand, the multi-body system dynamics research activities have contributed to a number of sub-areas of research. Most interested research field of multi-body dynamics is incorporated flexible effect into dynamic control equation[1-3]. For flexible multi-body system, in particular the movement system consisting of the small deformation objects , most of that use the relative described method, and introduce floating coordinates to decompose movement of systematic components, such as all nodes tangent coordinates, secant coordinates, or Trsserand coordinate and Bucken coordinates and so on[4]. Usually ,using the finite element method, the modal components method and other methods discrete elastic body relatively floating coordinates.Modal components method is based on dynamic sub-structure method of vibration analysis field of the modern structure[5], which greatly reduced generalized coordinates of kinetic equation, and can use that static modal recovery mode truncation error,and improve compute accuracy [6, 7]. when there exist a large component deformation, it would consider using the finite deformation theory to build system model [8]. On the multi-body system dynamics analysis , Currently, it has Kane equation, Roberson / Wittenburg system, variational methods, coordinates of the largest number , rotation matrix method and a forward recursive method of dynamic equation. The equation can be derived by Largrange equation, virtual work principle, principle of virtual power, Gibbs-Appells equations and Newton-Euler equations, For the ways of above ,it is existed controversy which is best [1,8]. On ways of describing multi-body structure, it have the correlative matrix of Wittenburg , access matrix [9] ,Huston's array of internal joint rigid [10] ,and Kim and Haug's recursive equation [11] , Shabana's recursive projection algorithm [12, 13],etc. For systems of non-trees , or constraint multi-body system ,the method of handling the constraint equations have the pseudo - inverse method, orthogonal-law, singular value decomposition and zero / tangent space law etc. It is very difficult to assess the merits of these methods of multi-body system dynamics analysis,because every way has each advantage and fault.It need continued study and exploration.

The paper [14] decompose components campaign by the rigid diaphragm floating coordinates of suitable description of the elastic deformation,and use components modal mothed to discrete elastic deformation, and establish hinge joints flexible multi-body system dynamics equations of the R / W system [9,14] starting from D'Alembert's Principle. Equation extends to the non-tree and constraints system. The author uses the singular value decomposition polycondensation to handle closure and constraint conditions, reduce the number of variables of the equation.

The paper[15-16] presents a modeling of joint hinge of the dynamics of flexible multi-body system by CAD, which is hinge joints recursive modeling method. It decompose components campaign by the rigid diaphragm floating coordinates of suitable description of the elastic deformation,and use components modal mothed to discrete elastic deformation, and stylized group set to establish dynamic equation that is expressed by hinge's relative coordinates and modal coordinates in flexible multi-body system with Chain, the tree and the closed-loop during the forward recursive process of kinematics. It has the advantages of the methods the traditional absolute coordinates and hinge's relative coordinates and overcome their deficiencies.

In the paper a forward recursive construction approach for dynamical equations of articulated flexible multibody systems with open loop construction are presented in the Wittenberg interconnected matrix systems.The motion of the flexible multibody systems with articulated joints has been resolved using the floating reference frames which are defined on the rigid states of flexible bodies,.Vibration modes of components and model coordinates are used to express the deformation of the flexible multibody systems with articulated joints. Because of independence of forward recursive construction approaches in the chain, the approach is very easy and has been developed as a method of parallel computing.

2 kinematics of Rotating Hinge system

2.1 Description of the object deformation

Considering the i body of flexible system in the small elastic deformation,its elastic deformation u_P

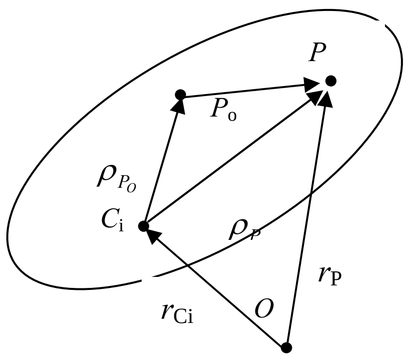

Figure 1 elastic body

can be expressed by elastic modal ψ_i and modal coordinates vector η_i thought using the finite element method as following: $u_P = \Psi\eta_i$ (2. 1)

Its displacement and angular are write by following forms: $u_P^d = \Psi_i^d \eta_i$; $u_P^r = \Psi_i^r \eta_i$ （2. 2）

