Cross section strength calculation of partial bonded prestress concrete beam on dynamic loading

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Abstract: - The degree of prestress is an important factor of the ductility of prestressed concrete structure [1] [2] [4]. The conception of degree of prestress and nominal section area of prestressed bar are introduced and the influence on ductility by the corresponded height of compression area is considered in this paper. Based on the equilibrium of prestressed concrete beam in flat bent, the calculation formula of cross section strength of partial bonded prestressed concrete beam on dynamic loading was deduced and the designs calculation example was given. This method is simple and convenient for design.

Key words: -partial bonded prestressed concrete beam; degree of prestress; strength calculation; design method

1 Introduction

With the national economy developing and the synthetic national power enhancing continuously, underground marketplace, underground route way, underground park, underground rumpus room etc are used increasingly. These underground engineering are with thin covering and great span, high using ratio and very good economic benefit during peacetime. But because of the poor ductility and brittle failure of traditional reinforced concrete under dynamic loadings, it cannot exert defenses action during wartime. Since Gulf War, it has been paid great attention to city's underground engineering construction. If the traditional reinforced concrete structure is still adopted, its cost and the structural geometrical size will greatly increase. Meanwhile the available space of construction will decrease [2] [3]. However the prestressed concrete structures possess all these characteristic: lesser section size, lower ratio of deflection-span, stronger load bearing capability, better anti-crack characteristic, higher ratio of space utilization and notable synthetic economic benefit. So the prestressed concrete structures should be considered in city's underground engineering with larger span and higher resistibility simultaneity. How to design the prestressed concrete member has become an urgent problem. The study on the design of prestressed concrete member under exploring loadings is very infrequence in both home and abroad [2]. So it is necessary to explore the design of prestressed concrete beam under dynamic loadings.

The prestressed concrete beam upon dynamic loadings is greatly influenced by the degree of pre-stress. It is necessary to consider the value of degree of prestress in design of prestressed concrete beam. Based on the equilibrium of prestressed concrete beam in flat bent and the conception of degree of prestress and nominal section area of prestressed bar, the calculation formula of cross section strength was deduced for partial bonded prestressed concrete beam on dynamic loading. The deduction of the calculation is only for rectangle section. But the design calculation of the cross section is applicable to I and T section types too [1] .

2 The basic assumption

The basic supposition of the strength calculation is as follows [1]:

1) The prestressed bars are only arranged on tensile region and not arranged on compressive region.

2) Deformation of partial banded prestressed concrete beam observes the assumption of plane section
3) The partial bonded prestressed concrete beam is all in elastic when external load is not added to the beam.
4) The stress of concrete in the beam is ignored when loadings is added on the beam.

3 The formula of strength calculation

3.1 Beam with single reinforcement

As in fig.1, b is the width of rectangle section, h is the height of rectangle section; h_p is the distance from the acting point of composite forces of prestressed bars to the top of the beam; A_p is the area of the cross section of prestressed bar; h_s is the distance of the acting point of composite forces of prestressed bars to the top of beam section; A_s is the area of non-prestressed bars in tensile region; h_0 is the distance of the acting point of composite forces of prestress and non- prestressed bars to the top of beam section; f_{cmd} is design flexural strength of concrete on dynamic loadings; x is the height of compression zone on equivalent rectangle; The stress on the section with bending moment M should meet:

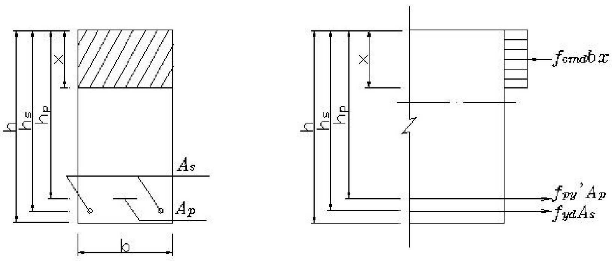

Fig.1 Guide to the size and stress of beam with single reinforcement

$$f_{cmd}bx = f_{py}A_p + f_{yd}A_s \quad (1)$$

$$M = f_{cmd}bx\left(h_0 - \frac{x}{2}\right) \quad (2)$$

Here: h_0 is the equivalent height of beam.