Where o-e_o is system coordinate, $e_o = [e_1^o, e_2^o, e_3^o]^T$ is orthogonal coordinate are fixed at it, $C_i - e_i$ is floating coordinate,action at the centre of mass C_i, $e_i = [e_1^i, e^i, e_3^i]^T$ is orthogonal coordinate fixed.P_0 is an arbitrary point at elastic body.The location after deformationis at point P. P—e^P is element coordinate,is fixed at point P, its coordinate vecter is $e_o = [e_1^P, e_2^P, e_3^P]^T$. derivative of the formulation (2.2) can be writed as following:

$$\dot{u}_P^d = \Psi_i^d \dot{\eta}_i \quad ; \quad \dot{u}_P^r = \Psi_i^r \dot{\eta}_i \qquad (2.3)$$

As shown in figure 1，displacement vector of an arbitrary point P at elastic body is expressed as:

$$r_P = r_{C_i} + \rho_{P_o} + u_P^d \qquad \{2.4\}$$

2.1 Movement description of elastic-body

According to formulation of displacement vector of the finite element nodes P at elastic body B_i,and considering formula (2.2),it has the velocity and angular velocity of point P in the inertial coordinate presented by following forms:

$$V^P = V^{C_i} + \omega^{C_i} \times (\rho_{P_O} + u_P^d) + \psi_i^d \dot{\eta}_i \qquad (2.5)$$

$$\omega^P = \omega^{C_i} + \psi_i^r \dot{\eta}_i \qquad (2.6)$$

That V^{C_i} and ω^{C_i} are respectively the centre of mass's speed and relative centoid's ration of elastomer B_i. Acceleration and angular acceleration of node p of the elastomer B_i as following:

$$a^P = a^{C_i} + \alpha^{C_i} \times (\rho_{P_O} + u_p^d) + \omega^{C_i} \times (\rho_{P_O} + u_p^d) + 2\omega^{C_i} \times \psi_i^d \dot{\alpha}_i + \psi_i^r \ddot{\eta}_i \qquad (2.7)$$

$$\alpha^P = \alpha^{C_i} + \omega^{C_i} \times \psi_i^r \dot{\eta}_i + \psi_i^r \ddot{\eta}_i \qquad (2.8)$$

That a^{C_i} and α^{C_i} are respectively speed of the centre of mass and relative ration at the centre of mas of elastic-body B_i.

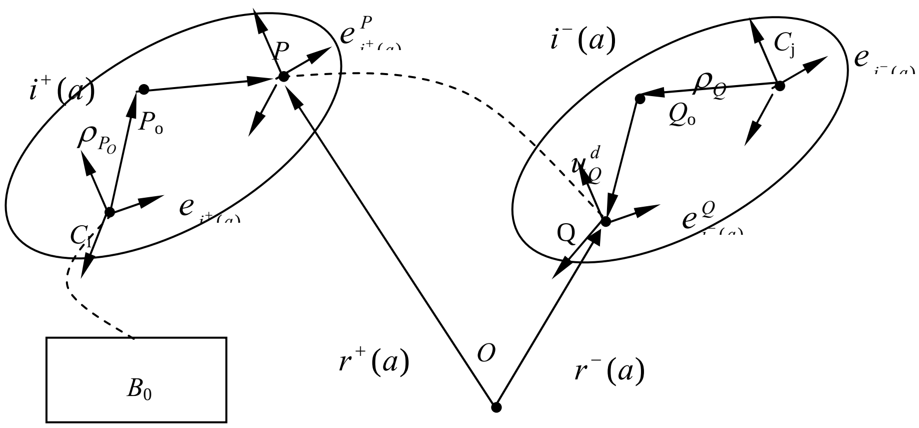

Figure2 hinge A and adjacent objects

2.2 The kinematics recurrence relations of joint rotation Hinge tree system

1.relative movement of adjacent objects

Considering one pair of adjacent objects B_i and B_j of flexible multi-body system, Assuming B_i is the internal objects of B_j, joint hinge of the two objects are recorded as a. For convenience, joint object B_i of two adjacent objects is recorded i^+（a）and the object B_j is recorded i^-（a）, and i^+（a）and i^-（a）is recorded incorporately $i^\pm$（a）. For rotational hinge, the joint point P and Q of two objects is the coincidence. Now hinge - of two objectst link firmly a vector-based P（Q）— $e^{P(Q)}_{i^\pm(a)}$， The vector connected to solid center is expressed as $e_{i^{\pm}(a)}$. As shown in figure 2, elastic rotational and angular velocity of vector-based relative respective objects are respectively following as:

$$e^{P(Q)}_{i^{\pm}(a)} = G'_{i^{\pm}(a)a} G_{i^{\pm}(a)a} e_{i^{\pm}(a)} ; \Omega_{i^{\pm}(a)a} = \psi^{T}_{i^{\pm}(a)a} \dot{\eta}_{i^{\pm}(a)} \quad (2.9)$$

$G'_{i^{\pm}(a)a}$ and $G_{i^{\pm}(a)a}$ are cosine matrix of elastic rotation direction and the cosine matrix of initial direction $e^{P(Q)}_{i^{\pm}(a)}$ and $e_{i^{\pm}(a)}$ of vector-based,. By using angular weight of hinge joints a , $G'_{i^{\pm}(a)a}$ is expressed as :

$$\mathbf{G}_{i^{\pm}(a)a} = \begin{bmatrix} c\varepsilon_2 c\varepsilon_3 & c\varepsilon_2 s\varepsilon_3 & -s\varepsilon_2 \\ s\varepsilon_1 s\varepsilon_2 c\varepsilon_3 - c\varepsilon_1 s\varepsilon_3 & s\varepsilon_1 s\varepsilon_2 s\varepsilon_3 + c\varepsilon_1 c\varepsilon_3 & s\varepsilon_1 c\varepsilon_2 \\ c\varepsilon_1 s\varepsilon_2 c\varepsilon_3 + s\varepsilon_1 s\varepsilon_3 & c\varepsilon_1 s\varepsilon_2 s\varepsilon_3 - s\varepsilon_1 c\varepsilon_3 & c\varepsilon_1 c\varepsilon_2 \end{bmatrix}_{i\pm(a)} \quad (2.10)$$

c and s of formula (2.10) represent sin and cos respectively. Considering that deformation of body is small elastic deformation, so $\sin\varepsilon_i = \varepsilon_i$, $\cos\varepsilon_i = 1$.formular (2.10) can be written as following forms:

$$\mathbf{G}'_{i^{\pm}(a)a} = \begin{bmatrix} 1 & \varepsilon_3 & -\varepsilon_2 \\ \varepsilon_1\varepsilon_2 - \varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1 \\ \varepsilon + \varepsilon_1\varepsilon_3 & \varepsilon_2\varepsilon_3 - \varepsilon_1 & 1 \end{bmatrix}_{i\pm(a)}$$

The relations of relative rotation between two ends of hinge a (the rigid relative movement between adjacent objects) can be given by kinematics. The rigid rotation for B_j (i-(a)) objects to objects B_i (i+(a)) is the relative rotation relations between two ends of hinge a, which is given by kinematics. The rigid rotation for B_j (i-(a)) objects to objects B_i (i+ (a)) is following form:

$$e^{Q}_{i^-(a)} = G_a e^{P}_{i^+(a)} \quad ; \Omega_a = p_a^T \dot{q}_a \qquad (2.11)$$

Where G_a is direction cosine matrix between two basis vector $e^{Q}_{i^-(a)}$ and $e^{P}_{i^+(a)}$. q_a is generalized coordinates of the hinge a. Parameters p_a is expression basis vector array of the hinge a at basis vector $e^{Q}_{i^-(a)}$ and $e^{P}_{i^+(a)}$.

Kinematics and geometric relationships of two adjacent objects is written as :

$$\omega_{i^-(a)} - \omega_{i^+(a)} = \Omega_{i^+(a)a} - \Omega_{i^-(a)a} + \Omega_a \quad a=1,2,\ldots,n \quad (2.12)$$

$$r_{i^-(a)} = r_{i^+(a)} \; or \quad r_{C_{i^+(a)}} + C_{C_{i^+(a)}} = r_{C_{i^-(a)}} + C_{C_{i^-(a)}} \qquad (2.13)$$

Where $r_{i^+(a)}$ and $r_{i^-(a)}$ is displacement vecter of hinge a of two adjacent objects i^+(a) and i^-(a), $C_{C_{i^+(a)}}$ and $C_{C_{i^-(a)}}$ present displacement vecter from the centre of mass $C_{i^+(a)}$ and $C_{i^-(a)}$ of two adjacent objects i^+ (a) and i^- (a) to the hinge a. As nodes of the hinge point a is a coincidence, displacement vecter of hinge point a of two adjacent objects i^+ (a) and i^- (a) is equal.