$$h_0 = \frac{f_{py}A_p h_p + f_{yd}A_s h_s}{f_{py}A_p + f_{yd}A_s} \quad (3)$$

A_g is defined which is the area of nominal prestressed bars.

$$A_g = \frac{A_p f_{py} + A_s f_{yd}}{f_{py}} \quad (4)$$

If λ is used to express the prestress degree values of partial banded prestressed concrete bending member under dynamic loadings.

$$\lambda = \frac{f_{py}A_p}{f_{py}A_p + f_{yd}A_s} \quad (5)$$

Eq (3) is simplified:

$$h_0 = (1-\lambda)h_s + \lambda h_p \quad (6)$$

Marked as:

$$\xi = \frac{x}{h_0} \quad , \quad \alpha = \xi\left(1 - \frac{\xi}{2}\right) \quad , \quad \nu = 1 - \frac{\xi}{2} \quad (7)$$

Eq (2) is simplified:

$$M = f_{cmd}bh_0^2\alpha$$

$$\alpha = \frac{M}{f_{cmd}bh_0^2} \quad (8)$$

It was found from Eq (7)

$$\xi = 1 - \sqrt{1-2\alpha} \quad (9)$$

$$\nu = \frac{1}{2}\left(1 + \sqrt{1-2\alpha}\right) \quad (10)$$

After ξ_b is obtained, we require $\xi \le \xi_b$ in order to ensure that the reinforced bars in bending member is proper, and meet the requirements of ductility and anti-earthquake. ξ ought to be satisfied much more rigorous condition as well. A great deal of information on dead loadings indicate that the ductility of beam is greatly influenced by the relatively height of compression region of prestressed concrete member. The design can be calculated as the design of beam with single reinforcement if ξ is smaller than or equal to 0.35. The design need recalculating for adding section area according to the design of single reinforced beam or keeping the size of section according to the design of the double reinforced beam if $\xi > \xi_b$.

Take Eq(4)into Eq(1)and Eq(2):

$$M = A_g f_{py} h_0 \left(1 - \frac{x}{h_0}\right)$$

Take Eq(7)into the above formula:

$$A_g = \frac{M}{f_{py} h_0 \nu}$$

(11)

It is found from Eq□4□and Eq□5□:

$$A_p = \lambda A_g$$

(12)

$$A_s = (1-\lambda) A_g \frac{f_{py}}{f_{yd}}$$

(13)

The limit height of compression region under dynamic loadings can be calculated by formulas:

$$\xi_b = \min(\xi_{sb}, \xi_{pb}) \qquad □14□$$

$$\xi_{sb} = \frac{h_s}{h_0} \frac{0.8}{1 + \frac{f_{yd}}{0.0033}} \qquad □15□$$

$$\xi_{pb} = \frac{h_p}{h_0} \frac{0.8}{1.6 + \frac{f_{py} - \sigma_{p0}}{0.0033 E_p}} \qquad □16□$$

Here ξ_{sb} is the nominal boundary relative height of compression region with the non-prestressed bars in post yield. ξ_{pb} is the nominal boundary relative height of compression region with the prestressed bars in post yield. σ_{p0} is the stress of prestressed bars when the concrete stress is zero where is at the acting point of composite forces of prestressed bars. σ_{pe} is initial effective stress of prestressed bars, which is usually smaller than σ_{p0}.