2. The recursive ralations of relative movement of objects

1) absolute angular velocity and absolute angular acceleration of body

Now considering absolute angular velocity and absolute angular acceleration of arbitrary body $B_{i^-(a)}$ of the system in inertial reference coordinate. absolute angular velocity ω_i of arbitrary body $B_{i^-(a)}$ of the system in inertial reference coordinate is sum of angular velocity ω_0 of the body B_0 and relative angular velocity of the pair of adjacent objects in the route along the body B_0 and the body $B_{i^-(a)}$ .Introduction of graph theory method, consider Pathway matric T_{ji} in the route between the body B_0 and the body $B_{i^-(a)}$,then the absolute angular velocity of arbitrary body $B_{i^-(a)}$ relative to the inertial reference coordinate system can be Written as following forms:

$$\omega_i = \omega_0 - \sum_{j=1}^{n} T_{ji} (\omega_{i^{\pm}(a)} + \Omega_{i^-(a)a} - \Omega_{i^+(a)a} - \Omega_a) \quad (j=1,2,\ldots,n)$$

Where : $\mathbf{\Omega}_{j^+(a)a}$ 、 $\mathbf{\Omega}_{j^-(a)a}$ and $\mathbf{\Omega}_{ja}$ can be written by formula（2.12）and （2.15）as following：

$$\mathbf{\Omega}_{j^-(a)a} = \boldsymbol{\psi}^r_{j^-(a)a}\dot{\eta}_j\,;\,\mathbf{\Omega}_{aj} = p^T_{aj}\dot{q}_{aj}$$

Therefore the absolute angular velocity of the arbitrary objects B_i relative to the inertial reference system can be written as following formula:

$$\omega_i = \omega_0 - \sum_{j=1}^{n} T_{ji}(\omega_{i^\pm(a)} + \psi^r_{i^+(a)a}\dot{\eta}_{ia} - \psi^r_{i^-(a)a}\dot{\eta}_{ia} - p^T_{ia}\dot{q}_{ia})$$

（j=1,2,…,n） （2.14）

Thinking about formula（2.11）,set the angular velocity of rigid movement Ω_{ja} of object B_j （i^-（a）） to object B_i（i^+（a））, Definition the partial derivative $\hat{\Omega}_{ja}$ that express Ω_{ja} relative of its Inscribed object,it is called relative angular acceleration of object i^-（a） to object i^+（a）.

$$\hat{\Omega}_{ja} = p^T_{ja}\ddot{q}_{ja} + \frac{\partial p^T_{ja}}{\partial q_{ra}}\dot{q}_{ra}\dot{q}_{ja} = p^T_{ja}\ddot{q}_{ja} + w_i$$

（j=1,2,…,n） （2.15）

Which w_i is definited by following formula:

$$w_i = \frac{\partial p^T_{ja}}{\partial q_{ra}}\dot{q}_{ra}\dot{q}_{ja} \qquad \text{（j,r=1,2,…,n）} \qquad \text{（2.16）}$$

Differentiating equations（2.14）with respect to time t yields the angular acceleration of the body B_i.

$$\dot{\omega}_i = \dot{\omega}_0 - \sum_{j=1}^{n} T_{ji}(\dot{\Omega}_{j^+(a)} - \dot{\Omega}_{j^-(a)a} + p^T_{ia}\ddot{q}_{ia} + w_i + \omega_{ja}\times\Omega_{ja})$$

（j=1,2,…,n） （2.17）

Where：

$$\dot{\Omega}_{i^+(a)a} = \psi^r_{i^+(a)a}\ddot{\eta}_{ia} + \omega_{i^+(a)a}\Omega_{i^+(a)a}$$

$$\dot{\Omega}_{i^-(a)a} = \psi^r_{i^-(a)a}\ddot{\eta}_{ia} + \omega_{i^-(a)a}\Omega_{i^-(a)a}$$

$\omega_{j(a)}$ is the rotation angular velocity of the fixed coordinate system of the hinge joints a.