3.2 Double reinforced beam

When the cross section strength design is according to the above method of single reinforcement, if $\xi > \xi_b$ and remaining the size of cross section, the beam must be designed according to the design of double reinforced beam which is placed compression bars. Figure 2 is the section of the double reinforced beam. a' is the distance from the acting point of composite forces of non-prestressed bars to the top of beam section. A_s' is the section area of non-prestressed bars in compression zone. The other parameters are the same as Figure 1. The stress on this section with bending moment M should meet:

$$f_{cmd} bx = f_{py} A_p + f_{yd} A_s - f_{yd}' A_s'$$

□17□

$$M = f_{cmd} bx (h_0 - \frac{x}{2}) + f_{yd}' A_s' (h_0 - a')$$

□18□

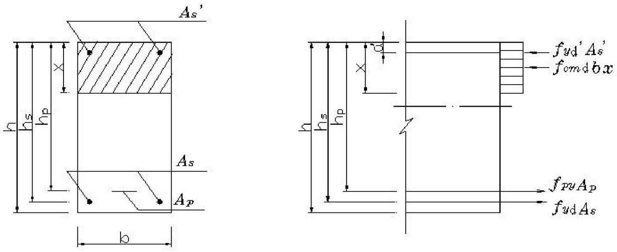

Fig. 2 Guide to the size and stress of beam with single reinforcement

The values of λ (the degree of pre-stress) of partial bonded prestressed concrete bending beam on dynamic loading can be obtained from its definition:

$$f_{py} A_p = \frac{\lambda}{1-\lambda} f_{yd} A_s$$

□19□

Take Eq□19□into Eq□17□:

$$f_{cmd} bx = \frac{f_{yd} A_s}{1-\lambda} - f_{yd}' A_s'$$

□20□

Divide A_s into A_{s1} and A_{s2}:

$$\frac{f_{yd} A_{s2}}{1-\lambda} = f_{yd}' A_s'$$

Namely: $A_{s2} = (1-\lambda) A_s' \frac{f_{yd}'}{f_{yd}}$ □21□

Then: $f_{cmd} bx = \frac{f_{yd} A_{s1}}{1-\lambda}$

Then: $A_{s1} = \frac{(1-\lambda) f_{cmd} bx}{f_{yd}}$ □22□

$$A_s = A_{s1} + A_{s2} = \frac{1-\lambda}{f_{yd}} \left(f_{cmd} bx + A_s' f_{yd}' \right)$$

□23□

It is found from Eq □19□□

$$A_p = \frac{\lambda}{f_{py}} \left(f_{cmd} bx + A_s' f_{yd}' \right)$$

□24□

It is found from Eq □18□□

$$A_s' = \frac{M - f_{cmd}bx\left(h_0 - \frac{x}{2}\right)}{f_{yd}'(h_0 - a')}$$ □25

□

In order to determine x, the conception of equivalent area of reinforced bars is introduced to define the equivalent area of reinforced bars (A):

$$A = A_s + A_s' + kA_p$$ □

26□

Here: k is the unit price ratio of the prestressed bars to non-prestressed bars.

The economical indexes of reinforced bars on the cross section are affected by the area of longitudinal reinforced bars. Through introduction of the conception of equivalent area of reinforced bars, the design of reinforced bars in bending member can be met, and the optimal cost of construction can be satisfied too. Then the member design can be optimized [2].

Take Eq□23□□□24□into Eq□26□:

$$A = \frac{1-\lambda}{f_{yd}}\left(f_{cmd}bx + A_s'f_{yd}'\right) + A_s' + k\frac{\lambda}{f_{py}}\left(f_{cmd}bx + A_s'f_{yd}'\right)$$