Formula (2.17) can de expanded as following forms：

$$\dot{\omega}_i = \dot{\omega}_0 - \sum_{j=1}^{n} T_{ji}(\psi^r_{i^+(a)a}\ddot{\eta}_{ia} + \omega_{i^+(a)a}\Omega_{i^+(a)a} - \psi^r_{i^-(a)a}\ddot{\eta}_{ia} - \omega_{i^-(a)a}\Omega_{i^-(a)a}$$

$$+ p^T_{ia}\ddot{q}_{ia} + w_i + \omega_{ja}\times\Omega_{ja}) \quad \text{（j=1,2,…,n）} \qquad \text{（2.18）}$$

Set： $\Omega_a = [\Omega_{1a} \ \cdots \ \Omega_{na}]^T$, $w = [w_1 \ \cdots \ w_n]^T$

$p_a = diag[p_{1a} \ \cdots \ p_{na}]$, $q_a = [q_{1a} \ \cdots \ q_{na}]^T$

$\eta_a = [\eta_1 \ \cdots \ \eta_n]^T$ （2.19）

Formula（2.14）and（2.18）being written into matric forms：

$$\omega_i = \omega_0 I_n - S^r T^T \dot{\eta}_a - t^T f\dot{q}_a \quad \text{（i=1,2,…,n）（2. 20）}$$

$$\dot{\omega}_i = \dot{\omega}_0 I_n - S^r T^T \ddot{\eta}_a - T^T f\ddot{q}_a - T^T\{\xi_i\} - T^T w$$

（i=1,2,…,n） （2.21）

Which：

$$S^r = S_{is}\psi^r_{is}\,,\quad f = diag[f_1 \ \cdot \ f_n]\,,\quad f_i = \omega_{i(ja)} \times p^T_{ja}$$

$$\xi_i = \omega_{j^+(a)a}\Omega_{j^+(a)a} - \omega_{j^-(a)a}\Omega_{j^-(a)a} + \omega_{i(ja)}\times\Omega_{ja}$$

（j=1,2,…,n） （2.22）

2) the the centre of mass velocity and acceleration of the body

Consider the distribution of jionts on objects : within the trees system, all joints Correlated with arbitrary object B_0 , Only with a special hinge become internal linkiage as Connectivity with the object B_0 .When use Rules Labeling, internal hinge is same with the serial number of the object, Credited O_i (figure 7.3).The others joints connencted with extranal object of the body B_i is called extranal hinge.

The vecter d_{ij} of Inscribed joints from the body $B_{i^+(a)}$ to he body $B_{i^-(a)}$ is written as form:

$$d_{ija} = -c_{ii} + c_{ij} - c_{jj} \qquad \text{（2.23）}$$

or: $d_{ija} = -(\rho_{O_i} + u^d_{O_i}) + (\rho_{P_0} + u^d_{P_0}) - (\rho_{Q_0} + u^d_{Q_0})$

Which $\rho_{O_i} = c^0_{ii}$, $\rho_{P_i} = c^0_{ij}$, $\rho_{Q_i} = c^0_{jj}$, c^0_{ii} , c^0_{ij} , c^0_{jj} is rigid body hinge vector of the hinge linkiage piont.So

$$d_{ija} = (-c^0_{ii} + c^0_{ij} - c^0_{jj}) + (-u^d_{O_i} + u^d_{P_0} - u^d_{Q_0}) \quad \text{（2.24）}$$

Set r_0 is displaecment vecter of the hinge O_1,then the vecter r_j the centre of mass of arbitrary body Relative hinge O_0 is the vecter sum of all pathway matric d_{kia} （k=1,2,…,n）from object B_0 to object $B_{i^-(a)}$.