Arrange and take Eq□17□into the above formula:

$$A\text{£½}\left(\frac{1\text{£-}\lambda}{f_{yd}} + k\frac{\lambda}{f_{py}}\right)f_{cmd}bx$$

$$+\left(1 + \frac{1-\lambda}{f_{yd}}f_{yd}' + k\frac{\lambda}{f_{py}}f_{yd}'\right)A_s'$$

$$= \left(\frac{1-\lambda}{f_{yd}} + k\frac{\lambda}{f_{py}}\right)f_{cmd}bx$$

$$+\left(1 + \frac{1-\lambda}{f_{yd}}f_{yd}' + k\frac{\lambda}{f_{py}}f_{yd}'\right)\frac{M - f_{cmd}bx(h_0 - \frac{x}{2})}{f_{yd}'(h_0 - a')}$$

For practical design, the unit price ratio of the prestressed bars to non-prestressed bars (k), the design strength of reinforced bars and twist steel and the degree of prestress are all constant. So A is the function of x [2].

$$\frac{dA}{dx} = f_{cmd}b\left[\left(\frac{1-\lambda}{f_{yd}} + k\frac{\lambda}{f_{py}}\right)\right.$$

$$\left.\text{£«}\left(1\text{£«}\frac{1\text{£-}\lambda}{f_{yd}}f'_{yd} + k\frac{\lambda}{f_{py}}f'_{yd}\right)\frac{(x - h_0)}{f'_{yd}(h_0 - a')}\right]$$

Let: $\frac{dA}{dx} = 0$

The height of equivalent rectangle of bending beam can be obtained when A is minimal value:

$$x = h_0 - \frac{f_{yd}'(h_0 - a')\left(\frac{1-\lambda}{f_{yd}} + k\frac{\lambda}{f_{py}}\right)}{1 + \frac{1-\lambda}{f_{yd}}f_{yd}' + k\frac{\lambda}{f_{py}}f_{yd}'}$$ □ 27 □

$$= \left\{1 - \left(1 - \frac{a'}{h_0}\right)\frac{f_{yd}'\left[(1-\lambda)f_{py} + k\lambda f_{yd}\right]}{f_{py}f_{yd} + k\lambda f_{yd}'f_{yd} + (1-\lambda)f_{py}f_{yd}'}\right\}h_0$$

$$\xi = \frac{x}{h_0}$$

$$= 1 - \left(1 - \frac{a'}{h_0}\right)\frac{f_{yd}'\left[(1-\lambda)f_{py} + k\lambda f_{yd}\right]}{f_{py}f_{yd} + k\lambda f_{yd}'f_{yd} + (1-\lambda)f_{py}f_{yd}'}$$

□28□

When □□□ grade reinforced bars is adopted, and the grade of reinforced bars in compression zone and pulling zone is the same ($f_{yd}' = f_{yd}$), equation (28) can be simplified:

$$\xi = 1 - (1 - \frac{a'}{h_0})\frac{(1-\lambda)f_{py} + k\lambda f_{yd}}{(2-\lambda)f_{py} + k\lambda f_{yd}}$$

□29□

When $\lambda = 0$, that is, the partial bonded prestressed concrete beam degenerates into common reinforced concrete member, equation (29) can be degenerated:

$$\xi = \frac{1}{2} + \frac{a'}{2h_0}$$ □

30□

Equation (30) is the same as formula common reinforced concrete beam. So the common reinforced concrete member is a special example to the partial bonded prestressed concrete beam [2,5].

If we get the height of relatively compression zone from Equation (28) and Equation (29): $\xi < 2a'/h_0$, choose $\xi = 2a'/h_0$; if $\xi > \min(0.35, \xi_b)$, choose $\xi = \min(0.35, \xi_b)$; if $2a'/h_0 \le \xi \le \min(0.35, \xi_b)$, then ξ remains as before. The parameters of A_s' , A_s and A_p can be obtained when taking $x = \xi h_0$ into Equation (23) □Equation (24).