$$r_i = \sum_{k=1}^{n} d_{kia} + r_0 + c_0 \quad （c_0 = c_{is}\big|_{i=0}） \quad （2.25）$$

$$r_i = \sum_{k=1}^{n} (-c_{ii} + c_{ij} - c_{jj}) + r_0 + c_0 \quad （2.26）$$

Differentiating equations（2.26）with respect to time t yields:

$$\dot{r}_i = \sum_{k-1}^{n} [-(\dot{\rho}_{O_{i0}} + \dot{u}^d_{Oi}) + (\dot{\rho}_{P_0} + \dot{u}^d_p) - (\dot{\rho}_{Q_0} + \dot{u}^d_Q)]$$

$$+ \dot{r}_0 + \dot{c}_0 \quad （i=1, 2, \cdots, n）（2.27）$$

Differentiating equations（2.31）with respect to time t yields:

$$\ddot{r}_i = \sum_{k-1}^{n} [-(\ddot{\rho}_{O_{i0}} + \ddot{u}^d_{Oi}) + (\ddot{\rho}_{P_0} + \ddot{u}^d_p) - (\ddot{\rho}_{Q_0} + \ddot{u}^d_Q)] + \ddot{r}_0 + \ddot{c}_0$$

$$（i=1, 2, \cdots, n）（2.28）$$

Introduction of graph theory method, make use of pathway matric T and correlation matrixin S ,the formula （2.26）、（2.27）and（2.28）can be written by blow expressions：

$$r_i = (r_0 + c_0)1_n - T^T S^{\mathbf{r}} \quad （2.29）$$

$$\dot{r}_i = (f \times T^T S^c T^T)\dot{q}_a + (S^r \times T^T S^c T^T - T^T S^d)\dot{\eta}_a + \chi' \quad （2.30）$$

$$\ddot{r}_i = (f \times T^T S^c T^T)\ddot{q}_a + (S^r \times T^T S^c T^T - T^T S^d)\ddot{\eta}_a + \chi \quad （2.31）$$

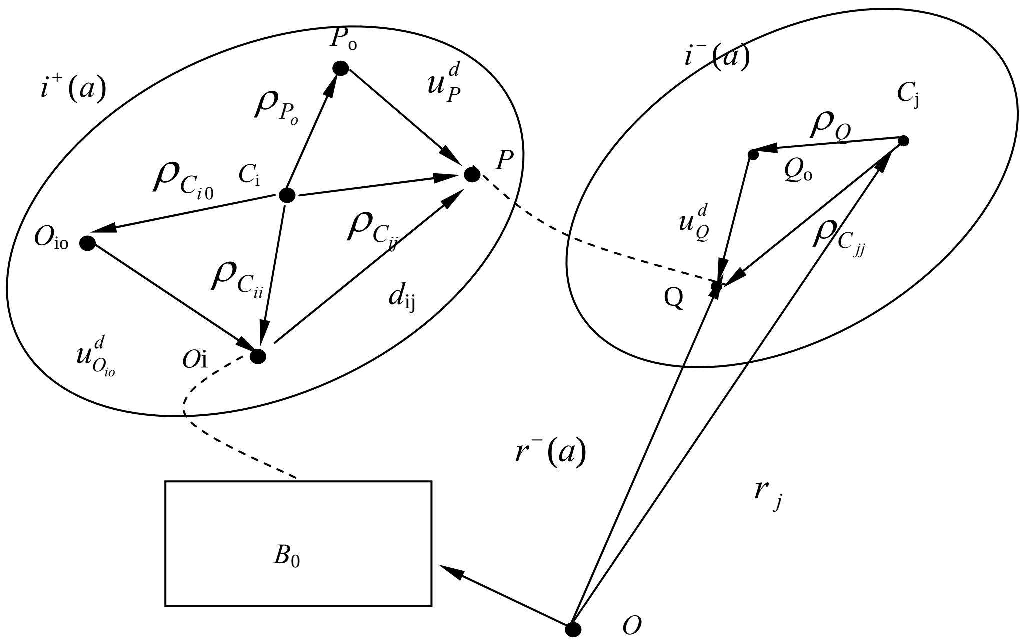

Figure2 the multbody system with arbitrary hinge linkiage

Where：

$$c_{ia} = (\rho_a + u_a)_i ; \; c_0 = c_{ia}\big|_{i=0} ; \; S^e = s_{ia}[\psi^d_{ia}]\dot{q}_a$$

$$S^c = s_{ia} c_{ia} ; \; S^d = s_{ia}[\psi^d_{ia}]^T ; \; \chi' = (\dot{r}_0 + \dot{c}_0)1_n - [\omega_0 \times (T^T S^c)]1_n$$

$$\chi = (\ddot{r}_0 + \ddot{c}_0)1_n - [\dot{\omega}_0 \times (T^T S^c)]1_n + (\xi_i \times T^T S^c T^T)$$

$$- T^T [\{\sum_{i=1}^{a} \omega_i \times (\omega_i \times s_{ia})\} - 2T^T S^e \times \omega_i] \quad （2.32）$$

3 dynamic equation of Flexible hinge system

3.1 dynamic equation of Flexible hinge system

Multi-body system can be divided into tree system and non-tree (including closed hinge) system by structure.Now building dynanics equations of the tree system. Assuming Tree systems are made up of n components B_i and n hinges O_i and naught objects whose Movement is known . Assuming that mass of B_i is m_i ,centye inertia tensor is I_i , displacement vecter of the centre of mass relatively fixed reference point is r_i, tatatoin angular velocity is ω_i. Resultant vector of the external force and resultant moment relatively the centre of mass is F_i^g, M_i^g, virtual work of internal force and elactic force is δP_i, By D'Alembert's principle,we have

$$\sum_{i=1}^{n}[\delta \dot{r}_i \bullet (m_i \ddot{r}_i - F_i^g) + \delta\omega_i \bullet (I_i \bullet \dot{\omega}_i + \varepsilon - M_i^g)] - \delta P = 0$$

（i =1,2,…,n）(3.1)

Which

$$\varepsilon_i = \omega_i \times (I_i \bullet \omega_i) \quad (i=1,2,\dots,n) \qquad (3.2)$$

virtual work of internal force and elactic force is δP_i

$$\delta P = \sum_{j=1}^{n}(\delta V_j \bullet F_j^a + \delta\Omega_j \bullet M_j^a) + \sum_{k=1}^{m}\delta V_k^e \bullet F_k^e + \delta W_i \qquad (3.3)$$

δV_j and $\delta\Omega_j$ are variation of the relative sliding speed and the relative rotational angular velocity linked external objects of the hinge O_j to the relative internal objects . F_j^a and M_j^a are Resultant vector of the active force action on internal objects and resultant moment relation to hinge O_i . δV_k^e is variation of relative velocity of the kth force element at force element hinge. F_k^e is force of external objects to internal objects.m is number of the force element. δW_i is virtual work of elactical force.

$$\delta W_i = -\{\delta a\}_i^T \Phi_i^T K_i \Phi_i \{\delta a\}_i = -\{\delta\eta\}_i^T K_i^* \{\delta\eta\}_i \qquad (3.4)$$

Which $\{\delta\eta\}_i$ is modal coordinate vecter, K_i^* is generalized stiffness matrix of the body. Substitution of formula (3.3) and (3.4) into (3.5) gives matrix equation as following:

$$\delta\dot{r} \bullet (m\ddot{r} - F^g) + \delta\boldsymbol{\omega}^T \bullet (I \bullet \dot{\boldsymbol{\omega}} + \boldsymbol{\varepsilon} - M^g) + \delta V^T \bullet F^a + \delta\Omega^T \bullet M^a) - \delta V^{eT} \bullet F^e \{\delta\eta\}^T K\{\delta\eta\} = 0 \qquad (3.5)$$

The symbol of matrix is defined as following forms:

$$m = diag[m_1 \cdots m_n], \quad I = diag[I_1 \cdots I_{\mathbf{n}}]$$

$$r = [r_1 \cdots r_n]^T, \quad \delta r = [\delta r_1 \cdots \delta r_n]^T$$

$$\omega = [\omega_1 \cdots \omega_n]^T, \quad \delta\omega = [\delta\omega_1 \cdots \delta\omega_n]^T$$

$$F^g = [F_1^g \cdots F_n^g]^T, \quad M^g = [M_1^g \cdots M_n^g]^T$$

$$\varepsilon = [\varepsilon_1 \cdots \varepsilon_n]^T, \quad \delta V = [\delta V_1 \cdots \delta V_{\mathbf{n}}]^T$$

$$\delta\Omega = [\delta\Omega_1 \cdots \delta\Omega_{\mathbf{n}}]^T, \quad F^\alpha = [F_{\mathbf{1}}^\alpha \cdots F_n^\alpha]^T$$

$$M^\alpha = [M_1^\alpha = M_n^\alpha]^T, \quad \delta V^e = [\delta V_1^e \cdots \delta V_m^e]^T$$

$$F^e = [F_1^e \cdots F_m^e]^T, \quad K = \phi^{\mathbf{T}} K^e \phi \qquad (3.6)$$