4 The calculation example of beam with single reinforcement

The maximum moment that added to the beam is $M = 6000000Nm$ of a certain continued beam of civil structure, in which partial bonded prestressed concrete was adopted. Its design is as follows:

1). the cross section of beam is a rectangle. The width of rectangle section: b=800mm the height of rectangle section: h=900mm, the concrete is C50. The design strength of concrete is $f_{cm} = 26MPa$, the dynamical improved modulus is $\gamma_{cd} = 1.5$. From these, the value of design strength of bending beam upon dynamic loadings can be obtained:

$f_{cmd} = \gamma_{cd} f_{cm} = 1.5 \times 26 = 39MPa$

2). the non-prestressed bars are grade reinforced bars. Its tensile design strength is $f_y = 310MPa$; its elastic modulus is $E_s = 2.1 \times 10^5 MPa$; its dynamical improved modulus is $\gamma_{sd} = 1.35$. From this point, the value of design strength of bending beam upon dynamic loadings can be obtained: $f_{yd} = \gamma_{sd} f_y = 1.35 \times 310 = 418.5MPa$; the prestressed bars is the twist steel in which its strength is $1860MPa$, its pulling design strength is $f_{py} = 1260MPa$, its elastic modulus is $E_p = 1.8 \times 10^5 MPa$, take $E_p = 1.8 \times 10^5 MPa$ and $h_p = 0.790m$.

3). Initial effective stress of prestressed bars in design: $\sigma_{p0} = 0.6 \times 1860 = 1116MPa$, the degree of prestress value $\lambda = 0.6$, then the effective section height is:

$h_0 = \lambda\ h_p + (1 - \lambda\ )h_s$

£½0.6 $\times 0.790 + (1 - 0.6) \times 0.865 = 0.820m$

4). Try to get the ξ

$$\alpha = \frac{M}{f_{cmd} b h_0^2} = \frac{6000000}{39 \times 10^6 \times 0.800 \times 0.820^2} = 0.286$$

$$\xi_{sb} = \frac{h_s}{h_0}\left(\frac{0.8}{1.0 + \frac{f_{yd}}{0.0033E_s}}\right) = \frac{0.865}{0.820}\left(\frac{0.8}{1.0 + \frac{418.5 \times 10^6}{0.0033 \times 2.1 \times 10^{11}}}\right)$$

$= 0.526$

$$\xi_p = \frac{h_p}{h_0}\left(\frac{0.8}{1.6 + \frac{f_{py} - \sigma_{pe}}{0.0033E_p}}\right)$$

$$\xi = \frac{0.790}{0.820}\left(\frac{0.8}{1.6 + \frac{1260\times10^6 - 1116\times10^6}{0.0033\times1.8\times10^{11}}}\right) = 0.418$$

$\xi = 1 - \sqrt{1 - 2\alpha} = 1 - \sqrt{1 - 2 \times 0.286} = 0.346$

If ξ ˝ $\min(0.35, \xi_b) = 0.35$, the bending beam can be designed according to the design of beam with single reinforcement;

5). The calculation of reinforced bars

$$\nu = 1 - \frac{\xi}{2} = 1 - \frac{0.346}{2} = 0.827$$

$$A_g = \frac{M}{f_{py} h_0 \nu} = \frac{600000}{1260 \times 10^6 \times 0.820 \times 0.827} = 0.007022m^2$$

$$A_p = \lambda \cdot A_g = 0.6 \times 7022 = 4213mm^2$$

Actual twist steel strand: 30 threadφj15, namely 5 bind 6φj15 twist steel strings.

Practically $A_p = 4200mm^2$

Actual twisted bars: 11Φ32, $A_s = 8847mm^2$

5 Conclusion

In this paper, based on the equilibrium of prestressed concrete beam in flat bend and introduced the conception of degree of prestress and nominal section area of prestressed bar, the calculation formula of cross section strength was deduced.for partial bonded prestressed concrete beam on dynamic loading Here is only for rectangle section. But the design calculation of the cross section types of I and T is applicable too. In simulation anti-exploding test, the prestressed concrete beam which is designed by this method was validated: it possessed well anti-exploding performance [2] [4]. This method is simple and convenient for design.

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