3.2 Dynanics equation of the flexible ratation joints system

Then joints of the mutlibody system is ratation hinge,set $\delta V = 0$ (3.7)

Due to the multi-body system exist kinematic constraints of the joints between objects and objects, $\delta\dot{r}$、$\delta\omega$、δV、$\delta\Omega$、δV^e in kinetic equations is not independent variational .They must be presented by variation $\delta\dot{q}$ and $\delta\dot{\eta}$ of generalized velocity.Base on formula of $\dot{r}$、ω and Ω (2.38)、(2.24) and (2.15) ,their variation can be written by following variational form:

$$\delta\dot{r}_i = (f \times TS^c T)^T \delta\dot{q} + (S^r \times TS^c T - TS^d)^T \delta\dot{\eta} \qquad (3.8)$$

$$\delta\omega_i = -S^r T^T \delta\dot{\eta} - T^T f \delta\dot{q} \qquad (3.9)$$

$$\delta\Omega_a = p^T \delta\dot{q} \qquad (3.10)$$

δV^e can be obtained by variation of the V^e [17]. Substitution of (2.33), (2.20) and (2.11) into(3.8)-(3.10) gives

$$\begin{aligned}\delta V^e &= -\delta\dot{\mathbf{r}}\ S^e - \delta\boldsymbol{\omega} \times C^{\mathbf{e}} \\ &= -\{(f \times TS^c T)^T \delta\dot{q} + (S^r \times TS^c T - TS^d)^T S^e \delta\dot{\eta} \\ &\quad + (S^r T^T \times C^e)\delta\dot{\eta} + (T^T f \times C^e)\delta\dot{q} \qquad (3.11)\end{aligned}$$

S^e and $C^{\mathbf{e}}$ respectively force elements Correlation matrix and weighted force element vecter. Substitution of (3.7)-(3.11) into(3.5) gives dynanics equation of the flexible ratation joints system

$$\begin{aligned}&\delta\dot{q}^T\{(f \times TS^c T) \bullet (m\ddot{r} - F^g) - Tf^T \bullet (I \bullet \dot{\omega} + \varepsilon \\ &- M^g) + p_a \bullet M^a) + (f \times TS^c T)S^{eT}\} + \\ &\delta\dot{\eta}^T\{(S^r \times TS^c T - TS^d) \bullet (m\ddot{r} - F^g) - S^{rT} T \\ &(I \bullet \dot{\omega} + \varepsilon - M^g) + (S^r \times TS^c T - TS^d)S^{eT} \\ &-(S^r T^T \times C^{\mathbf{e}})^{\mathbf{T}} - (T^T f \times C^{\mathbf{e}})^{\mathbf{T}} \bullet F^e - K\{\delta\eta\}\} = 0 \qquad (3.12)\end{aligned}$$

that

$$\begin{aligned}&(f \times TS^c T) \bullet (m\ddot{r} - F^g) - Tf^T \bullet (I \bullet \dot{\omega} + \varepsilon - M^g) \\ &+ p_a \bullet M^a) + (f \times TS^c T)S^{eT} = 0 \qquad (3.13)\end{aligned}$$

$$\begin{aligned}&(S^r \times TS^c T - TS^d) \bullet (m\ddot{r} - F^g) - S^{rT} T(I \bullet \dot{\omega} + \varepsilon \\ &- (S^r T^T \times C^e)^T - -M^g) + (S^r \times TS^c T - TS^d)S^{eT} \\ &(T^T f \times C^e)^{\mathbf{T}} \bullet F^e - K\{\delta\eta\}\} = 0 \qquad (3.14)\end{aligned}$$

Formula（3.13）and（3.14）is R/W equationa.

4 Conclusions

A forward recursive construction approach for dynamical equations of articulated flexible multibody systems with open loop construction are presented in the Wittenberg interconnected matrix systems. The motion of the flexible multibody systems with articulated joints has been resolved using the floating reference frames which are defined on the rigid states of flexible bodies. Vibration modes of components and model coordinates are used to express the deformation of the flexible multibody systems with articulated joints. Forward recursive construction approaches are used in kinetic analyzing. Kinetic equations and dynamics equations of the flexible body system with roll hinge and slide hinge are derived. The equations are fairly direct-vision and simple in resolution of the motion, straddling of elastic deformation of component and description of construction of system.

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